DOCUMENT RESUME

ED 342 657 SE 052 578

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|---|---|
| AUTHOR | Pagni, David, Ed. |
| TITLE | CAMP-LA: Book 4, Grades 7-8. Calculators and Mathematics Project. |
| SPONS AGENCY | California State Univ., Fullerton.; Los Angeles Unified School District, Calif.; National Science Foundation, Washington, D.C. |
| REPORT NO | ISBN-1-879853-07-8 |
| PUB DATE | 91 |
| NOTE | 363p.; For related documents, see SE 052 574-577. |
| AVAILABLE FROM | Cal State Fullerton Press, 2875 Orange Olive Road, Bldg. #2, Orange, CA 92665 ($20.95). |
| PUB TYPE | Guides - Classroom Use - Teaching Guides (For Teacher) (052) |
| EDRS PRICE | MF01 Plus Postage. PC Not Available from EDRS. |
| DESCRIPTORS | Algebra; Arithmetic; *Calculators; Classroom Techniques; *Curriculum Development; Curriculum Enrichment; Data Analysis; Decimal Fractions; Discovery Learning; Division; Educational Technology; Enrichment Activities; Evaluation Methods; Fractions; Functions (Mathematics); Geometry; Grade 7; Grade 8; Junior High Schools; Lesson Plans; Mathematical Enrichment; Mathematical Logic; *Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; Measurement; Multiplication; Number Concepts; Percentage; Probability; Problem Solving; Ratios (Mathematics); *Secondary School Mathematics; Small Group Instruction; Statistics; *Teaching Methods; Units of Study |
| IDENTIFIERS | Calculators and Mathematics Project CA; *Mathematics Framework for Calif Public Schools; NCTM Curriculum and Evaluation Standards; Patterns (Mathematics) |

ABSTRACT

The National Science Foundation funded Calculators and Mathematics Project, Los Angeles (CAMP-LA) developed curriculum materials focused solely on the use of calculators. The project was developed in three stages. The first stage studied the mathematics curriculums from different states and identified topics that are not included but should be if every student had a calculator, topics treated in too much detail, and topics no longer appropriate. Based on this information, CAMP-LA compiled a prototype curriculum organized by grade level to be consistent with the "California Mathematic Framework" strands. The second stage developed lessons to cover the topics through the curriculum. The third stage field tested these lessons in various parts of the country. This book is composed of lessons for grades 7-8 in the series. The introduction gives an overview of CAMP-LA, information on how to use the lesson plans, a discussion of assessment approaches, and a scope and sequence for the book. The remainder of the book is composed of 44 lessons in four chapters: Patterns and Functions, Logic/Statistics and Probability, Measurement/Geometry, and Number/Algebra. Each lesson is broken down into three sections. The three sections are labeled: "Grade", including grade level, strand, skill required, and purpose; "Management", including class organization, time frame, materials needed, vocabulary, and prerequisite skills; and "Lesson" including suggestions for directed instruction, guided practice, independent practice, evaluation, and home activity. (MDH)

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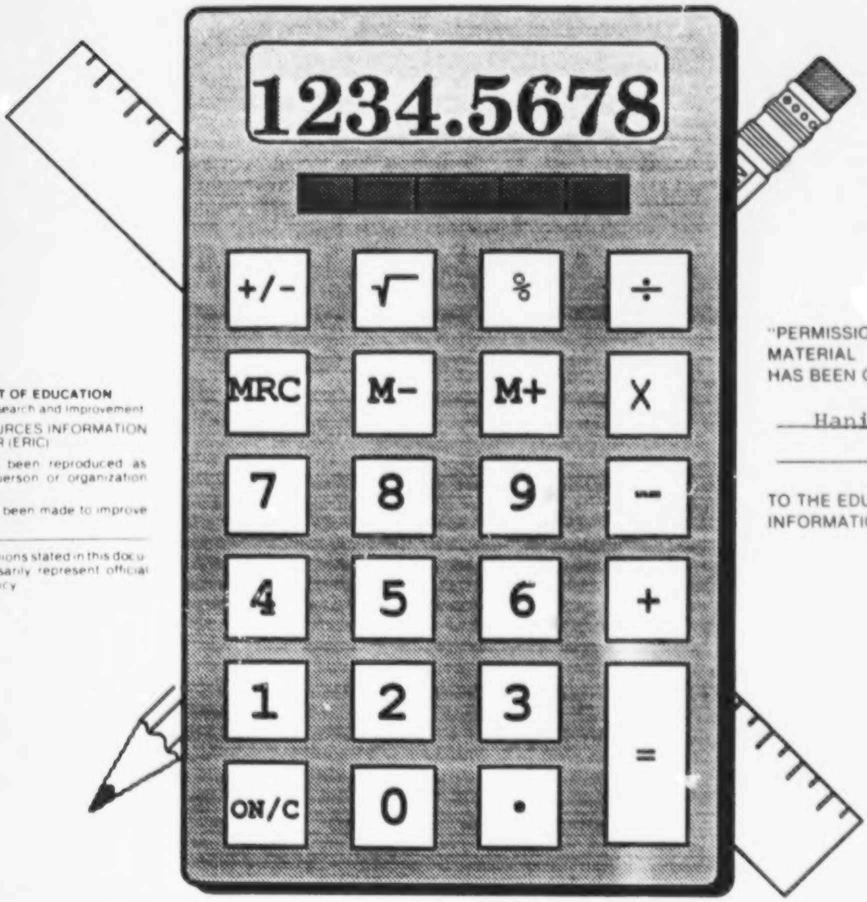
# CAMP - LA



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## BOOK 4
## GRADES 7 - 8

**Calculators and Mathematics Project,
Los Angeles (CAMP - LA)**

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Cal State Fullerton Press
First Edition

# CAMP-LA

## BOOK 4

## GRADES 7 - 8

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**Cal State Fullerton Press**

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The following mathematics lessons were produced by the Calculators and Mathematics Project, Los Angeles (CAMP-LA). The project was supported by California State University, Fullerton, Los Angeles Unified School District and the National Science Foundation (Grant #MDR - 8651616). However, the opinions, findings, conclusions or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the National Science Foundation. The lessons were developed around mathematics topics that could be taught or enhanced with the use of a calculator. In some cases the calculator is used to explore or learn a mathematical concept; in other cases, it is used as a computing tool. All lessons were field-tested in the Los Angeles Unified School District in a wide variety of school settings. Sample lessons have been used in workshops for teachers and other mathematics educators across the United States. The CAMP-LA lessons have always been well-received. The directors and writers of CAMP-LA believe that you and your students will find these lessons to be fun and challenging!

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**Printed in the United States of America.**
**ISBN: 1-879853-07-8**

# ADDITIONAL ACKNOWLEDGMENTS

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## Books by David Pagni:

CAMP- LA Book 1

CAMP- LA Book 2

CAMP- LA Book 3

CAMP- LA Book 4

Math Lessons for Grades K - 3

Math Lessons for Grades 3 - 5

Math Investigations for the Months

# PROJECT BACKGROUND

The Calculators and Mathematics Project, Los Angeles (CAMP-LA) was one of six projects[1] in the country funded by the National Science Foundation, Division of Materials Development and Research Instructional Materials Development Program, under a special program solicitation entitled "Materials for Elementary School Mathematics Instruction" in September, 1986. The special solicitation requested proposals that focused on the use of technology in elementary school mathematics.

Of these six projects, only CAMP-LA focussed its efforts soley on the use of calculators. The CAMP-LA philosophy is that every child should have access to a calculator at all times when investigating, studying, or learning mathematics.

The lesson development process spanned three stages. First, the project teams of writers and the two co-directors studied the mathematics curriculum guides from different states. They looked for:

- Topics not treated but which should be (assuming every child has a calculator)
- Topics treated in too much detail
- Topics no longer appropriate

Based on the results of this research, the CAMP-LA staff compiled a prototype curriculum organized around the strands of the *California Mathematic Framework*: Number, Measurement, Geometry, Patterns and Functions, Statistics and Probability, Logic, and Algebra. The CAMP-LA staff next isolated those topics that lent themselves to being taught with the use of a calculator. These topics were organized by grade level and became the CAMP-LA Calculator Continuum.

The second stage of the lesson development process was the writing of lessons that captured the essence of the Calculator Continuum. At this time, we decided to introduce a new strand, the Calculator Awareness strand for lessons designed to introduce students to the mechanics of operating a calculator. Of course, these lessons for introducing the calculator features are written in a mathematics context.

Drafts of lessons were written during the summer, 1987. During the following fall these skeletal lessons were evaluated to see which ones needed to be fleshed out, which needed to be deleted, and where in the Calculator Continuum additional lessons were needed.

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The third stage of the CAMP-LA lesson development process was the field testing of the lessons. Because of a nationwide interest in the project, a few lessons were field tested in schools in various parts of the country. However, all lessons were field tested in the Los Angeles Unified School District in a variety of school settings. The CAMP-LA field test teachers turned in written reports including samples of students' work for each lesson. The field test teachers also met with the project writers to discuss the strengths and weaknesses of the various lessons. The field testing went hand - in - hand with new lesson development throughout 1988, 1989, and 1990. During the summer and fall of 1990 the writing teams completed their work and the final editing was completed by David Pagni, Principal Investigator and Co-director of CAMP-LA.

**CAMP-LA Books**

| Book | Grade Level | Cost |
|---|---|---|
| Book 1 | K – 2 | $14.95 |
| Book 2 | 3 – 4 | $14.95 |
| Book 3 | 5 – 6 | $14.95 |
| Book 4 | 7 – 8 | $20.95 |

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[1] The six NSF funded projects were:

1) "A Revision of the Geometry and Measurement Strands, K-6" University of Georgia

2) "Calculators and Mathematics Project, Los Angeles"
California State University, Fullerton

3) "Development of a Logo–Based Geometry Curriculum"
Kent State University

4) "K–6 Supplementary Mathematics Materials for a Technological Society"
New York University

5) "Reckoning with Mathematics: Tools and Challenges for the Information Age"
Education Development Center

6) "Used Numbers: Collecting and Analyzing Real Data"
Technical Education Research Centers

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CAMP-LA Lesson Format .......... xiv
CAMP-LA Assessment .......... xvi
Scope and Sequence .......... xvii

## Chapter 1: *Patterns and Functions*

The study of patterns enables students to see order and predictability in many situations. Students have a powerful tool for solving problems when they understand patterns and functional relationships.

| Lesson | Title | Objectives | Page |
|---|---|---|---|
| 1 | Cut and Stack | A hands-on lesson using powers of 2. | 1 |
| 2 | An Ancient Oddity | Discover patters on an ancient stone tablet. | |
| 3 | Palatable Patterns | Explore the relationship between specific sums and divisibility by 11. | 14 |
| 4 | Digit Patterns | Explore some curious products. | 19 |
| 5 | Follow the Flow Chart 1 | Use flow charts to identify, extend and create number patterns. These flow charts each use one operation. | 23 |
| 6 | Follow the Flow Chart 2 | Use flow charts to identify, extend and create number patterns. These flow charts each use one operation. | 34 |
| | Chapter Assessment | | 40 |

## Chapter 2: *Logic/Statistics and Probability*

Logical reasoning develops as students identify attributes, recognize patterns, and use relationships to analyze mathematical situations. Students reason, make conjectures, and draw conclusions as they move from working with concrete materials to abstract thinking. Knowledge of statistics allows students to summarize what they know of the world and to make inferences about what they do not know. The study of probability enables students to indicate how certain they are about a prediction.

| Lesson | Title | Objectives | Page |
|---|---|---|---|
| 7 | Going to the Movies | Explore combinations of different priced movie tickets to organize and interpret data. | 43 |
| 8 | Number Claim | Use 4 digits and various operations to claim numbers on a chart. | 48 |
| 9 | M & M & M | Determine the mean, median and mode from a set of data. | 53 |
| 10 | Picture This | Investigate possible arrangements for creating new pictures. | 60 |
| 11 | Its All in the Game | Use Monopoly game board and the percent key to organize information for a circle graph. | 68 |
| 12 | Lets Party | Investigate combinations of friends you could invite to a party. | 81 |
| 13 | Carnival Booth | Investigate probabilities using gameboards. | 94 |
| 14 | Number Generation | Use the square root key to generate sets of data for exploration. | 104 |
| 15 | This Lesson Rules | A very challenging logic investigation using functions as a vehicle for exploration. | 113 |
|  | Chapter Assessment |  | 123 |

**Chapter 3:** ***Measurement/Geometry***

When we measure, we attach a number to a quantity using a unit which is chosen according to the properties of the quantity to be measured. Estimation plays an important role in the manipulation of non-standard and standard systems as well as conversion within and between systems of measurement. The study of geometry enables students to identify, describe, compare, and classify geometric figures. Students develop spatial sense and problem solving skills using geometric models.

| Lesson | Title | Objectives | Page |
|---|---|---|---|
| 16 | How Do You Spend Your Time | Gather and organize data with decimals, fractions, percents and units of time, and display on circle graphs. | 127 |
| 17 | I Have, Who Has | Find perimeters of various polygons. Builds reading and listening skills. | 134 |
| 18 | Its All In How You Look at It | Help students recognize that any of the three sides of a triangle can be thought of as the base when calculating area. | 143 |
| 19 | Lets Get It Right | Discover the relationship between the size of the sides of a triangle and the kind of triangle (right, obtuse or acute). | 153 |
| 20 | Heron's Formula | Investigate an alternate method for computing the area of a triangle using the square root key. | 160 |
| 21 | Long and Short of It | Explore the relationship between the perimeter and area of rectangles. Builds awareness of calculator limitations. | 170 |
| 22 | Another Fence on the Wall | Investigate the area of rectangles under a unique condition. | 177 |
| 23 | Wheels on the Bike Go Round and Round | Use circumference to make decisions about wheels. | 185 |

| Lesson | Title | Objectives | Page |
|---|---|---|---|
| 24 | Which Holds More? | Estimate and compute the volume of cylinders. | 189 |
| 25 | Chris' Up and Down Day | Converts metric and customary units using temperature formulas. | 194 |
| 26 | Scaling for Special Effects | Explores enlarging objects. First in a series of four lessons on scale factors. | 200 |
| 27 | More Scaling for Special Effects | Determine how area and volume are affected by changing scale factors. | 204 |
| 28 | Monumental Scaling | Find a scale factor to satisfy specific requirements. | 208 |
| 29 | Scaling Unkown Heights | Use scale factors to find missing information on heights and shadows. | 212 |
|  | Chapter Assessment |  | 216 |

## Chapter 4: *Number/Algebra*

Numbers are used to record and interpret information, solve problems, and to make decisions. Students develop number sense by being asked to make a choice among computational methods: estimation, mental arithmetic, paper and pencil, and the calculator. Algebra is studied informally to develop an understanding of abstract relationships. Students learn to express numerical relationships through the use of the variable.

| Lesson | Title | Objectives | Page |
|---|---|---|---|
| 30 | EZ Million Trivia Pursuit | Develop number sense and explore the meaning of 1,000,000 in problem situations. | 225 |
| 31 | Butcher Math | Apply decimal multiplication and division to pricing labels. | 235 |
| 32 | Get the Best Buy | Explore unit pricing. | 240 |
| 33 | Watch Your Money Grow | Problem solving exploration of powers using money as motivation. | 243 |

| Lesson | Title | Objectives | Page |
|---|---|---|---|
| 34 | Multiplication the Easy Exponential Way | Explore the use of powers as a multiplication tool. | 252 |
| 35 | Your Gain Your Loss | Explore how exercise and foods relate to weight loss and gain. | 263 |
| 36 | Piece of Cake | Use fractions and decimals to explore examples of infinite series. | 270 |
| 37 | Pardon My Dear Aunt Sally | Use the order of operations to compute. | 274 |
| 38 | Cautious Calories | Write and solve equations using the number of calories in meals. | 280 |
| 39 | What are my Numbers? | Solve a variety of problems using equations, logic, or guess and check strategies. | 284 |
| 40 | Multiple Madness | Find Least Common Multiples. | 288 |
| 41 | Famous Dates | Explore prime and composite numbers. Uses the square root key. | 293 |
| 42 | Interesting | Apply percent to show the growth of invested money. | 297 |
| 43 | Rock N' Math | Form a rock music group as a class project. | 306 |
| 44 | Dashing Down the Road | Compute the cost of painting stripes on the middle of a highway. | 319 |
|  | Chapter Assesment |  | 329 |

# CAMP-LA OVERVIEW

The **Calculators and Mathematics Project, Los Angeles** (**CAMP-LA**) provides materials for grades K-8 that integrate the calculator into the elementary school mathematics curriculum in a meaningful way.

**CAMP-LA** lessons support the philosophy expressed by the :

**Mathematical Sciences Education Board**
- **Everybody Counts**

**National Council of Teachers of Mathematics**
- **Agenda for Action,**
- **NCTM Standards**

**California State Department of Education**
- **Mathematics Framework**

**CAMP-LA** materials were written by classroom teachers and resource specialists. These materials are divided into four levels.

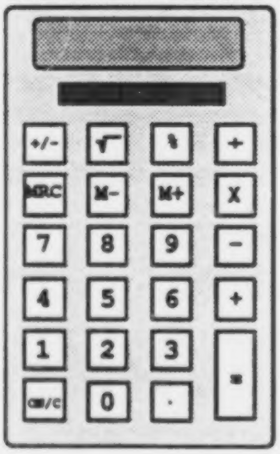
Grades K-2
Grades 3-4
Grades 5-6
Grades 7-8

The seventh and eighth grade lessons are printed in four chapters.

Patterns and Functions
Logic/Statistics and Probability
Measurement/Geometry
Number/Algebra

Meaningful assessment of student understanding is provided for all levels.

**CAMP-LA** lessons use calculators with the following features:



- Constant function for basic operations
- Clear/ Clear entry key(s)
- Memory Keys
- Square Root Key
- % Key (Recommended by not essential)

# FEATURES OF CAMP-LA LESSONS

**Calculators and Mathematics Project, Los Angeles Lessons:**

- Provide a challenging currriculum based on the assumption that every child has access to a calculator.
- Help students become confident and comfortable using the calculator as an effective tool for exploring mathematical concepts.
- Develop students' ability to choose how and when to use a calculator.
- Assist students to make the connection between the concrete and the abstract.
- Emphasize conceptual development, reasoning, numerical relationships, and application in real-life experiences.
- Help students use the language, symbols, and processes of mathematics to gain confidence with numbers.
- Encourage the discovery of patterns in our number system.
- Remove computational constraints so that students can focus on the processes of solving problems and develop problem-solving skills and strategies.
- Provide students opportunities to reason logically and develop an intellectual curiosity toward mathematics.
- Stimulate interest in mathematics.
- Involve students in cooperative learning groups to solve problems.

# CAMP-LA LESSON FORMAT

**MANAGEMENT**

A **Table of Contents** guides teachers in the selection of lessons. When lessons are integrated throughout the school year, students become familiar with the calculator and feel confident using it as a tool to learn mathematics. **CAMP-LA** lessons recognize that learning is often enhanced when the calculator is used with other learning materials. Lessons are models of how to incorporate calculators into the mathematics curriculum.

**CAMP-LA LESSON PLAN FORMAT**

All lessons in the **Calculators and Mathematics Project, Los Angeles** follow the same format.

**Teacher Information**

| | |
|---|---|
| **CAMP-LA** | **LESSON TITLE** |
| **GRADE LEVEL:** | Indicates appropriate grade levels. |
| **STRAND:** | Identifies content strand: Number, Measurement, Geometry, Patterns and Functions, Statistics and Probability, Logic, or Algebra. |
| **SKILL(S):** | States the skill developed in the lesson. |
| **MANAGEMENT** | |
| **CLASS ORGANIZATION:** | Recommends whole class, small groups, or pairs. |
| **TIME FRAME:** | Approximates the time needed to present the lesson. |
| **MATERIALS:** | Lists the materials necessary to implement the lesson. |
| **VOCABULARY:** | Identifies mathematical terms and other vocabulary used in the lesson. |
| **PREREQUISITE SKILLS:** | States entry skills needed for successful completion of the lesson. |

# Lesson Development

**LESSON**

**DIRECTED INSTRUCTION:**

Lessons are developed sequentially.

Suggestions for delivery of instruction include the use of:
- **Problem Solving**
- **Concrete Materials**
- **Cooperative Learning**
- **Mathematical Language**
- **Situational Lessons**

Questions are provided to:
- Stimulate critical thinking
- Focus on concepts to be developed
- Encourage student involvement

| | |
|---|---|
| **GUIDED PRACTICE:** | Students are provided practice under the teacher's guidance so that they can apply their mathematical knowledge independently. |
| **INDEPENDENT PRACTICE:** | Student activity sheets and teacher answer sheets are included. Student Activity Sheets are designed to encourage learning and understanding. |
| **EVALUATION:** | Evaluation methods are suggested to:<br>• Assess students' understanding of mathematical concepts.<br>• Bring mathematical closure to the lesson. |
| **HOME ACTIVITY/ EXTENSION:** | Home Activity Sheets and suggestions for Extension Activities provide additional opportunities for application of mathematical concepts in a variety of situations. |

# CAMP-LA ASSESSMENT

The purpose of assessment is to enhance learning and improve teaching. For the student, assessment indicates a measure of mathematical knowledge and power. For the teacher, it indicates how the instructional program should be modified. Teacher observation of students' actions and interactions gives information about mathematical knowledge, understanding of concepts, and ability to apply reasoning and analysis to solve problems.

Suggested **CAMP-LA** assessment items appear at the end of each chapter. The assessment items:

- have been written as models of assessment which support the major concepts presented in the **CAMP-LA** lessons;
- provide both open-ended and traditional assessment tasks;
- are meant to be done by pairs and/or small groups;
- indicate anticipated student responses for open-ended questions.

# SCOPE AND SEQUENCE

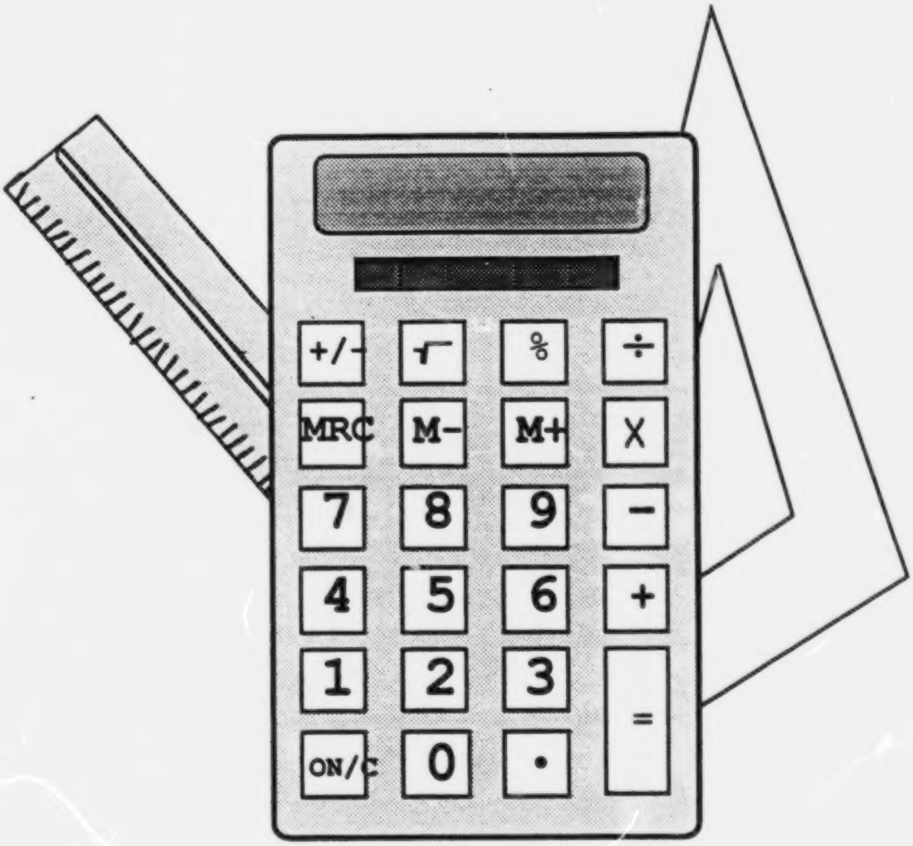
The Calculators and Mathematics Project, Los Angeles seventh and eighth grade lessons are listed in a **suggested order of presentation.** The columns represent three books which cover seven strands of mathematics. Lessons in each column are listed in an order that takes into account both the lesson's difficulty and the prerequisites that are required to successfully complete them.

| Patterns and Functions | Logic/ Statistics and Probability | Measurement/ Geometry | Number/ Algebra |
|---|---|---|---|
| Lesson 1<br>Cut and Stack | Lesson 7<br>Going to the Movies | Lesson 16<br>How Do You Spend Your Time? | Lesson 30<br>EZ Millions Trivia Pursuit |
| Lesson 2<br>An Ancient Oddity | Lesson 8<br>Number Claim | Lesson 17<br>I Have, Who Has | Lesson 31<br>Butcher Math |
| Lesson 3<br>Palatable Patterns | Lesson 9<br>M&M&M | Lesson 18<br>Its All in How You Look at It | Lesson 32<br>Get the Best Buy |
| Lesson 4<br>Digit Patterns | Lesson 10<br>Picture This | Lesson 19<br>Lets Get It Right | Lesson 33<br>Watch Your Money Grow |
| Lesson 5<br>Follow the Flow Chart 1 | Lesson 11<br>Its All in the Game | Lesson 20<br>Heron's Formula | Lesson 34<br>Multiplication the Easy Exponential Way |
| Lesson 6<br>Follow the Flow Chart 2 | Lesson 12<br>Lets Party | Lesson 21<br>Long and Short of It | Lesson 35<br>Your Gain, Your Loss |
| | Lesson 13<br>Carnival Booth | Lesson 22<br>Another Fence on the Wall | Lesson 36<br>Piece of Cake |
| | Lesson 14<br>Number Generation | Lesson 23<br>Wheels on the Bike Go Round and Round | Lesson 37<br>Pardon My Dear Aunt Sally |
| | Lesson 15<br>This Lesson Rules! | Lesson 24<br>Which Holds More? | Lesson 38<br>Cautious Calories |
| | | Lesson 25<br>Chris Up and Down | Lesson 39<br>What are My Numbers? |
| | | Lesson 26<br>Scaling for Special Effects | Lesson 40<br>Multiple Madness |
| | | Lesson 27<br>More Scaling for Special Effects | Lesson 41<br>Famous Dates |

| Patterns and Functions | Logic/ Statistics and Probability | Measurement/ Geometry | Number/ Algebra |
|---|---|---|---|
| | | Lesson 28<br>Monumental Scaling | Lesson 42<br>Interesting |
| | | Lesson 29<br>Scaling Unknown Heights | Lesson 43<br>Rock N' Math |
| | | | Lesson 44<br>Dashing Down the Road |

# CAMP-LA

Calculators and Mathematics Project, Los Angeles



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## Grades 7 - 8

# CHAPTER 1
## PATTERNS AND FUNCTIONS

# CUT AND STACK

**GRADE:** 7 - 8

**STRAND:** Patterns and Functions

**SKILL:** Identify and find powers of numbers.

**MANAGEMENT**
**CLASS ORGANIZATION:** Whole group, pairs

**TIME FRAME:** One math period

**MATERIALS:** Calculator, centimeter ruler, single sheets of paper, scissors, ream of paper for measuring, meter stick

**VOCABULARY:** Centimeter(cm), meter(m)

**PREREQUISITE SKILL:** Measure in centimeters, convert meters to centimeters, calculator constant function

**LESSON**

- **DIRECTED INSTRUCTION:**

Give each student one sheet of paper and Student Activity Sheet 1. Instruct the students using the following dialogue:

1. Cut the sheet of paper in half, and place one half on top of the other half in order to form a stack of paper. On the Activity Sheet, record 1 in Column A (Number of Times Paper is Cut and Stacked) and record 2 in Column B (Number of Pieces after Cut).
2. Cut the stack of paper in half again, place the cut paper in a single stack and count the pieces of paper in the stack. Record 2 (cuts) in Column A and 4 (pieces) in Column B.
3. Continue this process two more times (cut and stack). Each time, record the information in Columns A and B.
4. Estimate how many more times you would have to cut and stack in order to have 512 pieces of paper. What do you observe when you compare Column A and Column B? Students write responses on back of Student Activity Sheet or respond orally.
5. Follow the remaining directions without cutting paper.
6. Complete Column A and B.
7. Look at Column C. What is the pattern? Complete the next 7 lines of Column C.

8. Compare Column A with Column D. What is the pattern when you compare the number of times the paper is cut and stacked with the powers of 2?

9. Compare Columns A, B, C, and D. What are the patterns and relationships? Write what you observed about these columns.

10. Complete Column D of Activity Sheet 1. Discuss your results.

- **GUIDED PRACTICE:**

1. Take out a ream of paper and have 2 or 3 different students measure the height of the ream to the nearest millimeter. A ream contains 500 sheets of paper. Help the students figure out the approximate height of 1 sheet of paper (ream height ÷ 500). Students write this answer in the first line of Column E, then complete the rest of the Column. Once students have computed the approximate height of 1 sheet of paper, help students use the multiplication constant on the calculator to compute answers for column E.
   Note: 1 ream is about 5 cm thick.
   1 sheet is about .01 cm thick.

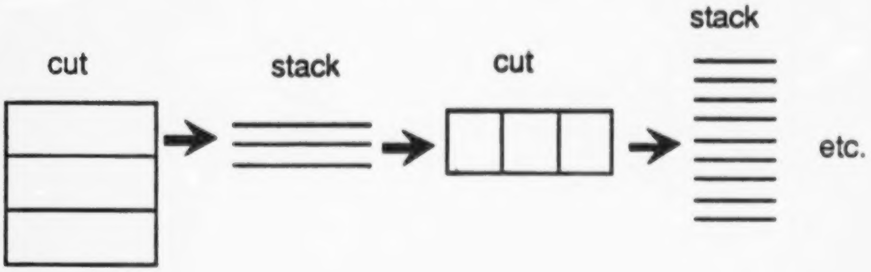
2. Check the results.

- **INDEPENDENT PRACTICE:**

Complete Student Activity Sheet 2. Discuss problem 1 in this activity to check for understanding before continuing on with the page. Part 3d, students research the height of the Empire State Building. Parts 3 e, f, g, students fill in and research the heights of other tall "things."

- **EXTENSION:** Check Student Activity Sheet

Pose the following question: "If we cut a piece of paper into 3 equal parts, stack the pieces, then continue to repeat the process, how high will the stack be in centimeters after 10 cuts?



Name:____________________

## CUT AND STACK

Teacher Answer Sheet
Student Activity Sheet 1

| A | B | C | D | E |
|---|---|---|---|---|
| Number of Times Paper Is Cut and Stacked | Number of Pieces after Cut | Powers of 2 | 2 to the nth Power | Height of Stack in cm |
| 0 | 1 | 1 | $2^{0}$ | .01 |
| 1 | 2 | 2 | $2^{1}$ | .02 |
| 2 | 4 | 2 x 2 | $2^{2}$ | .04 |
| 3 | 8 | 2 x 2 x 2 | $2^{3}$ | .08 |
| 4 | 16 | 2 x 2 x 2 x 2 | $2^{4}$ | .16 |
| 5 | 32 | 2x2x2x2x2 | $2^{5}$ | .32 |
| 6 | 64 | 2x2x2x2x2x2 | $2^{6}$ | .64 |
| 7 | 128 | 2x2x2x2x2x2 x 2 | $2^{7}$ | 1.28 |
| 8 | 256 | 2x2x2x2x2x2 x 2 x 2 | $2^{8}$ | 2.56 |
| 9 | 512 | 2x2x2x2x2x2 x 2 x 2 x 2 | $2^{9}$ | 5.12 |
| 10 | 1024 | ▨ | $2^{10}$ | 10.24 |
| 11 | 2048 | ▨ | $2^{11}$ | 20.48 |
| 12 | 4096 | ▨ | $2^{12}$ | 40.96 |
| 13 | 8192 | ▨ | $2^{13}$ | 81.92 |
| 14 | 16384 | ▨ | $2^{14}$ | 163.84 |
| 15 | 32768 | ▨ | $2^{15}$ | 327.68 |
| 16 | 65536 | ▨ | $2^{16}$ | 655.36 |
| 17 | 131072 | ▨ | $2^{17}$ | 1310.72 |
| 18 | 262144 | ▨ | $2^{18}$ | 2621.44 |

## CUT AND STACK
Teacher Answer Sheet
Student Activity Sheet 2

**USE THE RESULTS OF STUDENT ACTIVITY SHEET 1 TO ANSWER THE FOLLOWING QUESTIONS:**

1. Measure your height in centimeters, calculate how many times you would have to cut and stack the paper to make a stack higher than your height.

   Your height ____________________ cm

2. How many times would you have to cut and stack the paper in order to make a stack higher than the height of your classroom?____________

3. How many times would you have to cut and stack the paper in order to get a stack higher than the height of the following:

   (Hint: Extend the Cut and Stack chart.)

| | | Number of cuts needed |
|---|---|---|
| a. | Tallest Tree (126 m or 12600 cm)) | 24 cuts |
| b. | Great Pyramid (148 m or 14800 cm) | 24 cuts |
| c. | Sears Tower, Chicago (443 m or 44300cm) | 26 cuts |
| d. | Empire State Building (1414 ft or 43100 cm)<br>(1" = 2.54 cm) Students research height of the building. | 26 cuts |

e., f., g. Students research their own tall "things."

Answers will vary.

Name:____________________

## CUT AND STACK
Student Activity Sheet 1

| A | B | C | D | E |
|---|---|---|---|---|
| Number of Times Paper Is Cut and Stacked | Number of Pieces after Cut | Powers of 2 | 2 to the nth Power | Height of Stack in mm |
| 0 | 1 | 1 | $2^0$ | |
| | | 2 | $2^1$ | |
| | | 2 x 2 | $2^2$ | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | ▨ | | |
| | | ▨ | | |
| | | ▨ | | |
| | | ▨ | | |
| | | ▨ | | |
| | | ▨ | | |
| | | ▨ | | |
| | | ▨ | | |
| | | ▨ | | |

Name ______________________________

## CUT AND STACK

### Student Activity Sheet 2

**USE THE RESULTS OF STUDENT ACTIVITY SHEET 1 TO ANSWER THE FOLLOWING QUESTIONS:**

1. Measure your height in centimeters, calculate how many times you would have to cut and stack the paper to make a stack higher than your height.

   Your height ______________________ cm

2. How many times would you have to cut and stack the paper in order to make a stack higher than the height of your classroom? ______________

3. How many times would you have to cut and stack the paper in order to get a stack higher than the height of the following:

   (Hint: Extend the Cut and Stack Chart)

| | | Number of cuts needed |
|---|---|---|
| a. | Tallest Tree (126 m or 12600 cm)) | ______________ |
| b. | Great Pyramid (148 m or 14800 cm) | ______________ |
| c. | Sears Tower, Chicago (443 m or 44300cm) | ______________ |
| d. | Empire State Building, New York | ______________ |
| e. | ____________________________ | ______________ |
| f. | ____________________________ | ______________ |
| g. | ____________________________ | ______________ |

# AN ANCIENT ODDITY

**GRADE:** 7 - 8

**STRAND:** Patterns and Functions

**SKILL:** Discover the pattern relationship between consecutive odd numbers and numbers to the third power. (Cube numbers)

## MANAGEMENT

**CLASS ORGANIZATION:** Individual or pairs

**TIME FRAME:** One math period

**MATERIALS:** Calculator, scissors

**VOCABULARY:** Cubes and squares of numbers, exponential, archaeologist

**PREREQUISITE SKILL:** Powers of numbers

## LESSON

- **DIRECTED INSTRUCTION and GUIDED PRACTICE:**

Hand out Ancient Stone Tablet part 1 (Student Activity Sheet 1) and read the following motivating story to the class.

**Archaeologists found an old stone tablet buried in the ruins of a destroyed city. Over the centuries some of the numbers on the tablet were damaged.**

Your task is to figure out what the missing numbers are.

1. Tell the students to complete the blanks on the tablet by filling in the missing numbers to form a pattern. Assist students as needed by telling them the pattern is related to odd numbers.

- **INDEPENDENT PRACTICE:**

2. Teacher reads: **Years later the Archaeologists found the second part of the tablet.**
   Hand out Ancient Stone Tablet Part 2 (Student Activity Sheet 2).

3. Students use scissors to cut out the Ancient Stone Tablet. Place the second part of the tablet to the right of the first part.

4. Have students complete the numbers and discuss the patterns that were originally written on the tablets.

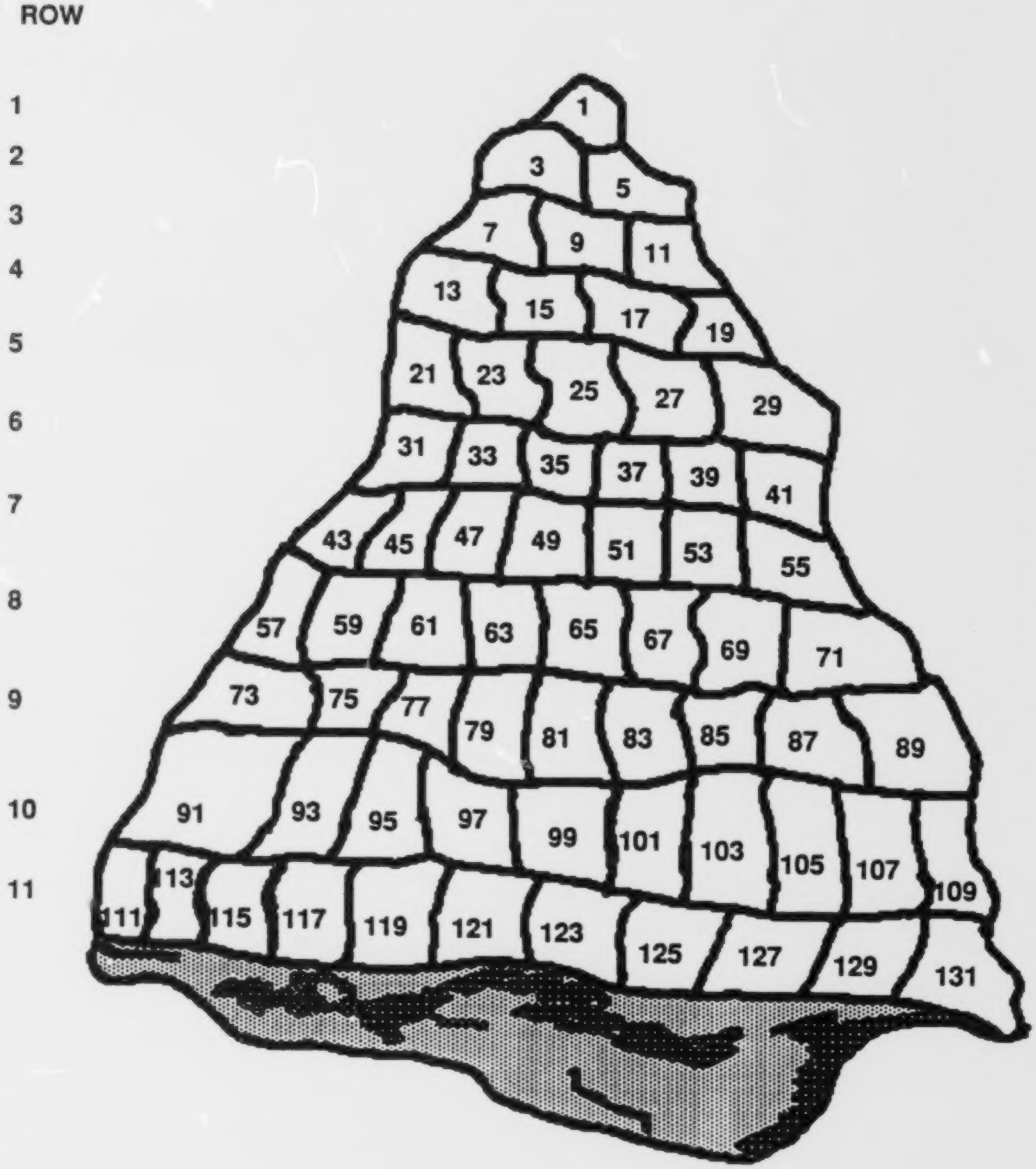
- **HOME ACTIVITY:**

Hand out Home Activity Sheet and have students complete the tablet and columns for homework.

## AN ANCIENT ODDITY

Teacher Answer Sheet
Student Activity Sheet 1

1. Complete the blanks on the tablet by filling in the missing numbers to form a pattern.
2. What do you notice about the numbers on this tablet?


ROW
1
2
3
4
5
6
7
8
9
10
11
1
3 5
7 9 11
13 15 17 19
21 23 25 27 29
31 33 35 37 39 41
43 45 47 49 51 53 55
57 59 61 63 65 67 69 71
73 75 77 79 81 83 85 87 89
91 93 95 97 99 101 103 105 107 109
111 113 115 117 119 121 123 125 127 129 131

## AN ANCIENT ODDITY

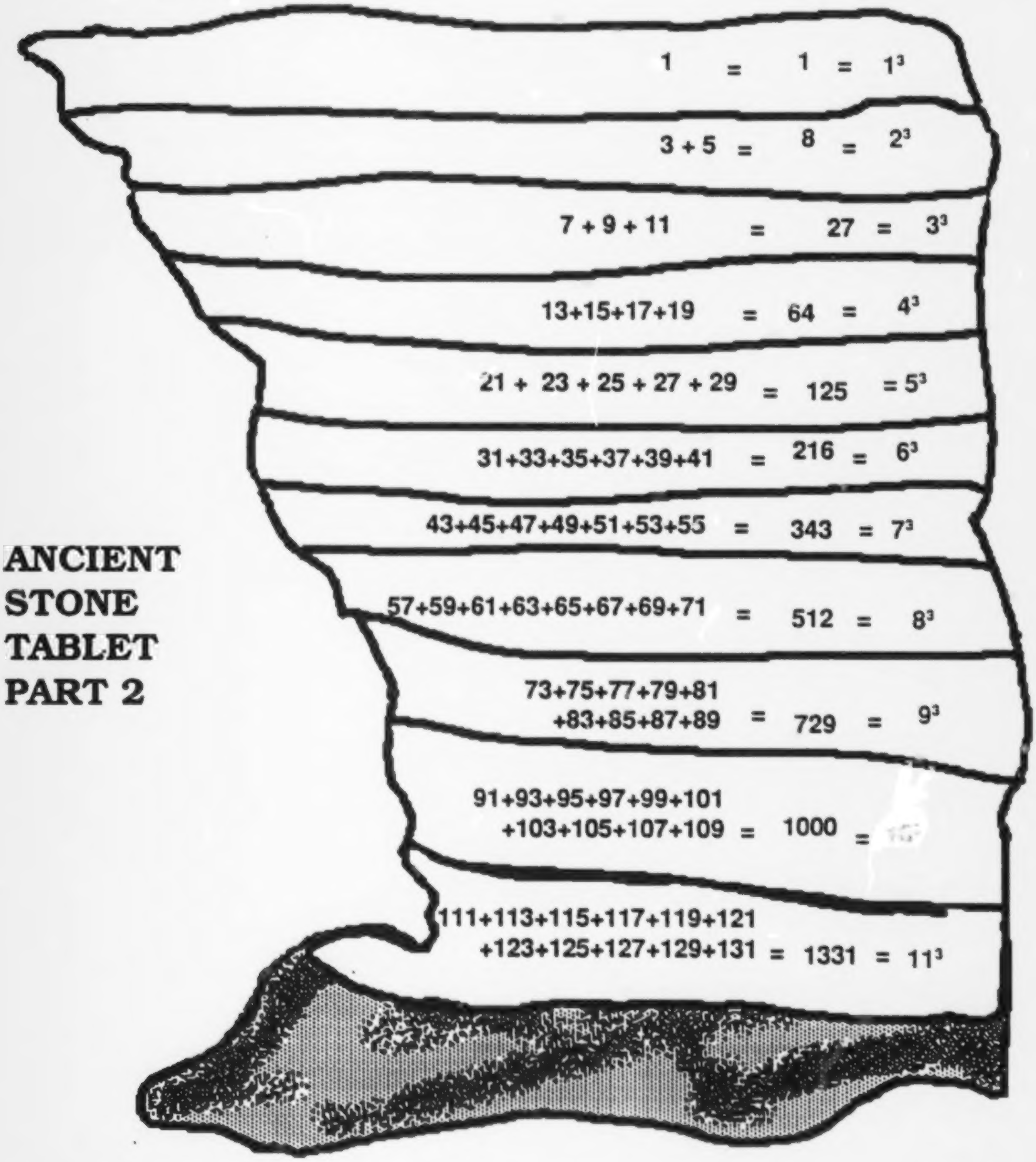
Teacher Answer Sheet- Student Activity Sheet 2

Years later Archaeologists found the second part of the tablet.

1. Cut out or place the two sections of the tablet together so the horizontal lines match.
2. Fill in the missing numbers to discover the pattern on the Ancient Tablet. What did you discover?

Teacher Note: The next to the last column consists of cube numbers. i.e. numbers found by multiplying a number by itself three times.
Example: 64 = 4 x 4 x 4

**ANCIENT STONE TABLET PART 2**



$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = 27 = 3^3$

$13+15+17+19 = 64 = 4^3$

$21 + 23 + 25 + 27 + 29 = 125 = 5^3$

$31+33+35+37+39+41 = 216 = 6^3$

$43+45+47+49+51+53+55 = 343 = 7^3$

$57+59+61+63+65+67+69+71 = 512 = 8^3$

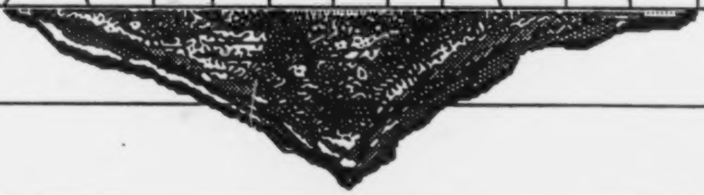
$73+75+77+79+81+83+85+87+89 = 729 = 9^3$

$91+93+95+97+99+101+103+105+107+109 = 1000 = 10^3$

$111+113+115+117+119+121+123+125+127+129+131 = 1331 = 11^3$

**AN ANCIENT ODDITY**

Teacher Answer Sheet- Home Activity Sheet

| Row | ANCIENT STONE TABLET | A | B |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 3 | 4 | $2^2$ |
| 3 | 1 3 5 | 9 | $3^2$ |
| 4 | 1 3 5 7 | 16 | $4^2$ |
| 5 | 1 3 5 7 9 | 25 | $5^2$ |
| 6 | 1 3 5 7 9 11 | 36 | $6^2$ |
| 7 | 1 3 5 7 9 11 13 | 49 | $7^2$ |
| 8 | 1 3 5 7 9 11 13 15 | 64 | $8^2$ |
| 9 | 1 3 5 7 9 11 13 15 17 | 81 | $9^2$ |
| 10 | 1 3 5 7 9 11 13 15 17 19 | 100 | $10^2$ |
| 11 | 1 3 5 7 9 11 13 15 17 19 21 | 121 | $11^2$ |
|  |  |  |  |
|  |  |  |  |

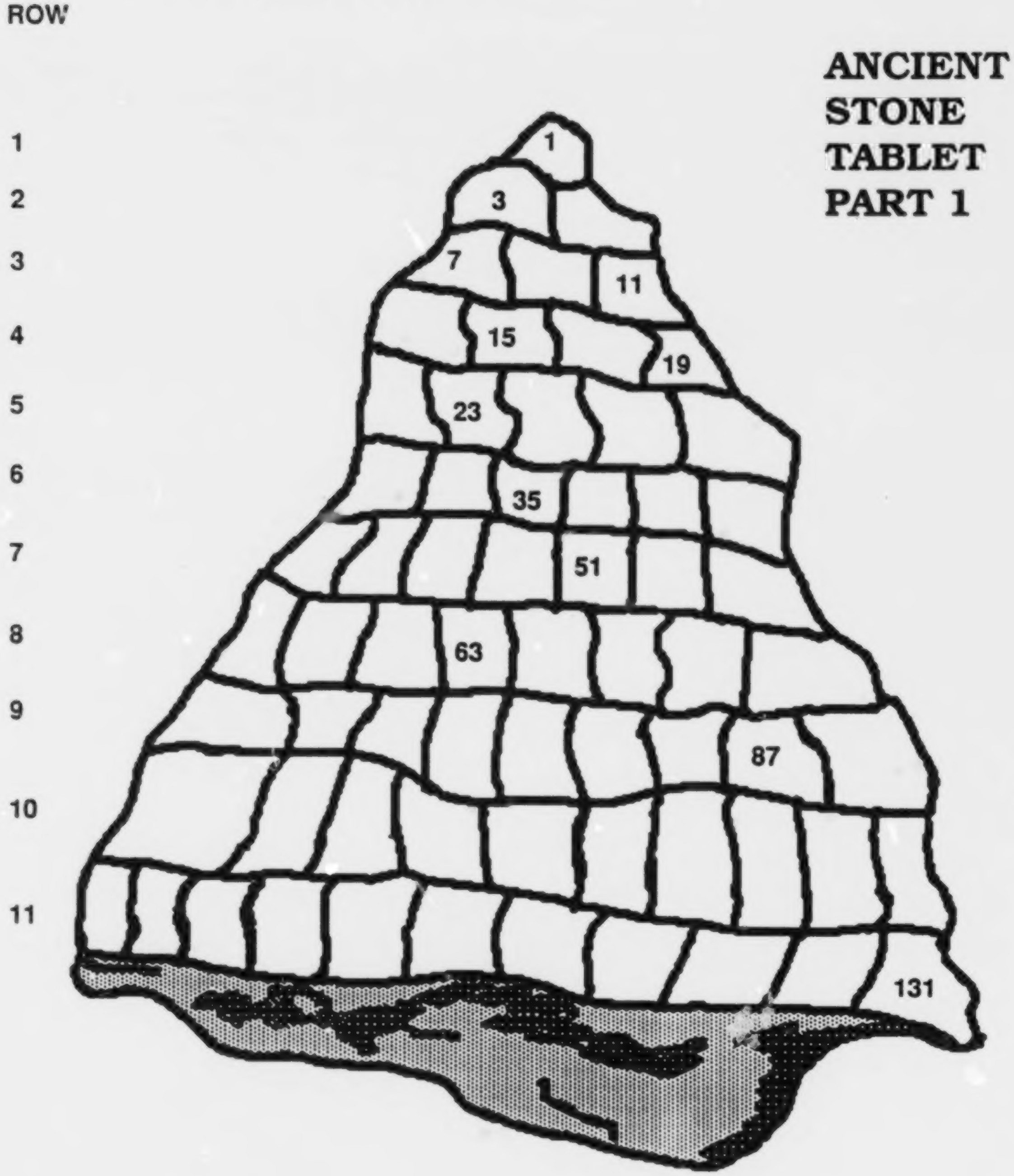
The sum of the numbers in each row of tablet is to be written in column A. The row number raised to the second power is placed in column B.
Example: 1+3+5 =9 which is $3^2$ or 3x3.
In row 25 you will have column A = 625 and column B = $25^2$

Name ____________________________

## AN ANCIENT ODDITY

### Student Activity Sheet 1

1. Complete the blanks on the tablet by filling in the missing numbers to form a pattern.
2. What do you notice about the numbers on this tablet?



**ANCIENT STONE TABLET PART 1**

ROW
1
2
3
4
5
6
7
8
9
10
11
1
3
7
11
15
19
23
35
51
63
87
131

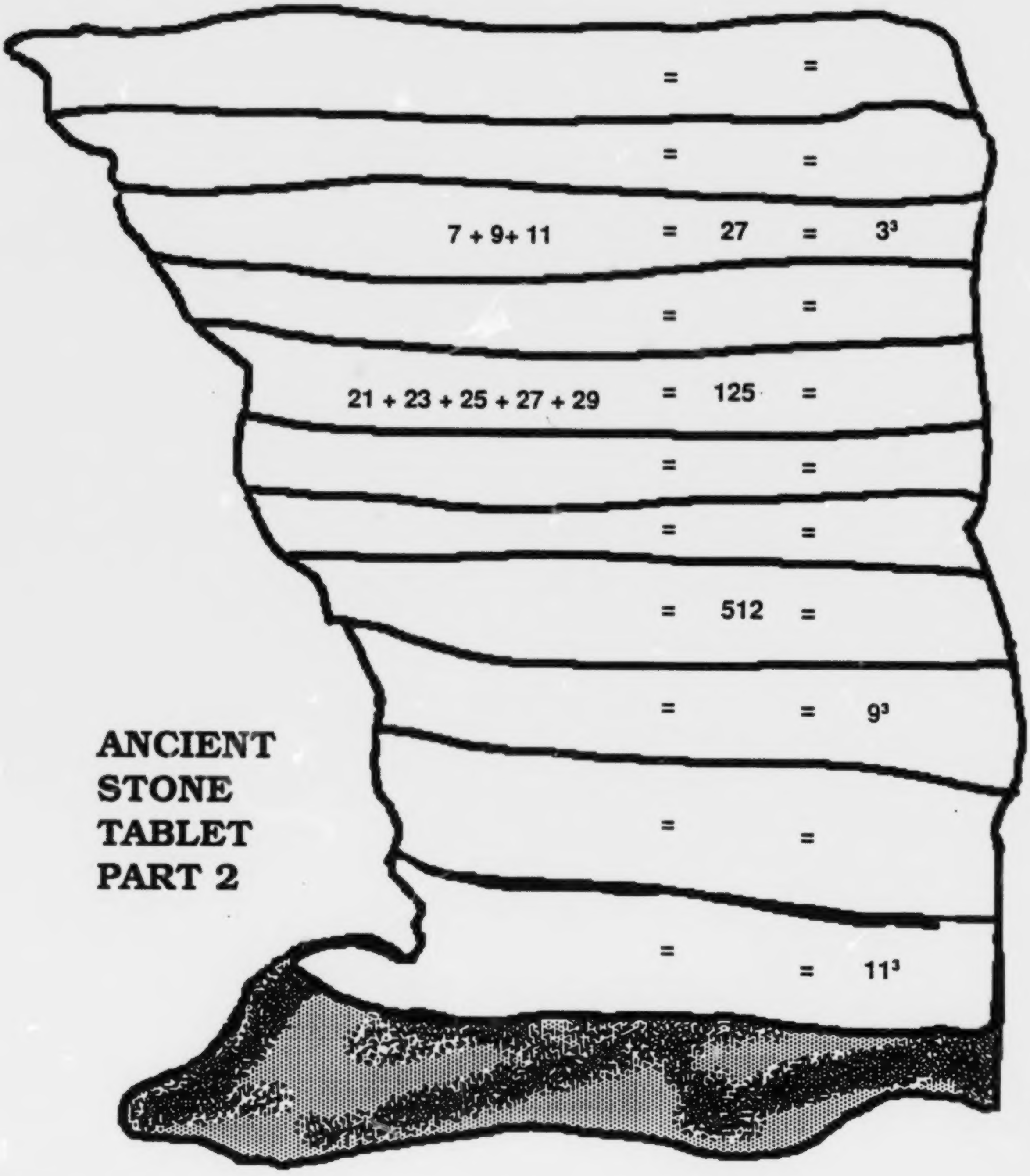
Name ______________________________

## AN ANCIENT ODDITY

Student Activity Sheet 2

Years later Archaeologists found the second part of the tablet.

1. Cut out or place the two sections of the tablet together so the rows line up.
2. Fill in the missing numbers to discover the pattern on the Ancient Tablet. What did you discover?



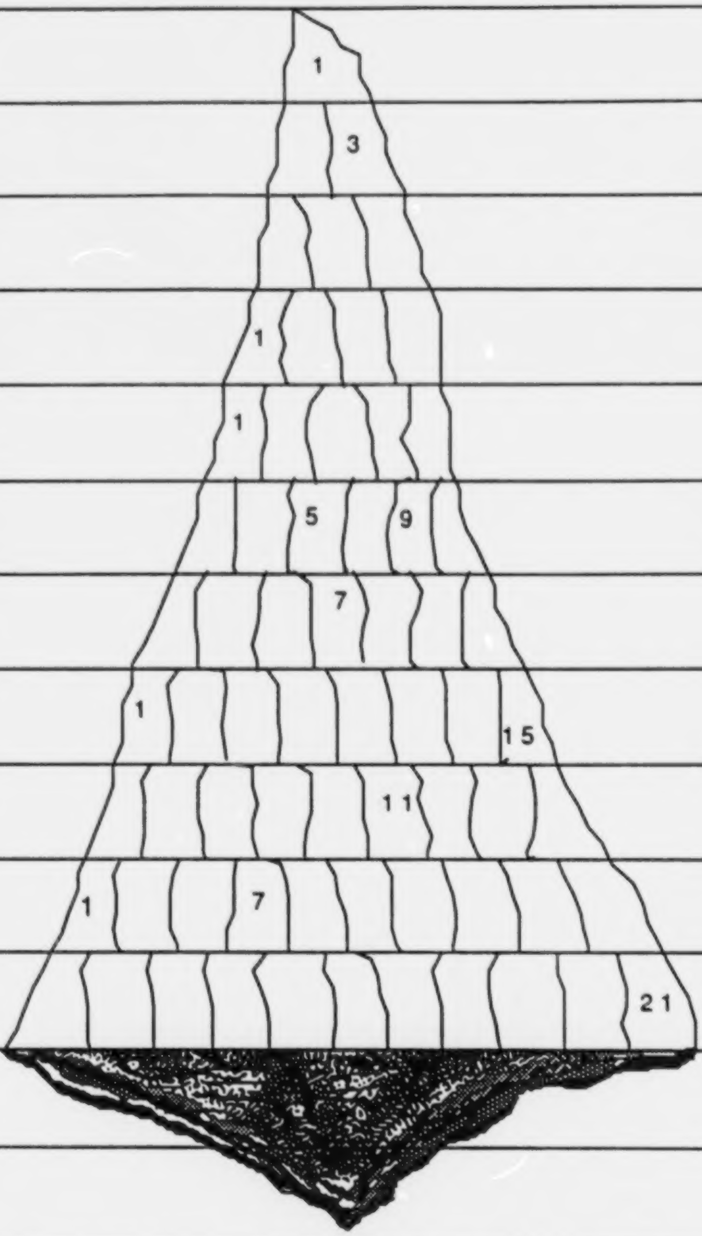
**ANCIENT STONE TABLET PART 2**

Name ______________________________

## AN ANCIENT ODDITY

Home Activity Sheet

1. Complete the blanks on the tablet by filling in the missing number to form a pattern.
2. Use the numbers of each row to complete column A and B.
   (Hint: What kind of numbers are found in the tablet.)



| Row | ANCIENT STONE TABLET | A | B |
|---|---|---|---|
| 1 | 1 | 1 | |
| 2 | 3 | | $2^2$ |
| 3 | | | |
| 4 | 1 | 16 | |
| 5 | 1 | | |
| 6 | 5 9 | 36 | |
| 7 | 7 | | |
| 8 | 1 15 | | |
| 9 | 11 | | $9^2$ |
| 10 | 1 7 | | |
| 11 | 21 | | |
| | | | |
| | | | |

What are the numbers for columns A and B in row 25? ________________

How does this tablet differ from the other Stone Tablets? ______________________________

______________________________________________________________

# PALATABLE PATTERNS

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Patterns and Functions |
| **SKILL:** | Add or divide a number by a one or a multi-digit number |

## MANAGEMENT

| | |
|---|---|
| **CLASS ORGANIZATION:** | Pairs |
| **TIME FRAME:** | One math period |
| **MATERIALS:** | Calculator, overhead transparency |
| **VOCABULARY:** | Quotient, reverse, digits, even numbers |
| **PREREQUISITE SKILL:** | Division |

## LESSON

- **DIRECTED INSTRUCTION/ GUIDED PRACTICE:**

  Lead students to discover, extend and create numbers patterns. Use an overhead projector to demonstrate.

- **INDEPENDENT PRACTICE:**

  Hand out Student Activity Sheet.

  Have students complete the charts and answer the question. Guide the students through the first few examples. Assist as needed. Discuss their observations such as: the quotients are all whole numbers.

- **EXTENSION:**

  Hand out and have students complete the Extension Activity Sheet. Students use numbers with an odd number of digits. Ask them if they will still get a quotient that is a whole number. After they answer, you can give them examples like 352, which will work out correctly. Then give 735 which will not work. (735 + 537) + 11 is not a whole number.

# PALATABLE PATTERNS
## Transparency

Work with a partner.

1. Choose any number with an even number of digits.
2. Reverse the order of the digits.
3. Use your calculator to add the two numbers.
4. Divide the sum by 11. Is the quotient a whole number? Will it always be a whole number?

| | | | |
|---|---|---|---|
| 56 | Addend | 5681 | Addend |
| +65 | Reverse addend | +1865 | Reverse Addend |
| 121 | | 7546 | |

$121 \div 11 = \boxed{11}$ $\qquad$ $7546 \div 11 = \boxed{686}$

The quotients 11 and 686 are whole numbers

| | Number | Reverse order | Sum | Sum ÷ 11 |
|---|---|---|---|---|
| Example 1 | 56 | 65 | 121 | 121 ÷ 11 = 11 |
| Example 2 | 5681 | 1865 | 7546 | 7546 ÷ 11= 686 |
| Example 3 | 2345 | | | |

# PALATABLE PATTERNS

## Student Activity Sheet
## Teacher Answer Sheet

Complete the chart.

| | Number | Reverse Order | Sum | Sum divided by 11 |
|---|---|---|---|---|
| | 5681 | 1865 | 7546 | 7546 ÷ 11 = 686 |
| 1 | 73 | 37 | 110 | 110 ÷ 11 = 10 |
| 2 | 97 | 79 | 176 | 176 ÷ 11 = 16 |
| 3 | 1112 | 2111 | 3223 | 3223 ÷ 11 = 293 |
| 4 | 3057 | 7503 | 10,560 | 10,560 ÷ 11 = 960 |
| 5 | 6935 | 5396 | 12,331 | 12,331 ÷ 11 = 1121 |
| 6 | 5009 | 9005 | 14,014 | 14,014 ÷ 11 = 1274 |
| 7 | 112,526 | 625,211 | 737,737 | 737,737 ÷ 11 = 67,067 |
| 8 | 727,575 | 575,727 | 1,303,302 | 1,303,302 ÷ 11 = 118,482 |
| 9 | 27,848,381 | 18,384,872 | 46,233,253 | 46,233,253 ÷ 11 = 4,203,023 |
| 10 | 67,132,203 | 30,223,176 | 97,355,379 | 97,355,379 ÷ 11 = 8,850,489 |

Choose numbers with an even number of digits and complete the chart.

| | Number | Reverse Order | Sum | Sum divided by 11 |
|---|---|---|---|---|
| 1 | Answers will vary | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |

What do you observe about the quotients? When the original number has an even number of digits, the number you get by finding the sum, as above, is always divisible by 11. You always get a whole number quotient.

Name ____________________

## PALATABLE PATTERNS

Student Activity Sheet

Complete the chart.

| | Number | Reverse Order | Sum | Sum divided by 11 |
|---|---|---|---|---|
| | 5681 | 1865 | 7546 | 7546 ÷ 11 = 686 |
| 1 | 73 | | | |
| 2 | 97 | | | |
| 3 | 1112 | | | |
| 4 | 3057 | | | |
| 5 | 6935 | | | |
| 6 | 5009 | | | |
| 7 | 112,526 | | | |
| 8 | 727,575 | | | |
| 9 | 27,848,381 | | | |
| 10 | 67,132,203 | | | |

Choose numbers with an even number of digits and complete the chart.

| | Number | Reverse Order | Sum | Sum divided by 11 |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |

What do you observe about the quotients? ____________________

____________________

____________________

____________________

Name ____________________

## PALATABLE PATTERNS
Extension Activity Sheet

Choose numbers with an odd number of digits and complete the chart.

| | Number | Reverse Order | Sum | Sum divided by 11 |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |

What do you observe about the quotients? ____________________

____________________

____________________

____________________

# DIGIT PATTERNS

**GRADE:** 7 - 8

**STRAND:** Patterns and Functions

**SKILL:** Identify, extend, and create number patterns.

## MANAGEMENT

**CLASS ORGANIZATION:** Pairs

**TIME FRAME:** One math period

**MATERIALS:** Calculator

**VOCABULARY:** Factor, product

**PREREQUISITE SKILL:** Multiplication

## LESSON

- **DIRECTED INSTRUCTION/GUIDED PRACTICE/INDEPENDENT PRACTICE:**

  Hand out Student Activity Sheet page 1.

  Have students work the page with their partner.
  Be sure students are aware that in the two problems for each example, the digits of the factors were each reversed and the digits in the products were reversed.

  For example 112 became 211
  113 became 311
  and 12656 became 65621

  Hand out Student Activity Sheet page 2.
  Have students complete the Student Activity Sheet with a partner.
  Students may want to use the constant feature of the calculator.

- **EVALUATION:**

  Teacher observation

## DIGIT PATTERNS

Student Activity Sheet page 2
Teacher Answer Sheet

Work with a partner and keep a record of your predictions.
Complete:

| | | A | | | B |
|---|---|---|---|---|---|
| | Factors | Product | Reverse Digits of Factors | Prediction of Product | Actual Product |
| 1 | 112 x 133 | 14,896 | 211 x 331 | | 69,841 |
| 2 | 112 x 223 | 24,976 | 211 x 322 | | 67,942 |
| 3 | 112 x 114 | 12,768 | 211 x 411 | | 86,721 |
| 4 | 112 x 122 | 13,664 | 211 x 221 | | 46,631 |
| 5 | 112 x 123 | 13,776 | 211 x 321 | | 67,731 |
| 6 | 112 x 113 | 12,656 | 211 x 311 | | 65,621 |
| 7 | 112 x 112 | 12,544 | 211 x 211 | | 44,521 |
| 8 | 112 x 232 | 25,984 | 211 x 232 | | 48,952 |
| 9 | 112 x 141 | 15,792 | 211 x 141 | | 29,751 |
| 10 | 112 x 111 | 12,432 | 211 x 111 | | 23,421 |

With your partner, find other factors that follow the same pattern.

These examples are very special because if you reverse the digits of the number in column A you obtain the number in column B.

Name ____________________

## DIGIT PATTERNS

Student Activity Sheet Page 1

Directions:

Work with a partner.

Example 1

- Look at the pair of equations. Use your calculator to find the product.

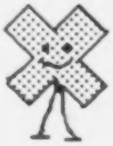
112 x 113 = 12656

211 x 311 = 65621

- Discuss with your partner the relationship of the factors and the products of the two equations.

Example 2

112 x 124 = 13888



- Reverse the digits in each of the factors.

- Predict and write the product.

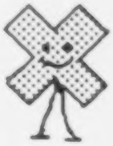
Prediction: ____________________________________

- Verify with your calculator.

Actual Product: ____________________________________

Example 3

102 x 112 = __________



- Reverse the digits in each of the factors.

- Predict and write the product.

Prediction: ____________________________________

- Verify with your calculator.

Actual Product: ____________________________________

Name ____________________

## DIGIT PATTERNS

Student Activity Sheet page 2

Work with a partner.
Complete:

| | Factors | Product | Reverse Digits of Factors | Prediction of Product | Actual Product |
|---|---|---|---|---|---|
| 1 | 112 x 133 | | | | |
| 2 | 112 x 223 | | | | |
| 3 | 112 x 114 | | | | |
| 4 | 112 x 122 | | | | |
| 5 | 112 x 123 | | | | |
| 6 | 112 x 113 | | | | |
| 7 | 112 x 112 | | | | |
| 8 | 112 x 232 | | | | |
| 9 | 112 x 141 | | | | |
| 10 | 112 x 111 | | | | |

Look at the factors used to make the patterns. With your partner, try to find other factors that follow the same pattern.

| | Factors | Product | Reverse Digits of Factors | Prediction of Product | Actual Product |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |

**0123456789**

# FOLLOW THE FLOW CHART 1

**GRADE:** 7 - 8

**STRAND:** Patterns and Functions

**SKILL:** Identify, extend, and create number sequences.

## MANAGEMENT

**CLASS ORGANIZATION:** Pairs

**TIME FRAME:** One or two math periods.

**MATERIALS:** Calculator, overhead calculator, overhead transparency

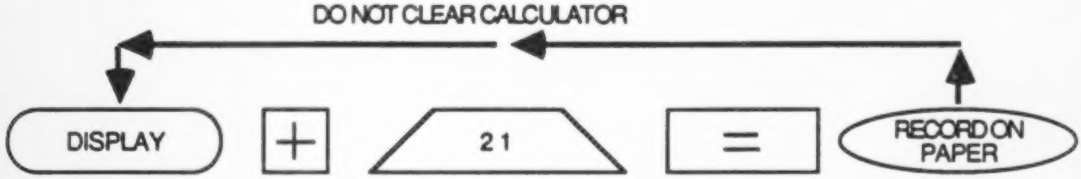
**VOCABULARY:** Sequence, flow chart

**PREREQUISITE SKILLS:** Understand a flow chart.

## LESSON

- **DIRECTED INSTRUCTION & GUIDED PRACTICE:**

  This lesson will help students create and recognize number sequences.
  Hand out Student Activity Sheet 1.
  Use flow chart transparency to help students follow the steps to complete problems 1 and 2. As they work through the sequence, have them predict the next number and then check it by following the directions on the flow chart.



[1] 56., 77., ____, ____, ____, ____, ____, ____

[2] 111., 132., ____, ____, ____, ____, ____, ____

- **INDEPENDENT PRACTICE:**

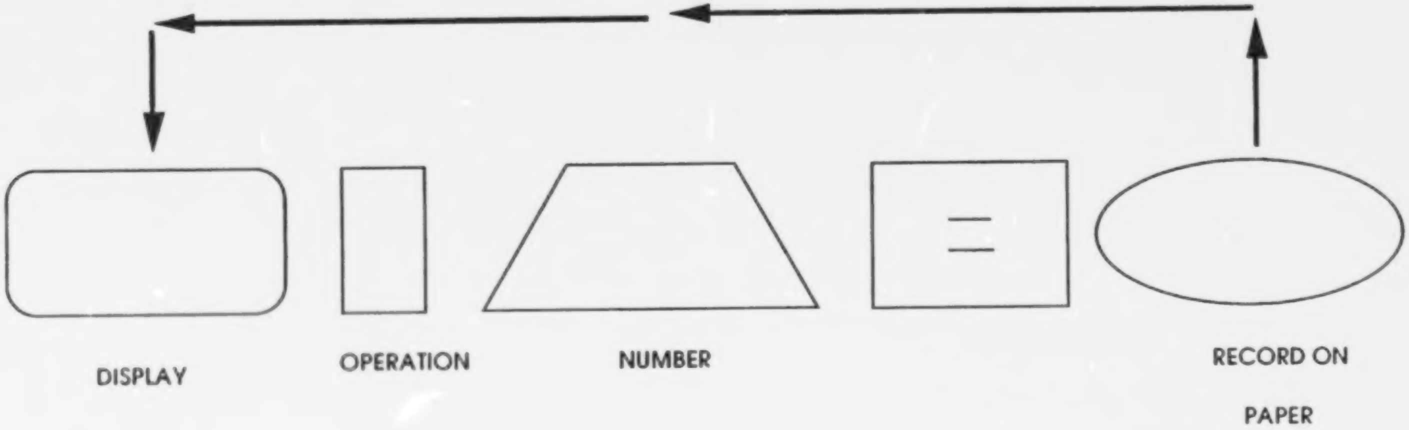
  Have students complete Student Activity Sheet 1. Discuss results, including their discoveries about the constant function.

  Hand out and have students complete Student Activity Sheets 2-4.

  Have students observe the sequences. They fill in the blanks on the flow charts with the missing operation and number and complete the sequences. Be sure students check to determine if their answers are reasonable. Have students or groups fill in their own flow charts to create sequences.

## FOLLOW THE FLOW CHART 1 OVERHEAD TRANSPARENCY

DO NOT CLEAR CALCULATOR


=
DISPLAY
OPERATION
NUMBER
RECORD ON
PAPER

STARTING NUMBER . ______ . ______ . ______ . ______ . ______ . ______ . ______

STARTING NUMBER . ______ . ______ . ______ . ______ . ______ . ______ . ______

STARTING NUMBER . ______ . ______ . ______ . ______ . ______ . ______ . ______

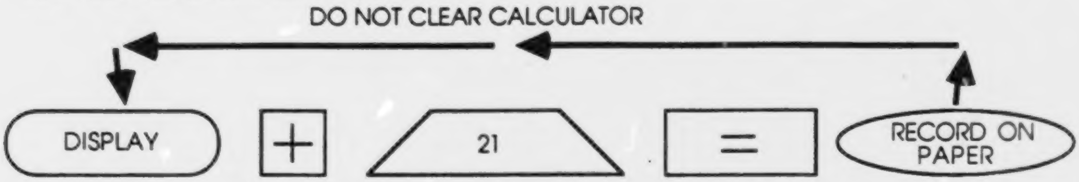
CAMP-LA

45

## FOLLOW THE FLOW CHART 1

Student Activity Sheet 1
Teacher Answer Sheet

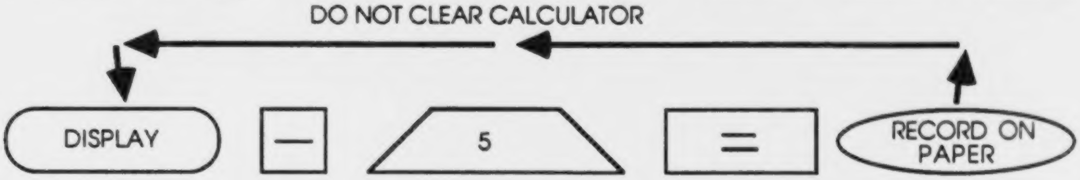
1. Enter the first number in the list on the calculator.
2. Follow the steps in the flow chart to develop a sequence.
3. Clear the calculator between problems.


DO NOT CLEAR CALCULATOR
DISPLAY
+
21
=
RECORD ON PAPER

A

1) 56 , 77 , 98 , 119 , 140 , 161 , 182 , 203

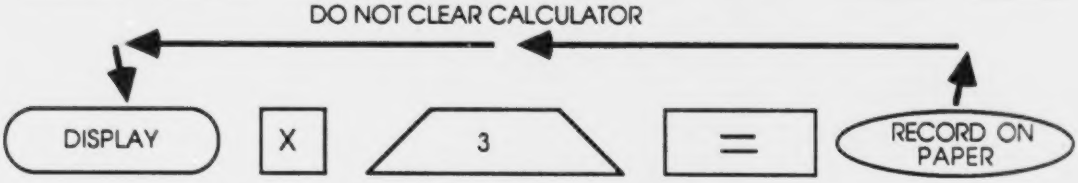
2) 111 , 132 , 153 , 174 , 195 , 216 , 237 , 258


DO NOT CLEAR CALCULATOR
DISPLAY
—
5
=
RECORD ON PAPER

B

3) 48 , 43 , 38 , 33 , 28 , 23 , 18 , 13

4) 102 , 97 , 92 , 87 , 82 , 77 , 72 , 67

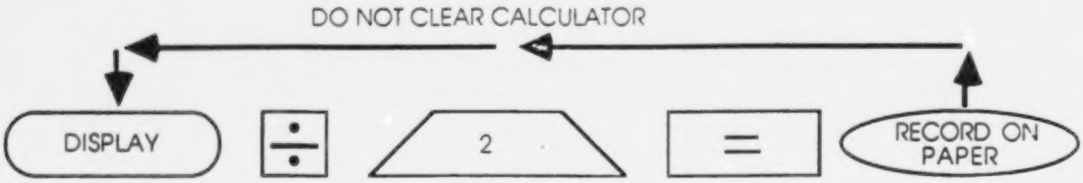

DO NOT CLEAR CALCULATOR
DISPLAY
X
3
=
RECORD ON PAPER

C

5) 12 , 36 , 108 , 324 , 972 , 2916 , 8748 , 26244

6) 123 , 369 , 1107 , 3321 , 9963 , 29889 , 89667 , 269001

## FOLLOW THE FLOW CHART 1
Student Activity Sheet 1
Teacher Answer Sheet



D

7) 3072 , 1536 , 768 , 384 , 192 , 96 , 48 , 24

8) 256 , 128 , 64 , 32 , 16 , 8 , 4 , 2

Use the following calculator shortcuts to check your answers to problems 1 - 8. The shortcuts use the calculator's constant function.

Check to see that the numbers displayed each time you press $\boxed{=}$ matched your recorded answers. Clear the calculator between problems.

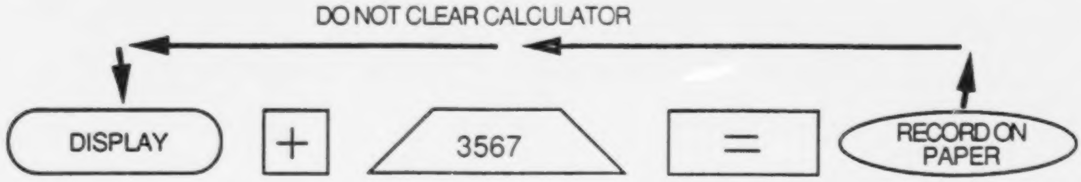
Sequence 1 Press 56 + 21 = = = = = = =
2 Press 111 + 21 = = = = = = =
3 Press 48 - 5 = = = = = = =
4 Press 102 - 5 = = = = = = =
5 Press 3 x 12 = = = = = = =
6 Press 3 x 123 = = = = = = =
7 Press 3072 ÷ 2 = = = = = = =
8 Press 256 ÷ 2 = = = = = = =

Explain what you discovered about how the constant function works for the different operations.

In multiplication, enter the constant multiplier before the operation symbol.
In addition, subtraction, and division, enter the constant number after the operation symbol.

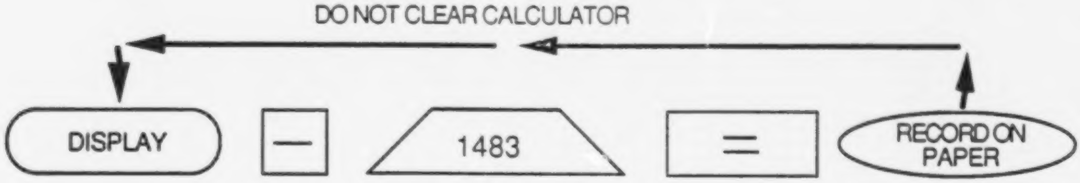
## FOLLOW THE FLOW CHART 1
Student Activity Sheet 2
Teacher Answer Sheet

Use the CONSTANT FUNCTION to complete the following sequences.


DO NOT CLEAR CALCULATOR
DISPLAY
+
3567
=
RECORD ON PAPER

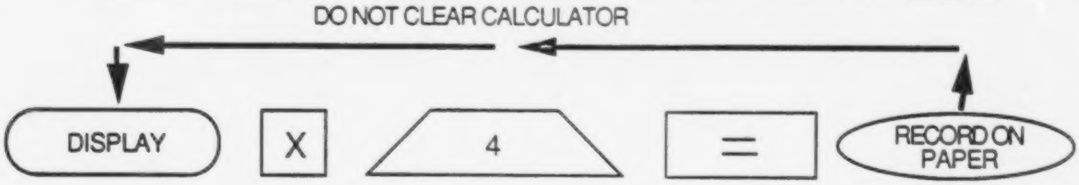
A

7 , 3574 , 7141 , 10708 , 14275 , 17842 , 21409


DO NOT CLEAR CALCULATOR
DISPLAY
—
1483
=
RECORD ON PAPER

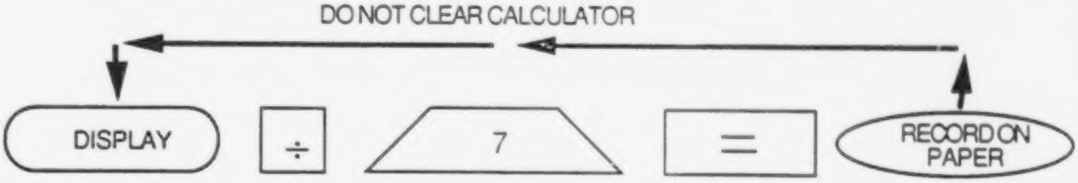
B

10,000 , 8517 , 7034 , 5551 , 4068 , 2585 , 1102


DO NOT CLEAR CALCULATOR
DISPLAY
X
4
=
RECORD ON PAPER

C

14 , 56 , 224 , 896 , 3584 , 14336 , 57344


DO NOT CLEAR CALCULATOR
DISPLAY
÷
7
=
RECORD ON PAPER

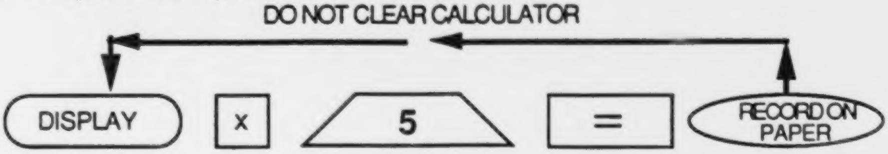
D

40353607 , 5764801 , 823543 , 117649 , 16807 , 2401 , 343

# FOLLOW THE FLOW CHART 1

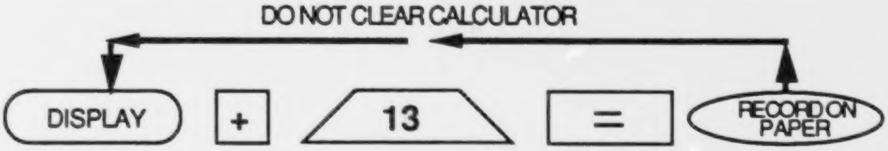
Student Activity Sheet 3
Teacher Answer Sheet

1. Look at the number sequence.
2. Fill in the flow chart to show how the sequences were created.
3. Complete the sequences under each flow chart.


DO NOT CLEAR CALCULATOR
DISPLAY
x
5
=
RECORD ON PAPER

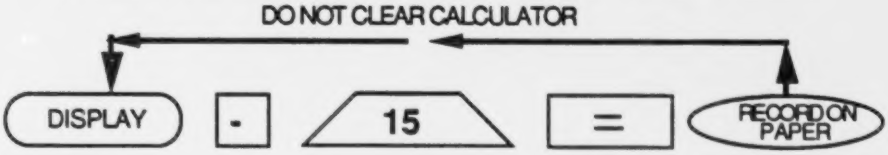
A

1) 1, 5, 25, 125, 625, 3125, 15625
2) 2, 10, 50, 250, 1250, 6250, 31250
3) 8, 40, 200, 1000, 5000, 25000, 125000


DO NOT CLEAR CALCULATOR
DISPLAY
+
13
=
RECORD ON PAPER

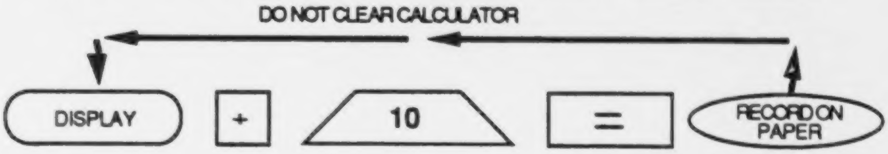
B

1) 28, 41, 54, 67, 80, 93, 106
2) 2070, 2083, 2096, 2109, 2122, 2135, 2148
3) 75, 88, 101, 114, 127, 140, 153


DO NOT CLEAR CALCULATOR
DISPLAY
-
15
=
RECORD ON PAPER

C

1) 100, 85, 70, 55, 40, 25, 10
2) 93, 78, 63, 48, 33, 18, 3
3) 250, 235, 220, 205, 190, 175, 160


DO NOT CLEAR CALCULATOR
DISPLAY
+
10
=
RECORD ON PAPER

D

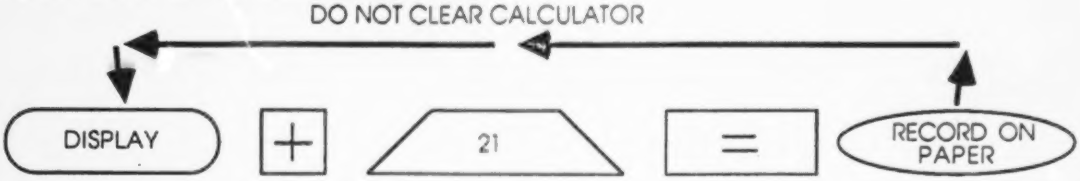
1) 10,000, 1,000, 100, 10, 1, 0.1, 0.01
2) 500,000, 50,000, 5,000, 500, 50, 5, .5
3) 280, 28, 2.8, .28, .028, .0028, .00028

Name ______________________

## FOLLOW THE FLOW CHART 1

Student Activity Sheet 1

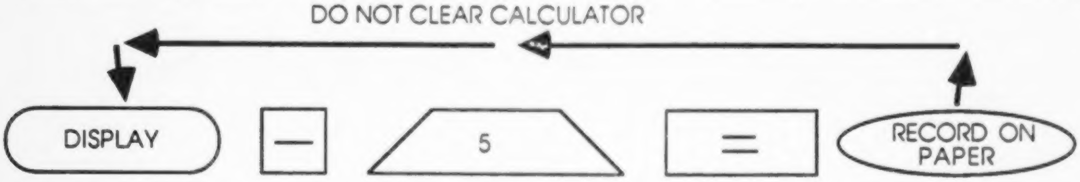
1. Enter the first number in the list on the calculator.
2. Follow the steps in the flow chart to develop a sequence.
3. Clear the calculator between problems.


DO NOT CLEAR CALCULATOR
DISPLAY
+
21
=
RECORD ON PAPER

A

1) 56, 77, ______, ______, ______, ______, ______, ______

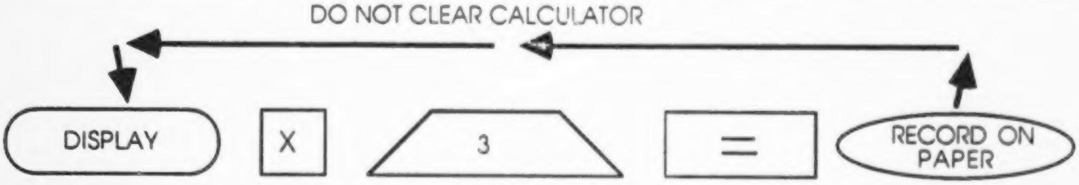
2) 111, 132, ______, ______, ______, ______, ______, ______


DO NOT CLEAR CALCULATOR
DISPLAY
—
5
=
RECORD ON PAPER

B

3) 48, ______, ______, ______, ______, ______, ______, ______

4) 102, ______, ______, ______, ______, ______, ______, ______


DO NOT CLEAR CALCULATOR
DISPLAY
X
3
=
RECORD ON PAPER

C

5) 12, ______, ______, ______, ______, ______, ______, ______

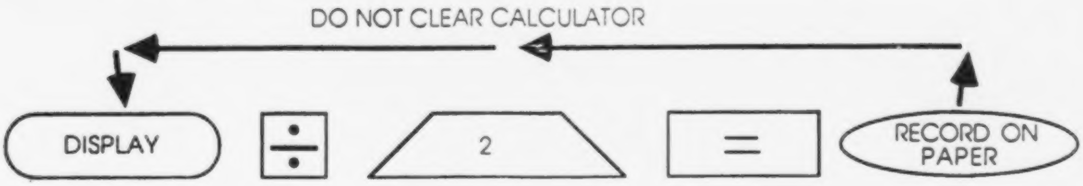
6) 123, ______, ______, ______, ______, ______, ______, ______

51

Name ____________________

## FOLLOW THE FLOW CHART 1

### Student Activity Sheet 1



D

7) 3072, ______, ______, ______, ______, ______, ______, ______

8) 256, ______, ______, ______, ______, ______, ______, ______

Use the following calculator shortcuts to check your answers to problems 1 - 8. The shortcuts use the calculator's constant function.

Check to see that the numbers displayed each time you press $\boxed{=}$ matched your recorded answers. Clear the calculator between problems.

| | | | | |
|---|---|---|---|---|
| Sequence | 1 | Press | 56 + 21 | = = = = = = = |
| | 2 | Press | 111 + 21 | = = = = = = = |
| | 3 | Press | 48 - 5 | = = = = = = = |
| | 4 | Press | 102 - 5 | = = = = = = = |
| | 5 | Press | 3 x 12 | = = = = = = = |
| | 6 | Press | 3 x 123 | = = = = = = = |
| | 7 | Press | 3072 ÷ 2 | = = = = = = = |
| | 8 | Press | 256 ÷ 2 | = = = = = = = |

Explain what you discovered about how the constant function works for the different operations. ______________________________________________

______________________________________________

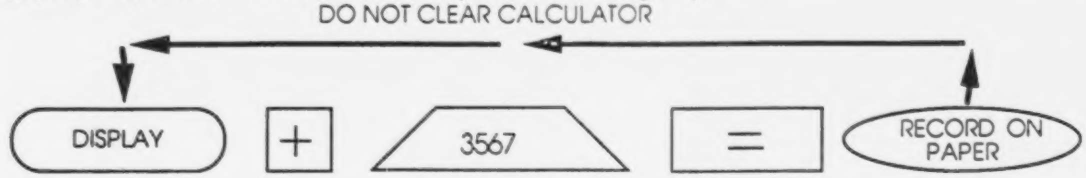
______________________________________________

Name ______________________

## FOLLOW THE FLOW CHART 1

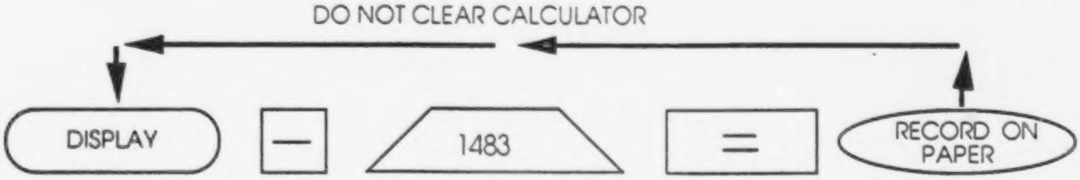
Student Activity Sheet 2

Use the CONSTANT FUNCTION to complete the following sequences.


DO NOT CLEAR CALCULATOR
DISPLAY
+
3567
=
RECORD ON PAPER

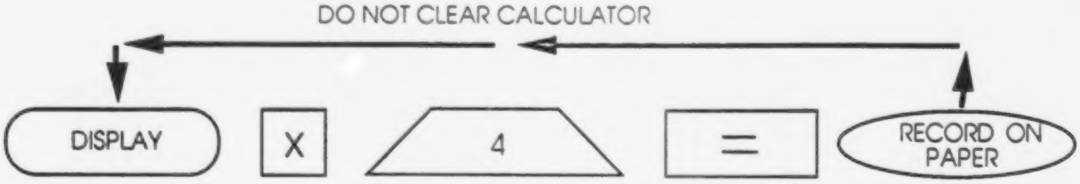
A

7 , ______ , ______ , ______ , ______ , ______ , ______


DO NOT CLEAR CALCULATOR
DISPLAY
—
1483
=
RECORD ON PAPER

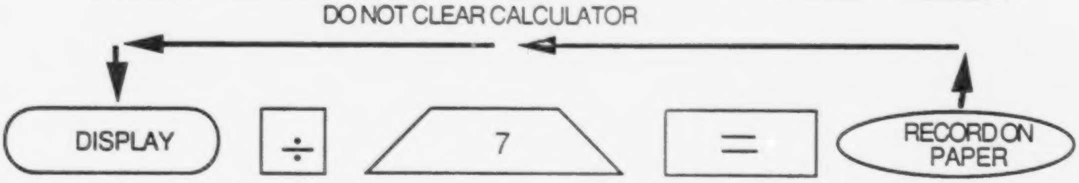
B

10,000 , ______ , ______ , ______ , ______ , ______ , ______


DO NOT CLEAR CALCULATOR
DISPLAY
X
4
=
RECORD ON PAPER

C

14 , ______ , ______ , ______ , ______ , ______ , ______


DO NOT CLEAR CALCULATOR
DISPLAY
÷
7
=
RECORD ON PAPER

D

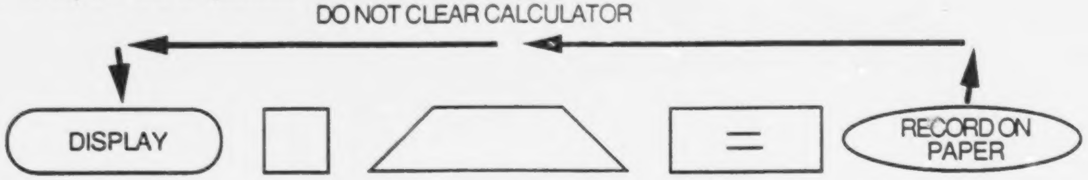
40353607 , ______ , ______ , ______ , ______ , ______ , ____

Name ____________________

## FOLLOW THE FLOW CHART 1

Student Activity Sheet 3

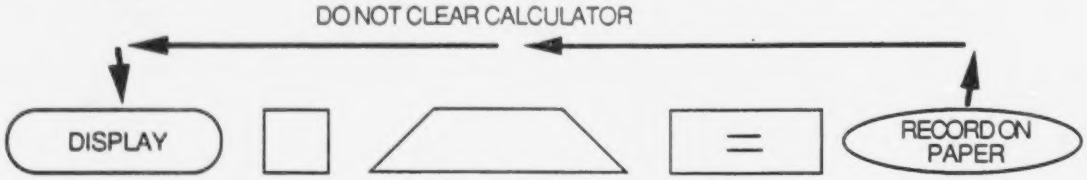
1. Look at the number sequence.
2. Fill in the flow chart to show how the sequences were created.
3. Complete the sequences under each flow chart.


DO NOT CLEAR CALCULATOR
DISPLAY
=
RECORD ON PAPER

A 1) 1, 5, 25, 125, 625, ____, ____

2) 2, 10, 50, ____, ____, ____, ____

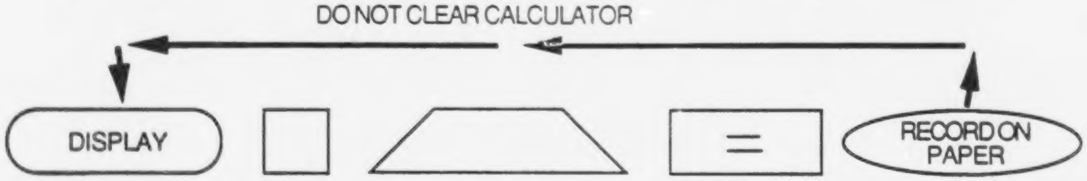
3) 8, ____, ____, ____, ____, ____, ____


DO NOT CLEAR CALCULATOR
DISPLAY
=
RECORD ON PAPER

B 1) 28, 41, 54, 67, 80, 93, ____

2) 2070, 2083, 2096, ____, ____, ____, ____

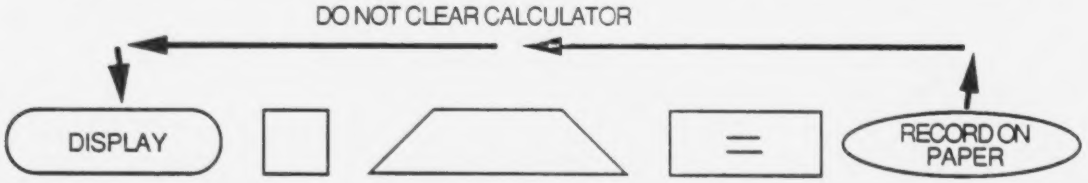
3) 75, ____, ____, ____, ____, ____, ____


DO NOT CLEAR CALCULATOR
DISPLAY
=
RECORD ON PAPER

C 1) 100, 85, 70, 55, 40, 25, ____

2) 93, 78, 63, ____, ____, ____, ____

3) 250, ____, ____, ____, ____, ____, ____


DO NOT CLEAR CALCULATOR
DISPLAY
=
RECORD ON PAPER

D 1) 10,000, 1,000, 100, 10, 1, 0.1, 0.01

2) 500,000, 50,000, 5,000, ____, ____, ____, ____

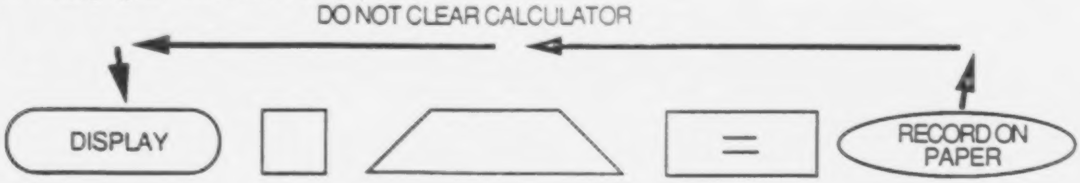
3) 280, ____, ____, ____, ____, ____, ____

Name____________________

## FOLLOW THE FLOW CHART 1

Student Activity Sheet 4

1. Work in cooperative groups.
2. Design your own number sequences.

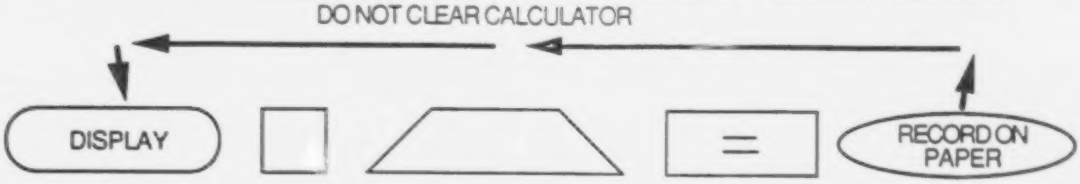

DO NOT CLEAR CALCULATOR
DISPLAY
=
RECORD ON PAPER

A

1) ______, ______, ______, ______, ______, ______, ______

2) ______, ______, ______, ______, ______, ______, ______

3) ______, ______, ______, ______, ______, ______, ______

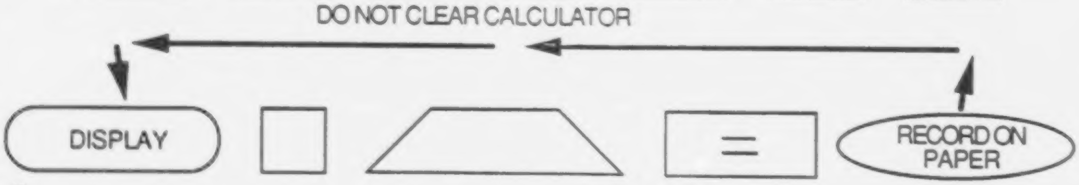

DO NOT CLEAR CALCULATOR
DISPLAY
=
RECORD ON PAPER

B

1) ______, ______, ______, ______, ______, ______, ______

2) ______, ______, ______, ______, ______, ______, ______

3) ______, ______, ______, ______, ______, ______, ______


DO NOT CLEAR CALCULATOR
DISPLAY
=
RECORD ON PAPER

C

1) ______, ______, ______, ______, ______, ______, ______

2) ______, ______, ______, ______, ______, ______, ______

3) ______, ______, ______, ______, ______, ______, ______

## FOLLOW THE FLOW CHART 2

**GRADE:** 7 - 8

**STRAND:** Patterns and Functions

**SKILL:** Identify, extend, and create number sequences.

**MANAGEMENT**

**CLASS ORGANIZATION:** Pairs

**TIME FRAME:** One or two math periods.

**MATERIALS:** Calculator, overhead calculator, overhead transparency

**VOCABULARY:** Sequence, flow chart, input, output

**PREREQUISITE SKILL:** Understand a flow chart. (If students do not have flow chart experience they should first complete Lesson 5.)

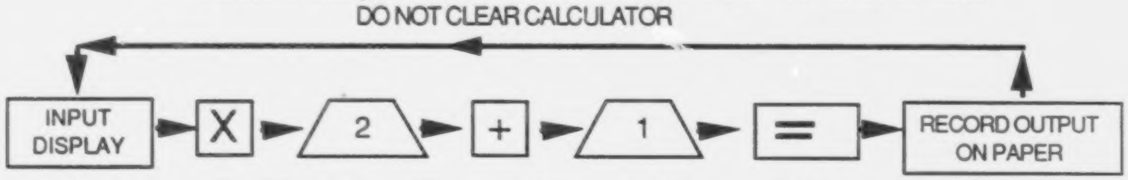
**LESSON**

- **DIRECTED INSTRUCTION:**

This lesson will help students understand sequences involving 2 operations.
Hand out Student Activity Sheet 1.
Use flow chart transparency to show students how to do problem A

Answer to A:

- Enter 11 in the calculator.
- After following all the operations in the flow chart you get an output of 23.
- Record the 23 below the flow chart.
- Use the 23 as the next input. Again follow the operations on the flow chart.
- Record the 47.
- Continue this process until each blank below the flow chart is filled.



1) 11 2) 23 3) 47 4) 95 5) 191 6) 383 7) 767

- **GUIDED PRACTICE:**

Students work problem B. Check their work by going over B on the overhead.

- **INDEPENDENT PRACTICE:**

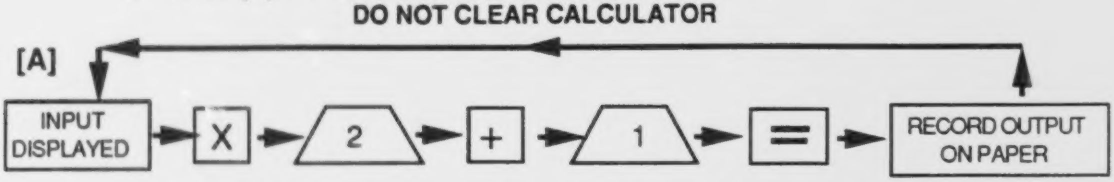
Students complete Student Activity Sheet 1(C-E) and check their work. Hand out Student Activity Sheet 2. It is more challenging than the previous sheet as it involves completing the sequence and the flow chart. Have students complete the sequences and then discuss their results.

## FOLLOW THE FLOW CHART 2

Student Activity Sheet 1
Teacher Answer Sheet

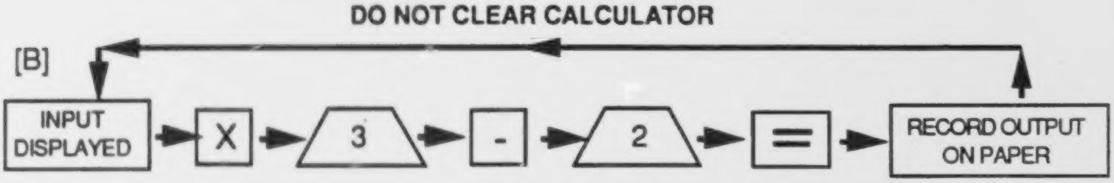
1. Enter the first number in the list on the calculator.
2. Follow the steps in the flow chart to develop a sequence.
3. Clear [C] the calculator between problems.

**[A]**


DO NOT CLEAR CALCULATOR
INPUT DISPLAYED
X
2
+
1
=
RECORD OUTPUT ON PAPER

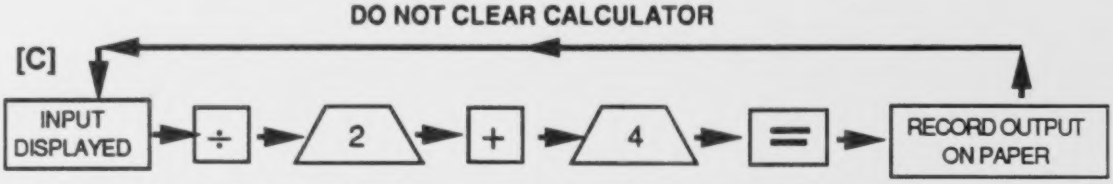
11 23 47 95 191 383 767

**[B]**


DO NOT CLEAR CALCULATOR
INPUT DISPLAYED
X
3
-
2
=
RECORD OUTPUT ON PAPER

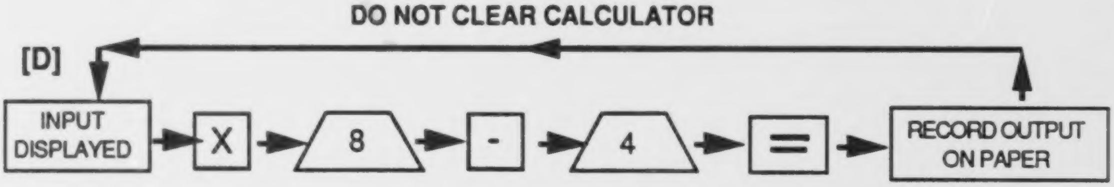
20 58 172 514 1540 4618 13852

**[C]**


DO NOT CLEAR CALCULATOR
INPUT DISPLAYED
÷
2
+
4
=
RECORD OUTPUT ON PAPER

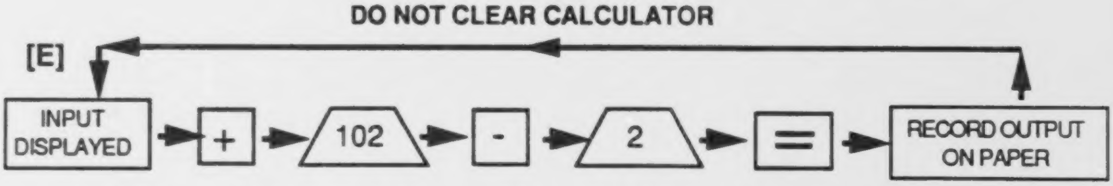
8200 4104 2056 1032 520 264 136

**[D]**


DO NOT CLEAR CALCULATOR
INPUT DISPLAYED
X
8
-
4
=
RECORD OUTPUT ON PAPER

3 20 156 1244 9948 79580 636636

**[E]**


DO NOT CLEAR CALCULATOR
INPUT DISPLAYED
+
102
-
2
=
RECORD OUTPUT ON PAPER

1200 1300 1400 1500 1600 1700 1800

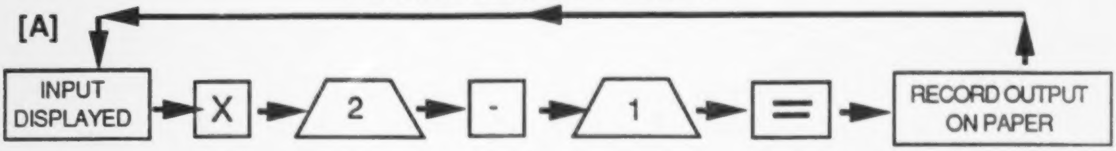
## FOLLOW THE FLOW CHART 2

Student Activity Sheet 2
Teacher Answer Sheet

1. Look at the number sequence.
2. Fill in the missing items on the flow charts.
3. Complete the sequences in each example using the flow chart.

**DO NOT CLEAR CALCULATOR**

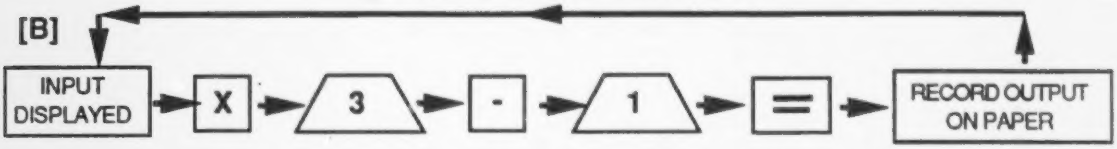
**[A]**


INPUT DISPLAYED
X
2
-
1
=
RECORD OUTPUT ON PAPER

8 15 29 57 113 225 449 897 1793

**DO NOT CLEAR CALCULATOR**

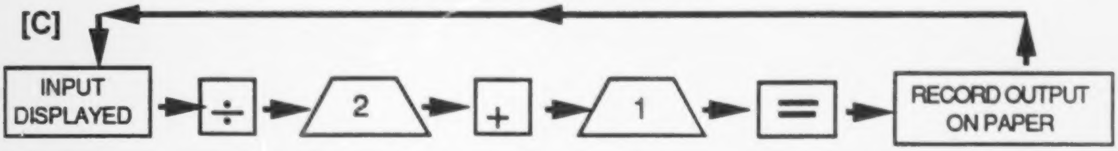
**[B]**


INPUT DISPLAYED
X
3
-
1
=
RECORD OUTPUT ON PAPER

8 23 68 203 608 1823 5468 16403 49208

**DO NOT CLEAR CALCULATOR**

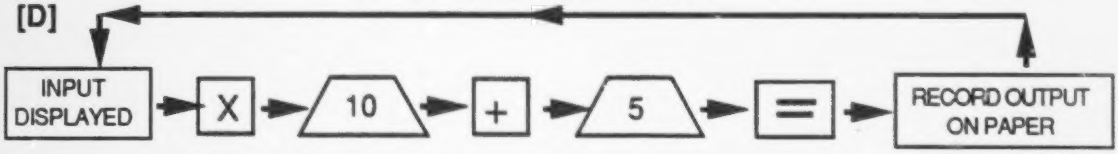
**[C]**


INPUT DISPLAYED
÷
2
+
1
=
RECORD OUTPUT ON PAPER

5378 2690 1346 674 338 170 86 44 23

**DO NOT CLEAR CALCULATOR**

**[D]**

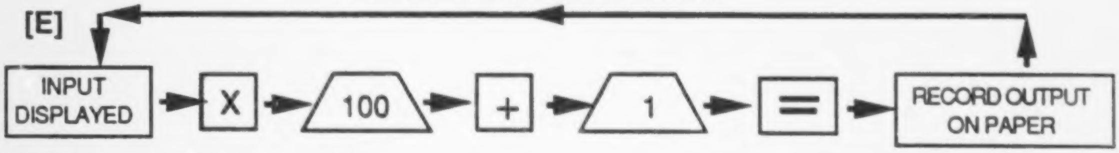

INPUT DISPLAYED
X
10
+
5
=
RECORD OUTPUT ON PAPER

1 15 155 1555 15555 155555 1555555 15555555

3 35 355 3555 35555 355555 3555555 35555555

**DO NOT CLEAR CALCULATOR**

**[E]**

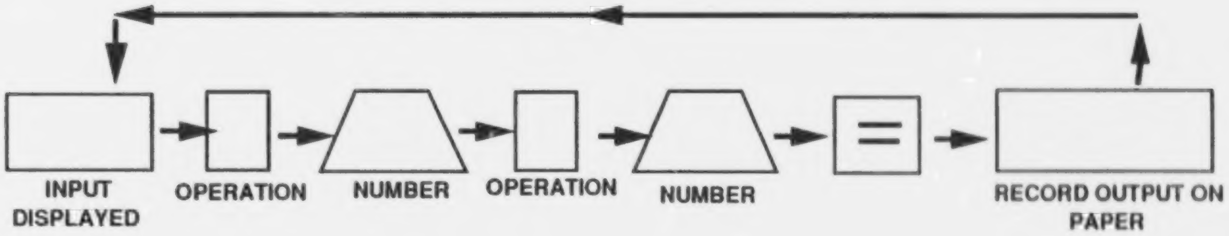

INPUT DISPLAYED
X
100
+
1
=
RECORD OUTPUT ON PAPER

1 101 10101 1010101 101010101 10101010101

3 301 30101 3010101 301010101 30101010101

## FOLLOW THE FLOW CHART 2 OVERHEAD TRANSPARENCY

DO NOT CLEAR CALCULATOR


INPUT DISPLAYED
OPERATION
NUMBER
OPERATION
NUMBER
=
RECORD OUTPUT ON PAPER

_____ _____ _____ _____ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____ _____ _____ _____ _____

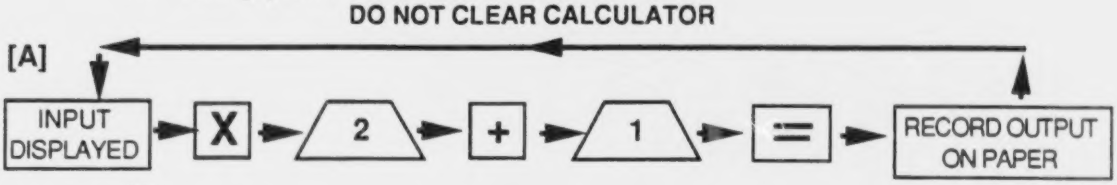
_____ _____ _____ _____ _____ _____ _____ _____ _____

Name_______________

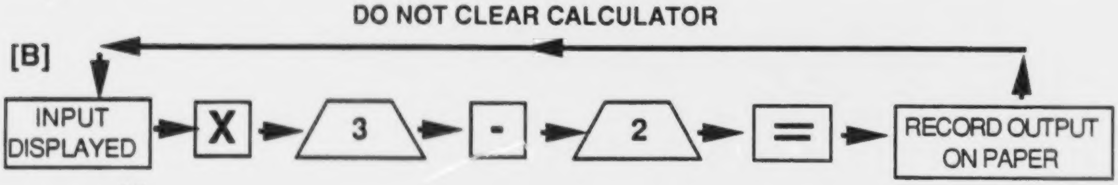
## FOLLOW THE FLOW CHART 2

Student Activity Sheet 1

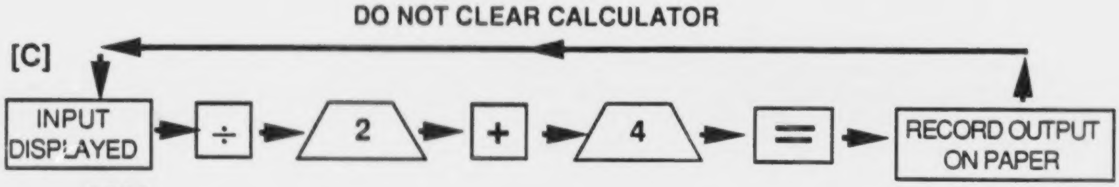
1. Enter the first number in the list on the calculator.
2. Follow all the steps in the flow chart to develop a sequence.
3. Clear [C] the calculator between problems.


DO NOT CLEAR CALCULATOR
[A]
INPUT DISPLAYED
X
2
+
1
=
RECORD OUTPUT ON PAPER

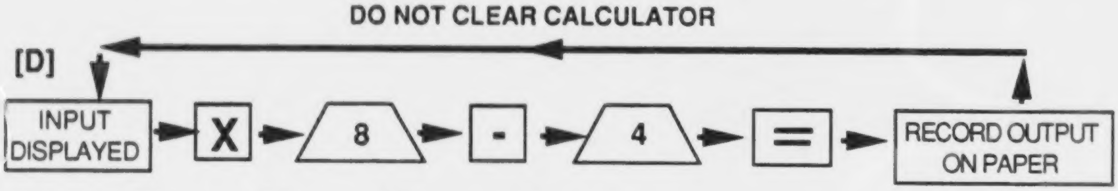
11 ____ ____ ____ ____ ____ ____


DO NOT CLEAR CALCULATOR
[B]
INPUT DISPLAYED
X
3
-
2
=
RECORD OUTPUT ON PAPER

20 ____ ____ ____ ____ ____ ____


DO NOT CLEAR CALCULATOR
[C]
INPUT DISPLAYED
÷
2
+
4
=
RECORD OUTPUT ON PAPER

8200 ____ ____ ____ ____ ____ ____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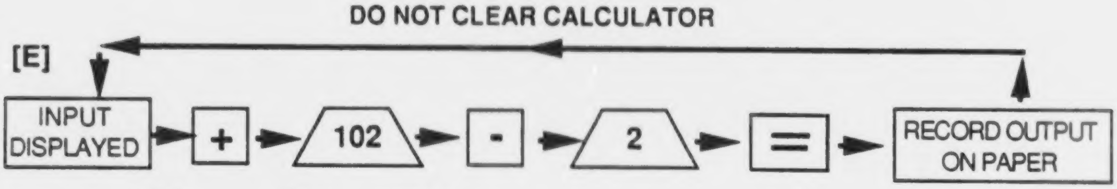
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# FOLLOW THE FLOW CHART 2

## Student Activity Sheet 2

1. Look at the number sequence.
2. Fill in the missing items in the flow charts.
3. Complete the sequences in each example using the flow chart.

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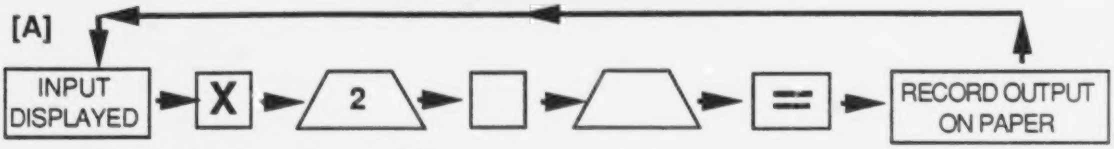
**[A]**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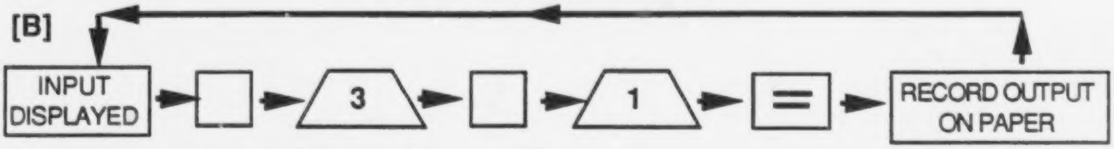
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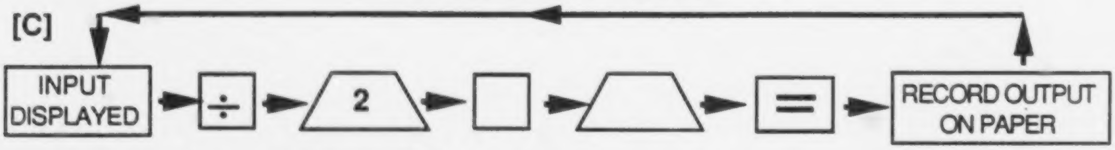
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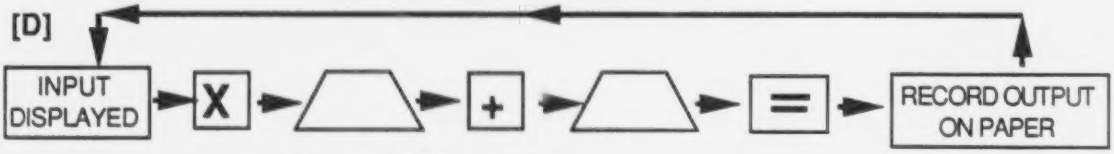
**[D]**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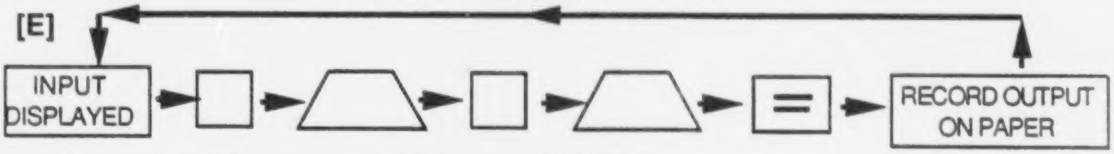
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3 301 ______ ______ ______ ______

62

# CHAPTER 1 ASSESSMENT:
## PATTERNS AND FUNCTIONS

1. What is the highest power of 2 that will fit on your calculator display?

   Student response:
   26. $2^{26} = 67108864$ fits on the calculator display, $2^{27}$ does not. (This assumes use of a calculator with an 8 digit display.)

2. a. Which do you think is larger $7^9$ or $9^7$? Estimate, then use the calculator.
   Student response: $7^9 = 40{,}353{,}607$ $9^7 = 4{,}782{,}696$ $7^9 > 9^7$

   b. Choose two different numbers for the base and power. Investigate, using your calculator, whether the smaller number as the base or the larger number as the base gives the greater answer. Record all results. Can you draw a conclusion?

   Student response should include examples showing that no conclusion can be reached. For example:

   $1^2 = 1$ is less than $2^1 = 2$
   $2^3 = 8$ is less than $3^2 = 9$
   $3^4 = 81$ is greater than $4^3 = 64$
   $4^5 = 1024$ is greater than $5^4 = 625$

3. Write the mathematical expressions that this calculator sequence will solve.

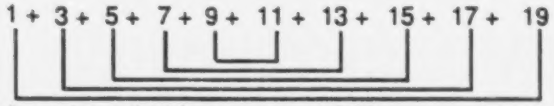
   5 x 1 = = = = = =

   Student response: $5^6$ or 5 x 5 x 5 x 5 x 5 x 5

4. Solve the following problem in as many different ways as you can. Explain all of your solutions.
   1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 =

   Student response:
   a. 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = $10^2$ = 100 by patterns from CAMP-LA lessons Strange Sequences and Ancient Oddity.

   b. Pairs of numbers that have a sum of 20.



   1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 5 x 20 = 100 by adding pairs of numbers that have a sum of 20.

   c. 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100 by normal addition.

   d. Pairing of numbers to simplify addition can also be used.

5. a. Choose 5 or more different 4-digit numbers. Complete the chart and answer the question below.

| 4-digit number | Number with digits reversed | Sum |
|---|---|---|
| | | |

b. What prime numbers are divisors of every one of the sums?

Student response should include a completed chart similar to the one below.

| 4-digit number | Number with digits reversed | Sum |
|---|---|---|
| 1234 | 4321 | 5555 |
| 7853 | 3587 | 11440 |
| 3579 | 9753 | 13332 |
| 9876 | 6789 | 16665 |
| 2468 | 8642 | 11110 |

6. Explain the numbers that appear in a calculator display when you do the following. Record, after each press of the equal sign.

a. 75 + 58 = = = = =
b. 1020 - 72 = = = = =
c. 1024 ÷ 2 = = = = =
d. 23 x 57 = = = =

Student response:

a. 75 + 58 = = = = = 133, 191, 249, 307, 365
When "=" is first pressed the calculator computes 75 + 58. Each additional time "=" is pressed, the calculator adds 58 to the number in the display.

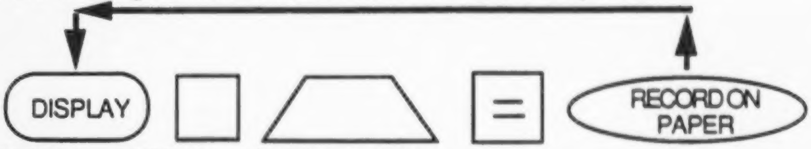
b. 1020 - 72 = = = = = 948, 876, 804, 732, 660
When "=" is first pressed the calculator computes 1020 - 72. Each additional time "=" is pressed, the calculator subtracts 72 to the number in the display.

c. 1024 ÷ 2 = = = = =
When "=" is first pressed the calculator computes 1024 ÷ 2. Each additional time "=" is pressed, the calculator divides the number in the display by 2.

d. 23 x 57 = = = =
When "=" is first pressed the calculator computes 23 x 57. Each additional time "=" is pressed, the calculator multiplies the number in the display by 23.

64

7. Fill in the missing flow chart items and continue the sequence.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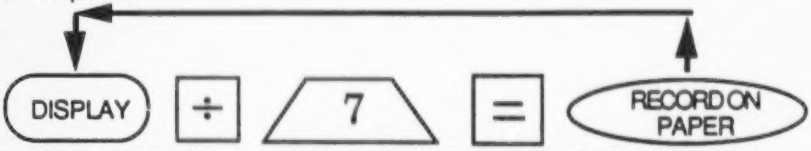
5764801. 823543. 117649. 16807. ____. ____. ____. ____. ____

Student response: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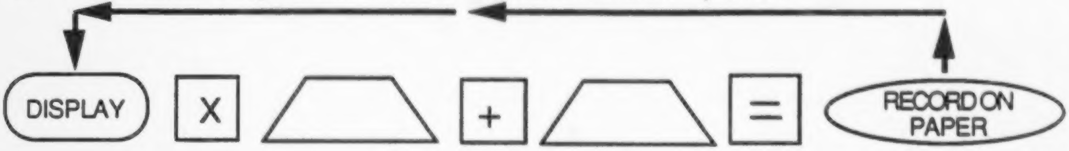
5764801. 8235543. 117649. 16807. 2401. 343. 49. 7. 1

8. Fill in the missing flow chart items and continue the sequence.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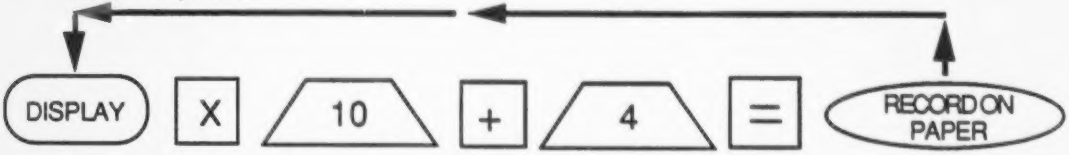
Student response: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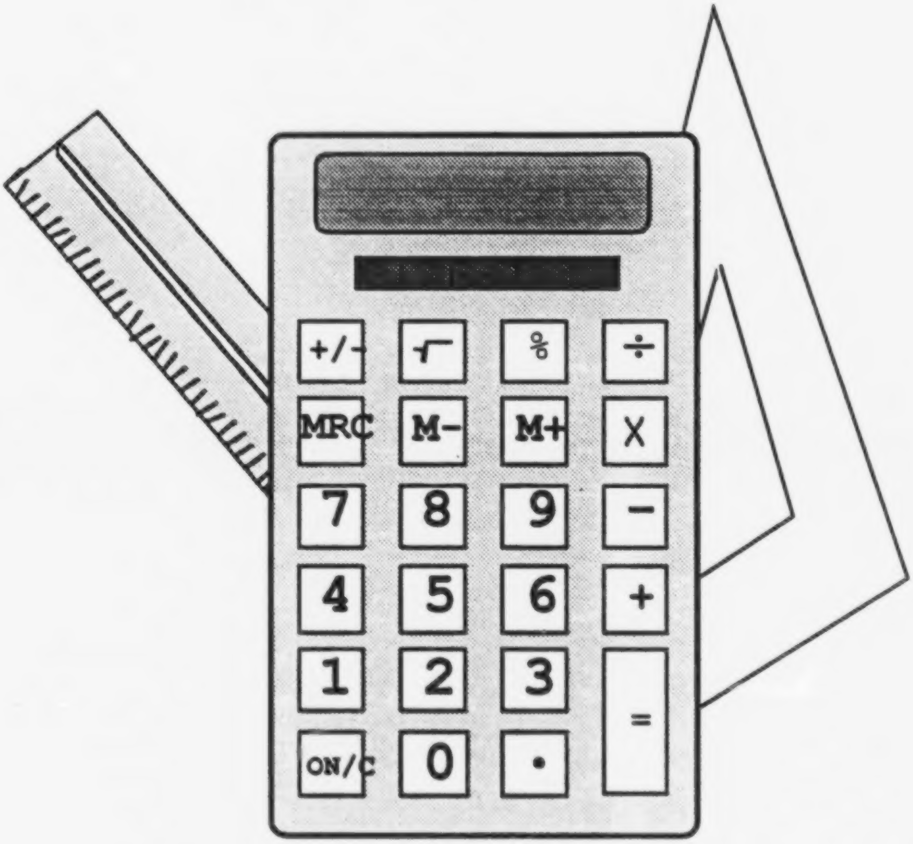
# CAMP-LA

**Calculators and Mathematics Project, Los Angeles**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## Grades 7 - 8

# CHAPTER 2
## LOGIC/ STATISTICS AND PROBABILITY

# GOING TO THE MOVIES

**GRADE:** 7 - 8

**STRAND:** Logic

**SKILL:** Organize and interpret data

**MANAGEMENT**

**CLASS ORGANIZATION:** Small group or pairs

**TIME FRAME:** One math period

**MATERIALS:** Calculator, overhead transparency

**PREREQUISITE SKILL:** Interpret data from a table

**LESSON**

- **DIRECTED INSTRUCTION:**

Tell the class this story. Some students from our school would like to take a field trip to the movies. Adults as well as children must attend. Adult tickets cost $5 and student tickets cost $3. Your job is to investigate what possible combinations of students and adults can attend if you must spend EXACTLY $300. Use an overhead transparency of Student Activity Sheet 1.

Ask, can these conditions be met if only 1 adult attends? Give them time to figure out that 1 adult ticket costing $5 would leave $295 for student tickets.

The answer to 295 ÷ 3 is not a whole number, so you can't spend exactly $295 on $3 student tickets.

There must be more than 1 adult. Ask, can there be exactly 2 adults? Give them time to work. Discuss that 2 adult tickets at $5 each leaves $290 for student tickets. 290 ÷ 3 is not a whole number so you can't spend exactly $290 on student tickets. There can't be 2 adults.
Ask, can there be exactly 3 adults? Allow time for students to work, then discuss that 3 adult tickets cost $15. There is $285 left for student tickets. 285 ÷ 3 = 95, so 95 student tickets could be purchased. Hand out the Student Activity Sheet 1. Tell them that the chart has already been filled in for 3 adults on the trip.

- **GUIDED PRACTICE:**

Ask students to see if there can be exactly 4, 5, 6 or more adults. Have them fill in successful solutions on their chart.
Ask students to find as many solutions as possible. Suggest that they look at the successful solutions on their chart to see if they can detect any patterns that will assist them in finding additional solutions. After they have spent sufficient time finding solutions, hand out Student Activity Sheet 2.

| Number of Adult tickets | Total Cost of Adult tickets | Number of Student Tickets | Total Cost of Student Tickets | Total Cost of All tickets |
|---|---|---|---|---|
| →5X→ | | →3X→ | | |
| 3 | $15 | 95 | $285 | $300 |
| 6 | $30 | 90 | $270 | $300 |
| 9 | $45 | 85 | $255 | $300 |
| 12 | $60 | 80 | $240 | $300 |
| 15 | $75 | 75 | $225 | $300 |
| 18 | $90 | 70 | $210 | $300 |
| 21 | $105 | 65 | $195 | $300 |
| 24 | $120 | 60 | $180 | $300 |
| 27 | $135 | 55 | $165 | $300 |
| 30 | $150 | 50 | $150 | $300 |
| 33 | $165 | 45 | $135 | $300 |
| 36 | $180 | 40 | $120 | $300 |
| 39 | $195 | 35 | $105 | $300 |
| 42 | $210 | 30 | $90 | $300 |
| 45 | $225 | 25 | $75 | $300 |
| 48 | $240 | 20 | $60 | $300 |
| 51 | $255 | 15 | $45 | $300 |
| 54 | $270 | 10 | $30 | $300 |
| 57 | $285 | 5 | $15 | $300 |

Ask students what patterns they observe in the chart. They should notice that in this form the first column increases by 3, the second by 15, the third decreases by 5, the fourth decreases by 15. They might say, as the student numbers get larger, the adult numbers get smaller. Give credit to any true observations.

- **INDEPENDENT PRACTICE:**

  Have students or groups of students complete Student Activity Sheet 2. Discuss the results with the class.

- **EVALUATION:**

  Have students or groups of students develop a similar situation. Write a chart recording all possible solutions, then write a set of conditions which narrows the choices down to a single solution.

## GOING TO THE MOVIES

Student Activity Sheet 2
Teacher Answer Sheet

(Hand out only after Page 1 has been completed and discussed)

| | Number of Adult tickets | Total Cost of Adult tickets | Number of Student Tickets | Total Cost of Student Tickets | Total Cost of All tickets |
|---|---|---|---|---|---|
| | →5X→ | | →3X→ | | |
| Z | 3 | $15 | 95 | $285 | $300 |
| Z | 6 | $30 | 90 | $270 | $300 |
| Z | 9 | $45 | 85 | $255 | $300 |
| Z | 12 | $60 | 80 | $240 | $300 |
| Z | 15 | $75 | 75 | $225 | $300 |
| | 18 | $90 | 70 | $210 | $300 |
| Y | 21 | $105 | 65 | $195 | $300 |
| Y | 24 | $120 | 60 | $180 | $300 |
| Y | 27 | $135 | 55 | $165 | $300 |
| Y | 30 | $150 | 50 | $150 | $300 |
| Y | 33 | $165 | 45 | $135 | $300 |
| X | 36 | $180 | 40 | $120 | $300 |
| X | 39 | $195 | 35 | $105 | $300 |
| X | 42 | $210 | 30 | $90 | $300 |
| X | 45 | $225 | 25 | $75 | $300 |
| X | 48 | $240 | 20 | $60 | $300 |
| X | 51 | $255 | 15 | $45 | $300 |
| X | 54 | $270 | 10 | $30 | $300 |
| X | 57 | $285 | 5 | $15 | $300 |

Use the data given below to eliminate possibilities from the chart.

1. There must be more students than adults. (Put an x in front of the rows for answers you must eliminate.)

2. The school must spend less than $100 on adult tickets. (Put a y in front of the rows for new answers you can eliminate.)

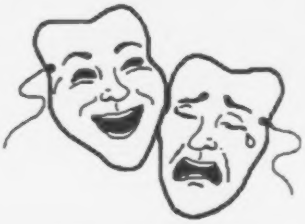
3. The bus holds only 89 passengers. (Put a z in front of the rows for new answers you can eliminate.)

4. How many adults and how many students are going on this trip.

   Adults 18 Students 70

5. Could you have arrived at his answer using only two clues above or were all three necessary? Only, clues 2 and 3 are needed.

Name ____________________



## GOING TO THE MOVIES

Student Activity Sheet 1

| Number of Adult tickets | Total Cost of Adult tickets | Number of Student Tickets | Total Cost of Student Tickets | Total Cost of All tickets |
|---|---|---|---|---|
| →5X→ | | →3X→ | | |
| 3 | $15 | 95 | $285 | $300 |
| | | | | $300 |
| | | | | $300 |
| | | | | $300 |
| | | | | $300 |
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## GOING TO THE MOVIES

Student Activity Sheet 2

| Number of Adult tickets | Total Cost of Adult tickets | Number of Student Tickets | Total Cost of Student Tickets | Total Cost of All tickets |
|---|---|---|---|---|
| →5X→ | | →3X→ | | |
| 3 | $15 | 95 | $285 | $300 |
| 6 | $30 | 90 | $270 | $300 |
| 9 | $45 | 85 | $255 | $300 |
| 12 | $60 | 80 | $240 | $300 |
| 15 | $75 | 75 | $225 | $300 |
| 18 | $90 | 70 | $210 | $300 |
| 21 | $105 | 65 | $195 | $300 |
| 24 | $120 | 60 | $180 | $300 |
| 27 | $135 | 55 | $165 | $300 |
| 30 | $150 | 50 | $150 | $300 |
| 33 | $165 | 45 | $135 | $300 |
| 36 | $180 | 40 | $120 | $300 |
| 39 | $195 | 35 | $105 | $300 |
| 42 | $210 | 30 | $90 | $300 |
| 45 | $225 | 25 | $75 | $300 |
| 48 | $240 | 20 | $60 | $300 |
| 51 | $255 | 15 | $45 | $300 |
| 54 | $270 | 10 | $30 | $300 |
| 57 | $285 | 5 | $15 | $300 |

Use the data given below to eliminate possibilities from the chart.

1. There must be more students than adults. (Put an **x** in front of the rows for answers you must eliminate.)

2. The school must spend less than $100 on adult tickets. (Put a **y** in front of the rows for new answers you can eliminate.)

3. The bus holds only 89 passengers. (Put a **z** in front of the rows for new answers you can eliminate.)

4. How many adults and how many students are going on this trip?

   Adults ____________________ Students ____________________

5. Could you have arrived at this answer using only two clues above or were all three necessary? ________________________________________

   ________________________________________

## NUMBER CLAIM

**GRADE:** 7 - 8

**STRAND:** Logic

**SKILL:** Practice computational operations +, -, x, ÷

### MANAGEMENT

**CLASS ORGANIZATION:** Small group

**TIME FRAME:** One math period

**MATERIALS:** Calculator

**VOCABULARY:** Digits, numeral, power, factorials

**PREREQUISITE SKILL:** Basic operations

### LESSON

- **DIRECTED INSTRUCTION:**

You are presented with a 4-digit number. Using the operations +, -, x, ÷, and the 4 digits of the number, the goal of your team is to "claim" as many numbers as you can on the Solution Tally Sheet.

**Rules:**

1. Form a mathematical expression using all 4 of the digits given.
2. You may not use a digit more than once unless it is used in the given number more than once.
3. You may use the operational symbols as few or as many times as necessary.

**Record Keeping:**

1. Write down the solutions you created for the numbers you wish to claim.
2. Write your initials next to each solution on the Solution and Scoring Sheet.
3. Initial each square you claim on the Solution Tally Sheet.
4. After 15 minutes we will stop and evaluate team progress.

**(Optional) Scoring:**

Teams earn 5 points for each number claimed. Teams earn 5 bonus points for each number claimed that no other team has.

- **GUIDED PRACTICE:**

The 4-digit numeral used in examples 1-4 is 1498.

**Example 1.** Write the mathematical expression:
$1 \boxed{+} 9 \boxed{+} 4 \boxed{+} 8 \boxed{=} 22$ Verify the solution. Explain to students how they are to record this information.

**Example 2.** $1 \boxed{\times} 9 \boxed{+} 4 \boxed{-} 8 \boxed{=} 5$ Verify, record and initial.

**Example 3.** 98 $\boxed{-}$ 14 $\boxed{=}$ 84 . Notice the same digits can be used to create 2 and 3 digit numerals.

**Example 4.** 8 $\boxed{x}$ 9 $\boxed{x}$ 4 $\boxed{\div}$ 1 $\boxed{=}$ 288 but that's not a number on the Solution Tally. Mental math and estimation lead to the conclusion that the answer would be too big for the chart.

Write down some other mathematical expressions and numbers generated. List solutions. (Allow 5-10 minutes and circulate to observe and assist.) Ask if there are any questions before handing out a Solution and Scoring Sheet and a Solution Tally Sheet to each team.

- **INDEPENDENT PRACTICE:**

  Hand out a Solution Tally Sheet and a Solution and Scoring Sheet to each team. Announce a four-digit numeral. You may wish to pick a date in history or the current year.

  Say, "Now work with other members of your group to claim as many numbers as you can in the next 15 to 30 minutes".

- **EVALUATION:**

  Call off numbers from the hundreds chart and have one student from each group verify the claim to that number (if using scoring option, have teams record scores).

  Cross off numerals on the overhead transparency as students cross them off on their team sheet.

  Ask people to share processes and strategies.

- **EXTENSION:**

  Have students make a list of numbers not claimed.

  Allow 10-15 minutes at the beginning of the next period to see if any additional solutions have been found.

  - Extension and/or Variations

    Teach $\sqrt{\phantom{x}}$, powers and factorials. Continue the process allowing these new operational symbols.

    Examples: $\sqrt{16} = 4$ $\qquad 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

    $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ (4 fractorial = 24)

  - Introduce INT concept from computer programming: INT concept takes any number and drops the decimal part.

    Examples: INT(8.75) = 8

    INT (3÷2) = INT (1.5) = 1

# SOLUTION TALLY SHEET

_____ _____ _____ _____

given numeral

TEAM MEMBERS:

____________________

____________________

____________________

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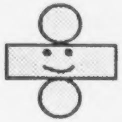
|  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |



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SQUARE AS YOU
CLAIM IT.

# SOLUTION AND SCORING SHEET

| | PROBLEM | I | PTS | BONUS | | PROBLEM | I | PTS | BONUS | | PROBLEM | I | PTS | BONUS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | 23 | | | | | 45 | | | | |
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| 6 | | | | | 28 | | | | | 50 | | | | |
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| 21 | | | | | 43 | | | | | 65 | | | | |
| 22 | | | | | 44 | | | | | 66 | | | | |

TEAM MEMBERS

TOTAL

## SOLUTION AND SCORING SHEET

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TEAM MEMBERS

TOTAL

## M AND M AND M

**GRADE:** 7 - 8

**STRAND:** Probability and Statistics

**SKILL:** Determine the mean, median and mode from a set of data.

### MANAGEMENT

**CLASS ORGANIZATION:** Individuals or groups

**TIME FRAME:** One or two math periods

**MATERIALS:** Calculator

**VOCABULARY:** Measures of central tendency: average, mean, median, mode

**PREREQUISITE SKILL:** Basic operations

### LESSON

- **DIRECTED INSTRUCTION:**
  - Hand out Student Activity Sheet 1. Without further explanation, ask the class to look at the prices of the cars for sale and answer question 1. Discuss student answers to questions 1.
  - Introduce the three different measures of central tendency: mean, median and mode. Work through part 1, problems 2-4 with the class, using the following information.
  - Discuss the fact that the median and mode are important measures of central tendency, even though the mean is most commonly used.
  - Use this formula to solve part 1, #2

**MEAN** (average) Add the numbers. Divide this total by the number of addends.

(7495 + 7250 + 7250 + 9000 + 7995) ÷ 5 = $7798.

Note: Students can use the M+ key to total the scores, the MCR ÷ 5 to find the mean.

Use this formula to solve part 1, #3

**MEDIAN** - List the numbers in order from least to greatest or greatest to least. If there is an ODD number of addends, the median is the middle number in the list:

7250, 7250, 7495, 7995, 9000

The median is 7495.

If there is an even number of addends, add the two middle numbers and divide their sum by two to find the median:

(7250, 7250, (7495, 7995), 9000, 9050)

7495 + 7995 = 15,490

15,490 ÷ 2 = 7745

The median is 7745

- Use this formula to solve part 1 #4

**MODE** - Identify the number which occurs more often than any other number in the list.
(Sometimes there is no mode and sometimes there is more than one)
7250, 7250, 7495, 7995, 9000
The mode is 7250 because it occurs more often than any other number in the list.

- **GUIDED PRACTICE:**

  Have students complete parts 2 and 3 of Student Activity Sheet 1. Upon completion, discuss answers. Let students justify their choice on #4. Give credit to ALL logical responses. Discuss the effect of the Ferrari price on the mean:
  Note: the Ferrari not only raises the mean, but also distorts it to a point where the mean may not necessarily be the best number to describe the average.
  Discuss student observations in part 3.

- **INDEPENDENT PRACTICE:**

  Hand out Student Activity Sheet 2. Have students complete parts 1 and 2 and then discuss their answers with the class.

- **HOME ACTIVITY:**

  Hand out Student Activity Sheet 3 to be completed as homework. In order to complete the assignment, students must collect data for 1 week and compute the mean, median, and mode.
  An optional activity would be to compute the class or group mean, mode, or median from their data.

- **EXTENSION:**

  Additional ideas for extension activities:
  1. Graph all data from the Student Activity Sheets in this lesson.
  2. Use the almanac to find the population of states beginning with the letter "w". Compute the mean, median and mode.
  3. Use the almanac to find the population of states beginning with the same letter as your state. Compute the mean, median and mode.
  4. Ask students how they could use a calculator to find the mean for the data below since these numbers have too many digits to fit in a calculator display.

  14,000,000 : 11,500,000,000 : 13,200,000,000 : 730,000,000

  Let them attempt the problem and then explain to you how they handled the zeroes in order to enable them to use the calculator.
  For example: they may say they removed a certain number of zeros from the end of each number and placed them back later.

## M AND M AND M

### Student Activity Sheet 1
### Teacher Answer Sheet

Looking at the cost of used cars in the classified section of the newspaper, you find that the following 1983 cars are listed:

| | | |
|---|---|---|
| 1983 | Chrysler | $7495 |
| 1983 | Ford Mustang | $7250 |
| 1983 | Ford T-Bird | $7250 |
| 1983 | Cadillac Sedan De Ville | $9000 |
| 1983 | Volkswagon | $7995 |

**Part 1**

1. Based on this information, about how much would you say a 1983 car costs? Student answers will vary.
2. What is the mean (average) price of the cars? $7798
3. What is the median price of the cars? $7495
4. What is the mode price of the cars? $7250

**Part 2**

The next day you look at the paper and find that the Volkswagon advertisement is no longer there, it has been replaced by an advertisement for a 1983 Ferrari selling for $75,000. To compute the following measure of central tendency substitute the 1983 Ferrari Price for the 1983 Volkswagon price. (Hint, if you used the M + key to total the car prices, use the M - key to subtract the VW price, then M + key to add the Ferrari price.)

1. What is the new mean? $21,199
2. What is the new median? $7495
3. What is the new mode? $7250
4. Which measure of central tendency (mean, median, or mode) best describes the data in this example? Why?

Give credit to any response as long as it has a logical rationale.

**Part 3**

Compare your Part 1 and Part 2 answers. What do you observe? Why did this happen?

The mean is higher but the median and mode remain the same. Give credit to any logical response.

# M AND M AND M

Student Activity Sheet 2
Teacher Answer Sheet

**Part I**
Teachers often use some measure of central tendency to compute grades.

If your math scores were 5%, 5%, 90%, 91%, 92%, 92%, and 92%, then:

1) What is your mean score to the nearest %? 67%

2) What is your median score to the nearest %? 91%

3) What is your mode score to the nearest %? 92%

4) If the teacher's grading scale is given by: 90-100 A, 80-89 B, 70-79 C, 60-69 D, what grade would the teacher probably give you?

mean D
median A-
mode A

5) Which average best describes the data? Why?

Allow all responses that are given with a reasonable justification.

**Part 2**
Choose 5 numbers to fit the following situations:

A. The mean is larger than the mode or median.
Answers will vary: An example is 1, 1, 1, 1, 2.

B. The median is larger than the mean or mode.
Answers will vary: An example is 1, 1, 5, 6, 7.

C. The mode is larger than the median or mean.
Answers will vary: An example is 1, 2, 3, 9, 9.

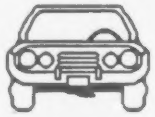
Name ____________________

## M AND M AND M

Student Activity Sheet 1

Looking at the cost of used cars in the classified section of the newspaper, you find that the following 1983 cars are listed:

| | | |
|---|---|---|
| 1983 | Chrysler | $7495 |
| 1983 | Ford Mustang | $7250 |
| 1983 | Ford T-Bird | $7250 |
| 1983 | Cadillac Sedan De Ville | $9000 |
| 1983 | Volkswagon | $7995 |



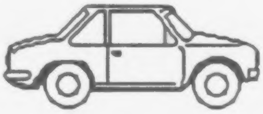
**Part 1**

1. Based on this information, about how much would you say a 1983 car costs? ____________________
2. What is the mean (average) price of the cars? __________
3. What is the median price of the cars? __________
4. What is the mode price of the cars? __________

**Part 2**

The next day you look at the paper and find that the Volkswagon advertisement is no longer there. It has been replaced by an advertisement for a 1983 Ferrari selling for $75,000. To compute the following measure of central tendency substitute the 1983 Ferrari Price for the 1983 Volkswagon price. (Hint, if you used the $\boxed{\text{M+}}$ key to total the car prices, use the $\boxed{\text{M-}}$ key to subtract the VW price, then $\boxed{\text{M+}}$ key to add the Ferrari price.)

1. What is the new mean? __________
2. What is the new median? __________
3. What is the new mode? __________



4. Which measure of central tendency (mean, median, or mode) best describes the data in this example? Why?

____________________________________________

____________________________________________

**Part 3**

Compare your Part 1 and Part 2 answers. What do you observe? Why did this happen?

____________________________________________

____________________________________________

____________________________________________

Name ____________________

## M AND M AND M

Student Activity Sheet 2

**Part I**

Teachers often use averages to compute grades.
Each test was worth 100 points.
If your math scores were 5%, 5%, 90%, 91%, 92%, 92%, and 92% then:

1) What is your mean score to the nearest %?

2) What is your median score to the nearest %?

3) What is your mode score to the nearest %?

4) If the teacher's grading scale is given by: 90-100 A, 80-89 B, 70-79 C, 60-69 D, what grade would the teacher probably give you?

mean ____________

median ____________

mode ____________

5) Which average best describes the data? Why?

________________________________________

________________________________________

________________________________________

________________________________________

**Part 2**

With the aide of your calculator:

Choose 5 numbers to fit the following situation:

A. The mean is larger than the mode or median.

__________ __________ __________ __________ __________

mean = __________ median = __________ mode = __________

B. The median is larger than the mean or mode.

__________ __________ __________ __________ __________

mean = __________ median = __________ mode = __________

C. The mode is larger than the median or mean.

__________ __________ __________ __________ __________

mean = __________ median = __________ mode = __________

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Name____________________

## M AND M AND M
Student Activity Sheet 3

Record the time spent eating breakfast, watching TV, and studying for 1 week.

| Record in Minutes 1 hour = 60 minutes | SUN | MON | TUES | WED | THURS | FRI | SAT | TOTAL MIN. |
|---|---|---|---|---|---|---|---|---|
| Time Spent Eating Breakfast | | | | | | | | |
| Time Spent Watching TV | | | | | | | | |
| Time Spent Studying | | | | | | | | |

At the end of the week, compute and record the mean, median, and mode for each row of data above, and record in the chart below.

| | MEAN | MEDIAN | MODE |
|---|---|---|---|
| Time Spent Eating Breakfast | | | |
| Time Spent Watching Television | | | |
| Time Spent Studying | | | |

If your mother asked you how many minutes a day you watch television, which would you chose to tell her, the mean, median or mode? ________________ Why?_______

______________________________________________________________

If your teacher asked you how many minutes a day you study, which would you report, the mean, median or mode? ____________Why?_________________________

______________________________________________________________

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# PICTURE THIS

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Logic |
| **SKILL:** | Investigate possible arrangements (permutations). |

## MANAGEMENT

| | |
|---|---|
| **CLASS ORGANIZATION:** | Individuals or pairs |
| **TIME FRAME:** | Two math periods |
| **MATERIALS:** | Calculator, Picture Sheets 1, 2, 3, scissors |
| **VOCABULARY:** | Combinations |

## LESSON

- **DIRECTED INSTRUCTION:**
  - Hand out picture sheets 1 and 2 **only**. Have students fold them at the solid line, then cut the pages along the dashed lines stopping at the fold. Place picture sheet 1 on top of picture sheet 2 .
  - Hand out Student Activity Sheet 1. Ask students to record how many different picture arrangements they can make. They may fold back a head, a midsection, or a bottom, or leave it in place in order to make their composite picture. Give them time to experiment.

POSSIBILITIES FOR CREATING A PICTURE.

| | Head | Midsection | Bottom |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 2 |
| 3 | 1 | 2 | 1 |
| 4 | 1 | 2 | 2 |
| 5 | 2 | 1 | 1 |
| 6 | 2 | 1 | 2 |
| 7 | 2 | 2 | 1 |
| 8 | 2 | 2 | 2 |

Note there are 2 possibilities for a head, 2 possibilities for a midsection, and 2 possibilities for a bottom.

There are 2 x 2 x 2 or 8 possible pictures arrangements.

After discussion, give students time to fill in missing combinations before going on.

- **GUIDED PRACTICE:**

  Hand out Picture Sheet 3 and have students place it behind Picture Sheets 1 and 2. Ask what picture combinations are now possible. Students record the possibilities on Student Activity Sheet 1. They should develop a strategy for listing all combinations. One plan would be to begin with Head #1, Midsection #1 and the various possible Bottoms (#1, #2, #3). Next, change to Midsection #2 combined with the Bottoms (#1, #2, #3). Change to Head #2 only after all Midsections have been used.

Possibilities for creating a picture.

| | head | midsection | bottom |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 2 |
| 3 | 1 | 1 | 3 |
| 4 | 1 | 2 | 1 |
| 5 | 1 | 2 | 2 |
| 6 | 1 | 2 | 3 |
| 7 | 1 | 3 | 1 |
| 8 | 1 | 3 | 2 |
| 9 | 1 | 3 | 3 |
| 10 | 2 | 1 | 1 |
| 11 | 2 | 1 | 2 |
| 12 | 2 | 1 | 3 |
| 13 | 2 | 2 | 1 |
| 14 | 2 | 2 | 2 |
| 15 | 2 | 2 | 3 |
| 16 | 2 | 3 | 1 |
| 17 | 2 | 3 | 2 |
| 18 | 2 | 3 | 3 |
| 19 | 3 | 1 | 1 |
| 20 | 3 | 1 | 2 |
| 21 | 3 | 1 | 3 |
| 22 | 3 | 2 | 1 |
| 23 | 3 | 2 | 2 |
| 24 | 3 | 2 | 3 |
| 25 | 3 | 3 | 1 |
| 26 | 3 | 3 | 2 |
| 27 | 3 | 3 | 3 |

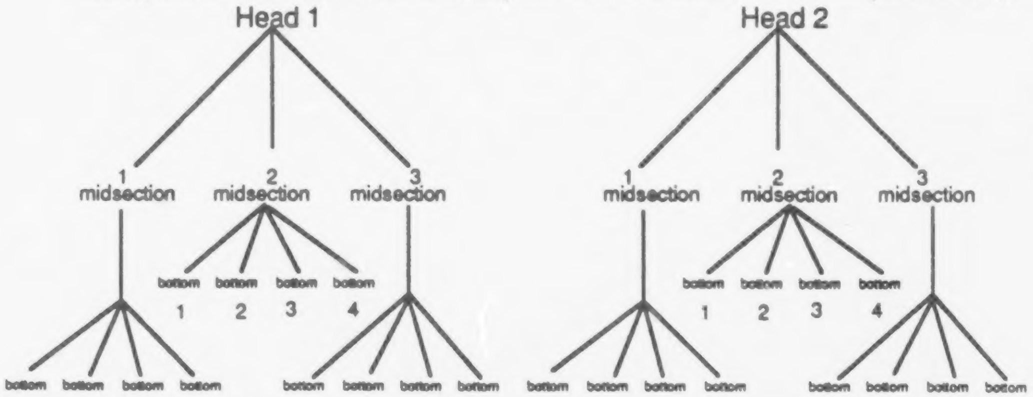
Note: There are now 3 possibilities for a head, 3 possibilities for a midsection, and 3 possibilities for a bottom.

$3 \times 3 \times 3 = 27$

There are 27 possible pictures arrangements.

Discuss that if you know how many possibilities there are for choosing a head, a midsection, and a bottom you can multiply those three numbers to find the total number of picture arrangements possible.

Example: If there were **2 heads, 3 midsections**, and **4 bottoms** there would be $2 \times 3 \times 4 = 24$ picture possibilities. This is true since each of the 2 heads can be joined with each of 3 midsections ($2 \times 3 = 6$ ways) and each of those 6 arrangements can be attached to any of the 4 bottoms ($6 \times 4 = 24$ possibilities).


Head 1
1 midsection
2 midsection
3 midsection
bottom 1
bottom 2
bottom 3
bottom 4
bottom
bottom
bottom
bottom
bottom
bottom
bottom
bottom
Head 2
1 midsection
2 midsection
3 midsection
bottom 1
bottom 2
bottom 3
bottom 4
bottom
bottom
bottom
bottom
bottom
bottom
bottom
bottom

- **INDEPENDENT PRACTICE:**
  - Hand out Student Activity Sheet 2. Remind students that the number of possible heads x number of possible midsections x number of possible bottoms = number of picture arrangements.
  - Tell students that a publisher wants them to create a flip book of people pictures.

1. They don't have to have the same number of choices for heads, midsections, or bottoms.
2. They need at least 1 head, at least 1 midsection, and at least 1 bottom.
3. The number of heads plus the number of midsections plus the number of bottoms must equal exactly 100. Different combinations will yield a different number of picture arrangements.

Using the calculator, students explore how to create the maximum and then the minimum number of picture arrangements.

An example of using 100 choices is listed on first line of Student Activity Sheet 2.

After selecting the number of heads, midsections and bottoms be sure they total 100. There are 20 x 20 x 60 = 24,000 arrangements . Remind students to analyze each answer by comparing it to previous answers to help them arrive at the best response.

Let students have time to experiment. Do not give answers to questions until they have made some discoveries themselves.

- **EXTENSION:**

In question 3, after making the generalization that the maximum number is created by the most even split of the choices (e.g. 33, 33, 34), and the minimum by the most lopsided split of the choices (98, 1, 1), ask them to apply this generalization to the following totals for possible picture arrangements.

**150** possible picture arrangements [Maximum: 50 x 50 x 50 = 125,000; Minimum: 1 x 1 x 148 = 148]
**90** possible picture arrangements [Maximum: 30 x 30 x 30 = 27,000; Minimum: 1 x 1 x 88 = 88]
**200** possible picture arrangements [Maximum: 66 x 67 x 67 = 291,852; Minimum: 1 x 1 x 198 = 198]
**1000** possible picture arrangements [Maximum: 333 x 333 x 334 = 37,036,926; Minimum: 1 x 1 x 998 = 998]

Name ____________________

## PICTURE THIS
Student Activity Sheet 1

List possible picture arrangements for picture sheets 1 and 2.

| | Head | Midsection | Bottom |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |
| 12 | | | |

COMBINING PICTURE SHEETS 1 AND 2

_______ Possibilities for "Heads"

_______ Possibilities for "Midsections"

_______ Possibilities for "Bottoms"

There are __________ possible pictures.

COMBINING PICTURE SHEETS 1, 2 AND 3

_______ Possibilities for "Heads"

_______ Possibilities for "Midsections"

_______ Possibilities for "Bottoms"

There are __________ possible pictures.

List possible picture arrangements for picture sheets 1, 2, and 3.

| | Head | Midsection | Bottom |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |
| 12 | | | |
| 13 | | | |
| 14 | | | |
| 15 | | | |
| 16 | | | |
| 17 | | | |
| 18 | | | |
| 19 | | | |
| 20 | | | |
| 21 | | | |
| 22 | | | |
| 23 | | | |
| 24 | | | |
| 25 | | | |
| 26 | | | |
| 27 | | | |

Name ________________

## PICTURE THIS

Student Activity Sheet 2

| Number of Heads | Number of Midsections | Number of Bottoms | Total | Number of possible picture arrangements |
|---|---|---|---|---|
| 20 | 20 | 60 | 100 | 24,000 |
| | | | 100 | |
| | | | 100 | |
| | | | 100 | |
| | | | 100 | |
| | | | 100 | |
| | | | 100 | |
| | | | 100 | |
| | | | 100 | |
| | | | 100 | |
| | | | 100 | |
| | | | 100 | |

1. Which distribution gives the maximum number of possible picture arrangements?

   ________________________________________

   What do you observe about this solution? ________________________

   ________________________________________

2. Which distribution(s) give the minimum number of possible picture arrangements?

   ________________________________________

   What do you observe about this solution? ________________________

   ________________________________________

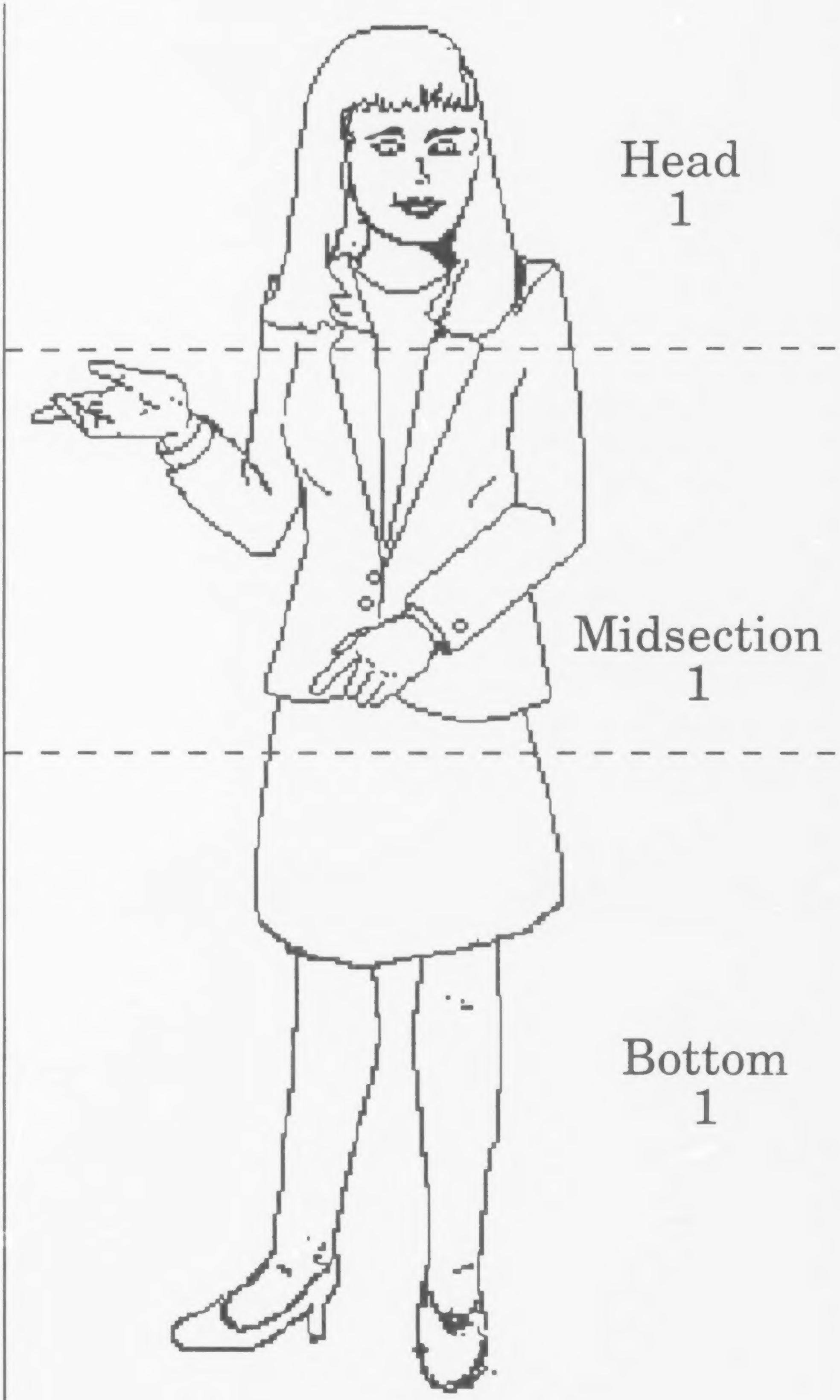
3. If you were given a different number of choices, how would you arrange them to get the maximum and minimum number of possible picture arrangements? ____________

   ________________________________________

Picture Sheet 1

CUT ALONG DASHED LINE

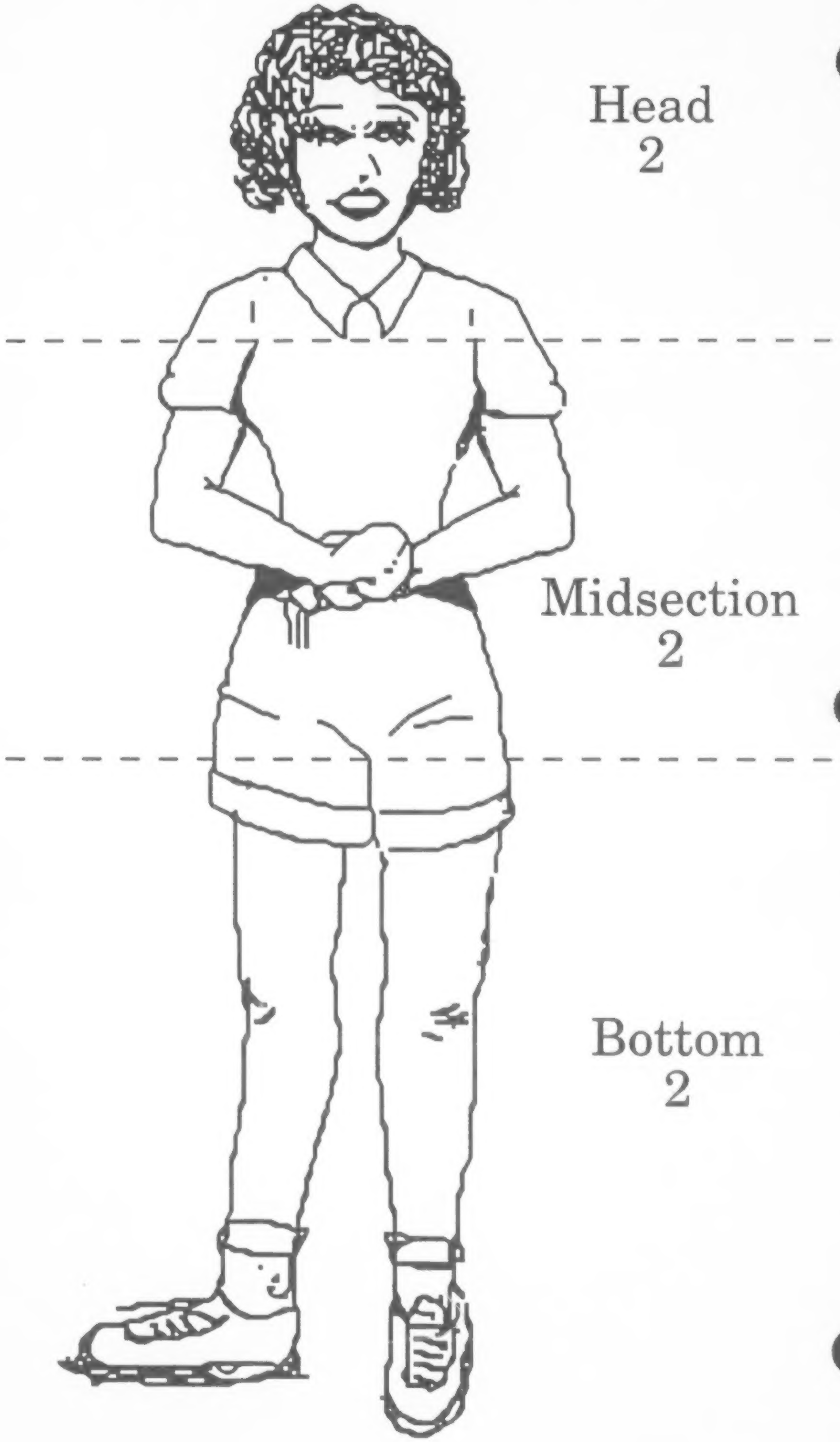
BEND ON SOLID LINE


Head
1
Midsection
1
Bottom
1

Picture Sheet 2

CUT ALONG DASHED LINE

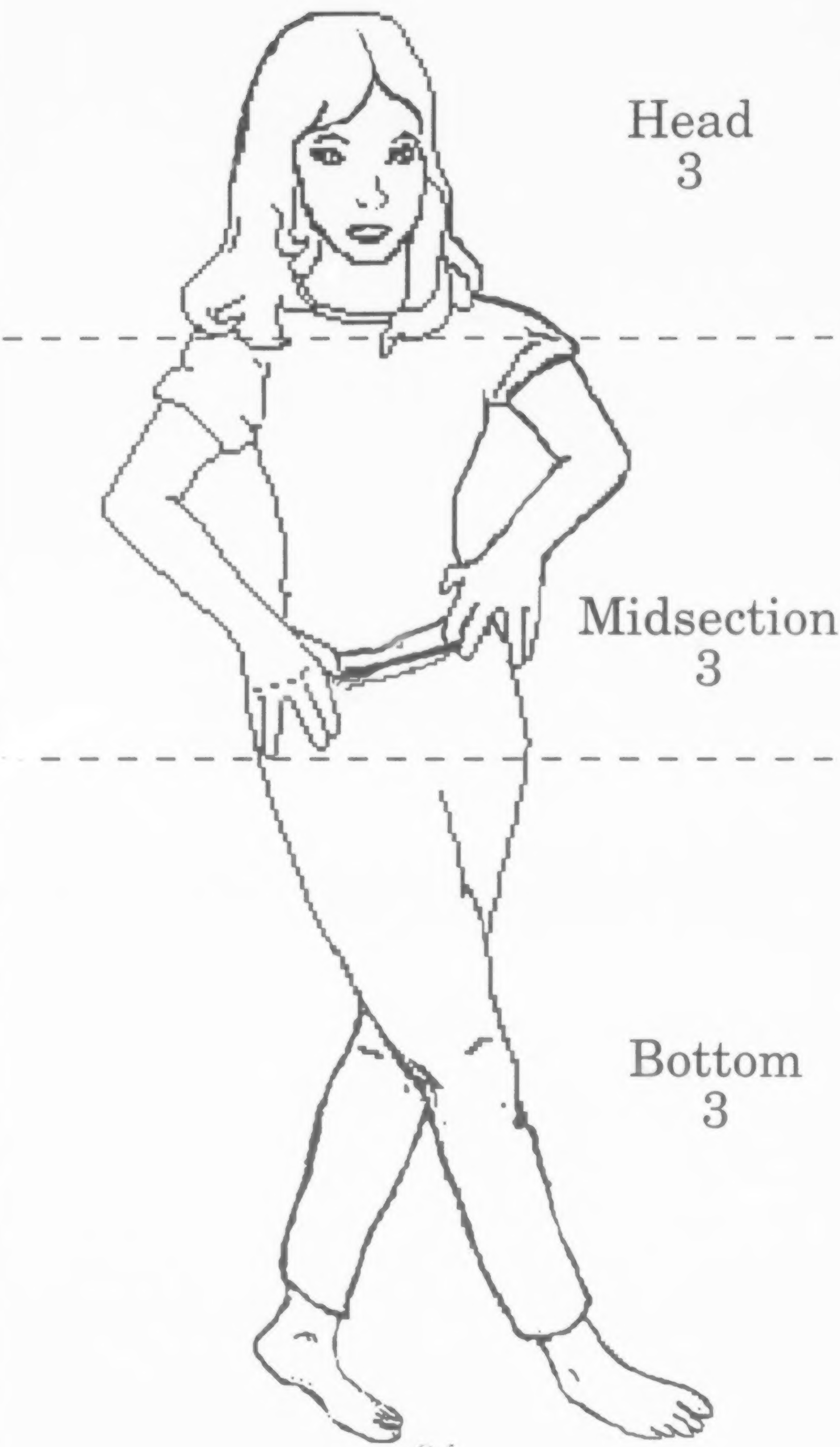
BEND ON SOLID LINE


Head
2
Midsection
2
Bottom
2

Picture Sheet 3

CUT ALONG DASHED LINE

BEND ON SOLID LINE


Head
3
Midsection
3
Bottom
3

# IT'S ALL IN THE GAME

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Logic |
| **SKILL:** | Organize information in a circle graph, convert fractions to percent. |
| **MANAGEMENT**<br>**CLASS ORGANIZATION:** | Pairs or cooperative groups of 4 |
| **TIME FRAME:** | One or two math periods |
| **MATERIALS:** | Calculator, protractor, compass, ruler, gameboards for each group, or transparency of gameboard |
| **VOCABULARY:** | Sector, central angle, radius, protractor, category |
| **PREREQUISITE SKILL:** | Measure angles using protractor |

## LESSON

- **DIRECTED INSTRUCTION/GUIDED PRACTICE:**

Part 1:
- Distribute Student Activity Sheet 1 after a discussion of the properties of the gameboard. Use an overhead transparency of the gameboard.
- A MONOPOLY gameboard is made up of many spaces on which you could land. Depending on where you land, you can buy property, utilities, or railroads, draw a card, or get a penalty.
- Hand out Student Activity Sheet 1. Have students complete Part 1. Discuss.

Part 2:
- What part of the MONOPOLY gameboard is taken up by the various types of spaces? A circle graph will be used to compare the portions of the gameboard taken up by each category.
- Read the directions for Part 2.
- Students complete Part 2 independently.

Part 3:
- Read directions and guide students to complete part 3. Use the percent key.

Part 4:
- Review how to use a protractor. Students complete part 4 graphing independently.

- **INDEPENDENT PRACTICE:**
  - Hand out Student Activity Sheet 2. Students complete the sheet independently.

- **EXTENSION:**
  - Hand out Extension Activity Sheet.
  - Create a circle graph to show the portion of the gameboard occupied by various categories.
  - Use the format above.

Option:
- Students may choose a different gameboard and use the format above to create a circle graph.

## IT'S ALL IN THE GAME
Student Activity Sheet 1
Teacher Answer Sheet

Part 1: Look at the MONOPOLY gameboard. How many total spaces are there to land on? 40 Now count and record the number of different types of spaces there are.

| Number | Category |
|---|---|
| 4 | Railroads |
| 22 | Property (on which you can build) |
| 2 | Utilities |
| 6 | Pick a Card (Chance Or Community Chest) |
| 3 | Penalty and tax (includes go to jail) |
| 1 | GO (collect $200) |
| 2 | Free Parking and Just Visiting |
| Total 40 | |

Part 2. To create a circle graph you need to know what percent of the total gameboard is taken up by the various types of spaces. You can use your calculator to determine percent as follows:

$$\text{PERCENT} = \frac{\text{number of spaces in a category} \times 100}{\text{total number of spaces}}$$

(Round all answers to the nearest tenth)

Use this method to compute the percent for each category and record your answers below:

| Percent | Category |
|---|---|
| 10 % | Railroads |
| 55 % | Property (on which you can build) |
| 5 % | Utilities |
| 15 % | Pick a Card (Chance Or Community Chest) |
| 7.5 % | Penalty and tax (includes go to jail) |
| 2.5 % | GO (collect $200) |
| 5 % | Free Parking and Just Visiting |
| Total 100 % | |

Check to see that the total is 100%.

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## IT'S ALL IN THE GAME

Student Activity Sheet 1 page 2
Teacher Answer Sheet

Part 3. Copy the percents from Part 2 into the Percent column of the chart below. You now can compute the information necessary to design a circle graph. Some things you need to know:

1. A circle contains 360°.
2. To determine how many degrees for each category, multiply 360° by the percent taken up by that category.

Steps:

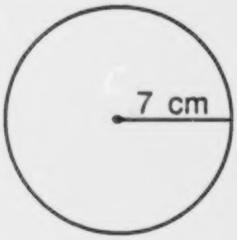
- Enter 360 on your calculator.
- Press $\boxed{x}$ key.
- Press the percent for Railroads.
- Press the $\boxed{\%}$ key.
- Enter this number, rounded to the nearest whole degree.
- Repeat this process for the other categories.

Enter the degrees per category below:

| Percent | Degrees | Category |
|---|---|---|
| 10 % | 36 | Railroads |
| 55 % | 198 | Property (on which you can build) |
| 5 % | 18 | Utilities |
| 15 % | 54 | Pick a Card (Chance Or Community Chest) |
| 7.5 % | 27 | Penalty and tax (includes go to jail) |
| 2.5 % | 9 | GO (collect $200) |
| 5 % | 18 | Free Parking and Just Visiting |
| 100 % | Total 360° | |

Add the degrees in each category. They should total approximately 360°. If not, check your calculations.

Part 4. 1. Use your compass to draw a circle with a radius of approximately 7 cm. Draw a radius from the center of the circle.



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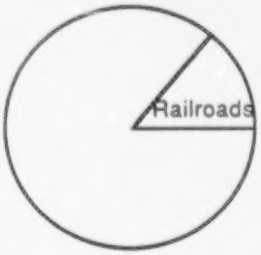
### IT'S ALL IN THE GAME

Student Activity Sheet 1 page 3
Teacher Answer Sheet

2. Lay the protractor along this radius and mark off the degrees you calculated for Railroads. Draw another radius at this point.



You have created a **central angle,** (an angle whose vertex is the center of the circle) that is equal to the degrees taken up by Railroads.

3. Lay the protractor along the radius you just drew. Mark the degrees for the next category, Property. Draw another radius. Label the sector appropriately.
4. Continue creating angles for all the categories listed in Part 3.
5. When the circle graph is complete, give it a title. Summarize results below.

The largest sector is taken up by __Property______________

The smallest sector is taken up by __Go__________________

Two sectors that are the same size are _Utilities_ and _Free Parking & Just Visiting.

6. Write two statements about your circle graph.

______Answer Will Vary________________________________

______________________________________________________

______________________________________________________

______________________________________________________

______________________________________________________

# IT'S ALL IN THE GAME

Student Activity Sheet 2
Teacher Answer Sheet

In a MONOPOLY game there are various amounts of money.

| Denomination | Number of Bills | Percent | Degrees |
|---|---|---|---|
| a. Five hundreds | 20 | 10% | 36 |
| b. Hundreds | 30 | 15% | 54 |
| c. Fifties | 40 | 20% | 72 |
| d. Tens | 50 | 25% | 90 |
| e. Fives | 60 | 30% | 108 |
| | Total 200 | Total 100% | Total 360° |

1. Complete the chart above.
2. Use the information to create a circle graph.
3. When the circle graph is completed, give it a title.
4. Write two statements about your circle graph.

# IT'S ALL IN THE GAME
## Extension Activity Sheet
## Teacher Answer Sheet

In the game SCRABBLE, you build words on a gameboard. There are 225 total spaces on the board.

1. There are five different types of spaces as follows:
   The center square counts as a double word score.

| Number of Spaces | Category |
|---|---|
| 24 | double letter |
| 12 | triple letter |
| 16 | double word |
| 8 | triple word |
| 165 | no value |
| Total 225 | |

2. Record the number of spaces for each category on the line provided.
3. Use this information to create a circle graph as you did with the MONOPOLY game. Remember to first convert the number of spaces for each category to a percent of the total number of spaces.

$$\text{PERCENT} = \frac{\text{number of spaces in a category} \times 100}{\text{total number of spaces}}$$

(Round all answers to the nearest percent)

Record percents for each category on chart below.

4. Find out how many degrees of the circle to use for each sector. Record on chart below.

Degrees = 360 [x] [percent for category] [%]

(Round all answers to the nearest degree)

| Percent | Degrees | Category |
|---|---|---|
| 11 % | 40° | double letter |
| 5 % | 18° | triple letter |
| 7 % | 25° | double word |
| 4 % | 14° | triple word |
| 73 % | 263° | no value |
| Total 100 % | Total 360° | |

5. Draw the circle graph and give it a title.

6. Write two statements about your circle graph. ______________________

______________________________________________

Name ____________________

## IT'S ALL IN THE GAME

Student Activity Sheet, page1

Part 1: Look at the MONOPOLY gameboard. How many total spaces are there to land on? _________ Now count and record the number of different types of spaces there are.

| Number | Category |
|---|---|
| | Railroads |
| | Property (on which you can build) |
| | Utilities |
| | Pick a Card (Chance Or Community Chest) |
| | Penalty and tax (includes go to jail) |
| | GO (collect $200) |
| | Free Parking and Just Visiting |
| Total | |

Part 2. To create a circle graph you need to know what percent of the total game board is taken up by the various types of spaces. You can use your calculator to determine percent as follows:

$$\boxed{\text{PERCENT} = \frac{\text{number of spaces in a category} \times 100}{\text{total number of spaces}}}$$

(Round all answers to the nearest tenth)

Use this method to compute the percent for each category and record your answers below:

| Percent | Category |
|---|---|
| % | Railroads |
| % | Property (on which you can build) |
| % | Utilities |
| % | Pick a Card (Chance Or Community Chest) |
| % | Penalty and tax (includes go to jail) |
| % | GO (collect $200) |
| % | Free Parking and Just Visiting |
| Total________% | |

Check to see that the total is 100%.

Name ____________________

## IT'S ALL IN THE GAME

Student Activity Sheet 1, page 2

Part 3. Copy the percents from Part 2 into the Percent column of the chart below. You now can compute the information necessary to design a circle graph. Some things you need to know:

1. A circle contains 360°.
2. To determine how many degrees for each category, multiply 360° by the percent taken up by that category.

Steps:

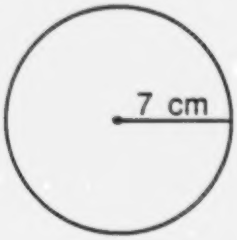
- Enter 360 on your calculator.
- Press $\boxed{x}$ key.
- Press the percent for Railroads.
- Press the $\boxed{\%}$ key.
- Enter this number, rounded to the nearest whole degree.
- Repeat this process for the other categories.

Enter the degrees per category below:

| Percent | Degrees | Category |
|---|---|---|
| % | | Railroads |
| % | | Property (on which you can build) |
| % | | Utilities |
| % | | Pick a Card (Chance Or Community Chest) |
| % | | Penalty and tax (includes go to jail) |
| % | | GO (collect $200) |
| % | | Free Parking and Just Visiting |
| Total_______% | Total________ | |

Add the degrees in each category. They should total approximately 360°. If not, check your calculations.

Part 4. 1. Use your compass to draw a circle with a radius of approximately 7 cm. Draw a radius from the center of the circle.

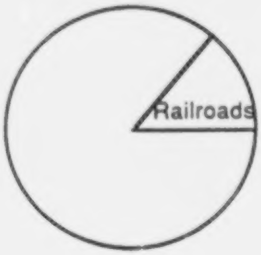


Name ____________________

## IT'S ALL IN THE GAME

Student Activity Sheet 1, page 3

2. Lay the protractor along this radius and mark off the degrees you calculated for Railroads. Draw another radius at this point.



You have created a **central angle,** (an angle whose vertex is the center of the circle) that is equal to the degrees taken up by Railroads.

3. Lay the protractor along the radius you just drew. Mark the degrees for the next category, Property. Draw another radius. Label the sector appropriately.
4. Continue creating angles for all the categories listed in Part 3.
5. When the circle graph is complete, give it a title. Summarize results below.

The largest sector is taken up by ______________________

The smallest sector is taken up by ______________________

Two sectors that are the same size are ____________ and ___________ .

6. Write two statements about your circle graph.

____________________________________________________________

____________________________________________________________

____________________________________________________________

____________________________________________________________

____________________________________________________________

Name ____________________

## IT'S ALL IN THE GAME

### Student Activity Sheet 2

In a MONOPOLY game there are various amounts of money.

| Denomination | Number of Bills | Percent | Degrees |
|---|---|---|---|
| a. Five hundreds | 20 | | |
| b. Hundreds | 30 | | |
| c. Fifties | 40 | | |
| d. Tens | 50 | | |
| e. Fives | 60 | | |
| | Total ______ | Total ______ | Total ______ |

1. Complete the chart above.
2. Use the information to create a circle graph.
3. When the circle graph is completed, give it a title.
4. Write two statements about your circle graph.

______________________________________________

______________________________________________

______________________________________________

______________________________________________

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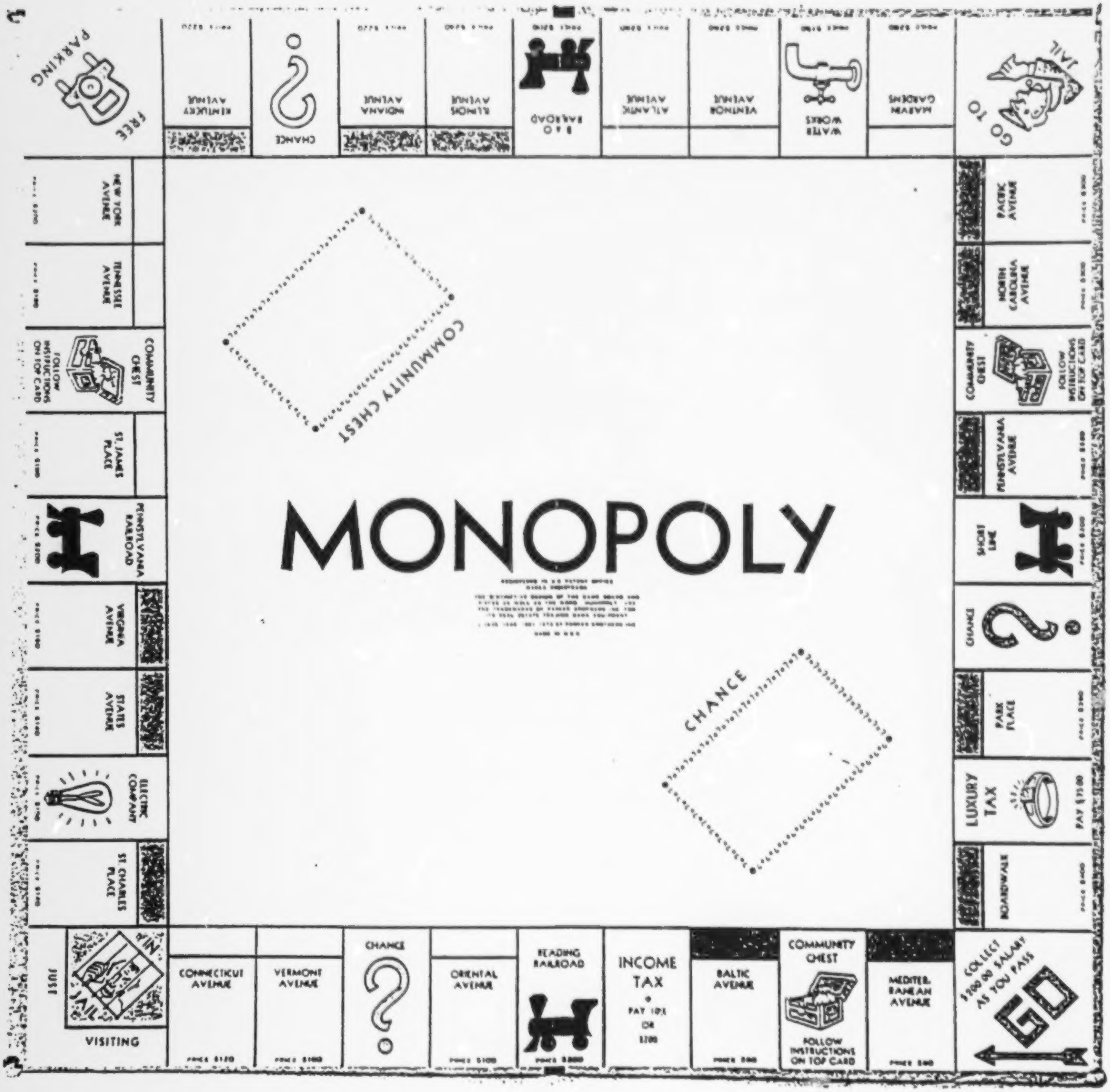
MONOPOLY
FREE PARKING
KENTUCKY AVENUE
CHANCE
INDIANA AVENUE
ILLINOIS AVENUE
B & O RAILROAD
ATLANTIC AVENUE
VENTNOR AVENUE
WATER WORKS
MARVIN GARDENS
GO TO JAIL
PACIFIC AVENUE
NORTH CAROLINA AVENUE
COMMUNITY CHEST
FOLLOW INSTRUCTIONS ON TOP CARD
PENNSYLVANIA AVENUE
SHORT LINE
CHANCE
PARK PLACE
LUXURY TAX
PAY $75.00
BOARDWALK
COLLECT $200.00 SALARY AS YOU PASS
GO
MEDITERRANEAN AVENUE
COMMUNITY CHEST
FOLLOW INSTRUCTIONS ON TOP CARD
BALTIC AVENUE
INCOME TAX
PAY 10% OR $200
READING RAILROAD
ORIENTAL AVENUE
CHANCE
VERMONT AVENUE
CONNECTICUT AVENUE
JUST VISITING
IN JAIL
ST. CHARLES PLACE
ELECTRIC COMPANY
STATES AVENUE
VIRGINIA AVENUE
PENNSYLVANIA RAILROAD
ST. JAMES PLACE
COMMUNITY CHEST
FOLLOW INSTRUCTIONS ON TOP CARD
TENNESSEE AVENUE
NEW YORK AVENUE
COMMUNITY CHEST
CHANCE

NAME ____________

## IT'S ALL IN THE GAME

### Extension Activity Sheet

In the game SCRABBLE you build words on a gameboard. There are 225 total spaces on the board.

1. There are five different types of spaces as follows:
The center square counts as a double word score.

| Number of Spaces | Category |
|---|---|
| | double letter |
| | triple letter |
| | double word |
| | triple word |
| | no value |
| Total ______ | |

2. Record the number of spaces for each category on the line provided.
3. Use this information to create a circle graph as you did with the MONOPOLY game. Remember to first convert the number of spaces for each category to a percent of the total number of spaces.

$$\text{PERCENT} = \frac{\text{number of spaces in a category} \times 100}{\text{total number of spaces}}$$

**(Round all answers to the nearest percent)**

Record percents for each category on chart below.

4. Find out how many degrees of the circle to use for each sector. Record on chart below.

Degrees = 360 [x] [percent for category] [%]

**(Round all answers to the nearest degree)**

| Percent | Degrees | Category |
|---|---|---|
| % | | double letter |
| % | | triple letter |
| % | | double word |
| % | | triple word |
| % | | no value |
| Total ______ % | Total ______ | |

5. Draw the circle graph and give it a title.
6. Write two statements about your circle graph. ____________________

____________________________________________

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SCRABBLE® is a registered trademark in the United States of Hasbro, Inc.

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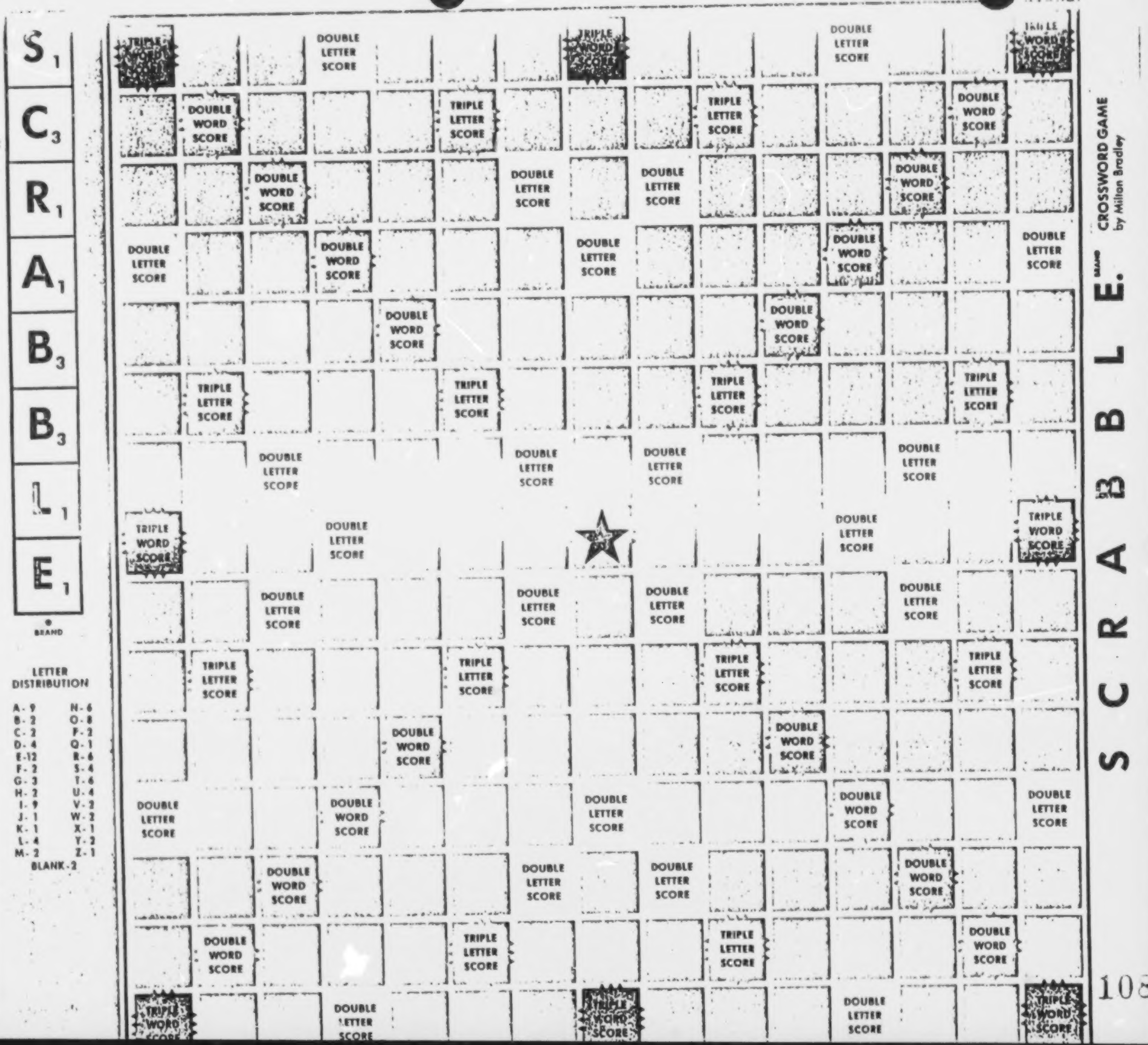
S 1
C 3
R 1
A 1
B 3
B 3
L 1
E 1
BRAND
TRIPLE WORD SCORE
DOUBLE WORD SCORE
TRIPLE LETTER SCORE
DOUBLE LETTER SCORE
SCRABBLE BRAND
CROSSWORD GAME
by Milton Bradley

LETTER DISTRIBUTION

| | |
|---|---|
| A - 9 | N - 6 |
| B - 2 | O - 8 |
| C - 2 | P - 2 |
| D - 4 | Q - 1 |
| E - 12 | R - 6 |
| F - 2 | S - 4 |
| G - 3 | T - 6 |
| H - 2 | U - 4 |
| I - 9 | V - 2 |
| J - 1 | W - 2 |
| K - 1 | X - 1 |
| L - 4 | Y - 2 |
| M - 2 | Z - 1 |

BLANK - 2

# LET'S PARTY

**GRADE:** 7 - 8

**STRAND:** Probability and Statistics

**SKILL:** Investigate combinations (subsets of a set of people or objects).

**MANAGEMENT**

**CLASS ORGANIZATION:** Whole class

**TIME FRAME:** One math period

**MATERIALS:** Calculator

**VOCABULARY:** Factorial, combinations

**PREREQUISITE SKILL:** Multiplication

**LESSON**

- **DIRECTED INSTRUCTION/ GUIDED PRACTICE:**

Your parents said that you can have a group of friends over. You are excited and you ask how many friends you may invite. They tell you to invite only 3 of your friends. You have 6 close friends, so you are faced with the problem of which 3 to invite. What are your possible choices? Hand out Student Activity Sheets 1 and 2 and help the students work through them. Discuss organized ways to list the combinations other than trial and error. For example, suppose the names are A, B, C, D, E, F and you could choose 4. One strategy would be to list them alphabetically. ABCD, ABCE, ABCF, ABDE, ABDF, ABEF, ACDE, ACDF, ACEF, ADEF, BCDE, BCDF, BCEF, BDEF, and CDEF.
Students need not follow this strategy, but they should at least be thinking of an organized way of listing.

- **DIRECTED INSTRUCTION:**

Tell students that it is useful to know how many combinations there are in a problem before trying to list them all. If you know how many there are then you know when your list is complete. There is an easier way to find the number of combinations than first making the long lists. It has to do with using factorials. Factorials are indicated by a number followed by an exclamation (!) sign. A number followed by an exclamation mark means that the number is multiplied by the next smallest whole number times the next smallest whole number, and so on, all the way down to 1.

For example:

5! means 5 x 4 x 3 x 2 x 1 which equals 120 (5 factorial = 120)
3! means 3 x 2 x 1 which equals 6

Have students verify 5! = 120 by multiplying 5 x 4 x 3 x 2 x 1.

- **GUIDED PRACTICE:**

Hand out and have students complete Student Activity Sheet 3. Discuss results. Say that factorials help us to find the number of combinations. On John's list (Student Activity Sheet 1) he found that with 6 friends the number of combinations of 3 he could write down was 20. To find this number using a mathematical rule, we use the factorials from the chart in Student Activity Sheet 3 and the following rule.

The number of combinations that can be made from 6 friends choosing 3 at a time is equal to $\frac{6!}{3! \times (6-3)!}$ We can use the calculator and mental math to compute this :

$$\frac{6!}{3! \times (6-3)!} = \frac{6!}{3! \times 3!} \qquad \text{Since } 6 - 3 = 3$$

$$= \frac{720}{6 \times 6} \qquad \text{Since } 6! = 720, \text{ and } 3! = 6$$

$$= \frac{720}{36} \qquad \text{Since } 6 \times 6 = 36$$

$$= 20 \qquad \text{Since } 720 \div 36 = 20$$

If □ = the number of friends you can choose from

Δ = the number of friends you can select at a time

then the formula becomes:

The number of combinations that can be made from □ friends choosing Δ at a time is equal to $\frac{\square!}{\Delta! \times (\square - \Delta)!}$

Have the students write this formula on the bottom of their Factorial Chart (Student Activity Sheet 3).

Then use the formula to find the number of combinations that can be made from 6 friends selected 4 at a time.

Check to see they worked the problem the following way:

$$\frac{6!}{4! \times (6-4)!} = \frac{6!}{4!\ 2!} = \frac{720}{24 \times 2} = \frac{720}{48} = 15$$

- **INDEPENDENT PRACTICE:**

  Hand out and have students complete Student Activity Sheet 4.

  Note: A problem like number 6 can be done by using the factorial chart (Student Activity Sheet 3),

$$\frac{10!}{6!(10-6)!} = \frac{10!}{6!\ 4!} = \frac{3628800}{720 \times 24} = \frac{3628800}{17280} = 210$$

  or by writing it out using the meaning of factorial and reducing.

$$\frac{10!}{6!(10-6)!} = \frac{10!}{6!\ 4!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times 2 \times \cancel{1}}{\cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1} \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = \frac{10 \times 9 \times \cancel{8} \times 7}{\cancel{4} \times 3 \times \cancel{2} \times 1}$$

$$= \frac{10 \times 9 \times 7}{3 \times 1} = \frac{10 \times \cancel{9}^{3} \times 7}{\cancel{3} \times 1}$$

$$= \frac{10 \times 3 \times 7}{1} = 210$$

- **HOME ACTIVITY:**

  Hand out and have students complete the Home Activity Sheet parts A-D.

Home Activity Sheet Answer for part B.

B. Assume you were only allowed to watch 3 of them. Using factorials, your chart from Student Activity Sheet 3, and your calculator, compute how many different combinations of three programs you could watch.
(Write the formula below and show your computations)

$$\frac{7!}{3! \times (7-3)!} = \frac{7!}{3!\ 4!} = \frac{5040}{6 \times 24} = 35$$

# LET'S PARTY

## Student Activity Sheet 1

## Teacher Answer Sheet

If John had to decide which 3 of his six friends to invite to his house. He first makes a list of the possible combinations and decides which combination of friends he would most like to invite.

Note: To find combinations, form a list where the order you write the list of names does not matter. Combinations are subsets of the original set of names.

John has 6 friends: Arnold, Betty, Cathy, David, Terri, and Frank. John made an organized list of combinations using 3 of his friends. The combination of Arnold, David, and Frank is the same as David, Arnold, and Frank. The combination of Arnold, David, and Terri would be a different combination.

**Complete John's list by filling in the blanks:**

| Combination # | Names | Names | Names |
|---|---|---|---|
| 1. | Arnold | Betty | Cathy |
| 2. | Arnold | Betty | David |
| 3. | Arnold | Betty | Terri |
| 4. | Arnold | Betty | Frank |
| 5. | Arnold | Cathy | David |
| 6. | Arnold | Cathy | Terri |
| 7. | Arnold | Cathy | Frank |
| 8. | Arnold | David | Terri |
| 9. | Arnold | David | Frank |
| 10. | Arnold | Terri | Frank |
| 11. | Betty | Cathy | David |
| 12. | Betty | Cathy | Terri |
| 13. | Betty | Cathy | Frank |
| 14. | Betty | David | Terri |
| 15. | Betty | David | Frank |
| 16. | Betty | Terri | Frank |
| 17. | Cathy | David | Terri |
| 18. | Cathy | David | Frank |
| 19. | Cathy | Terri | Frank |
| 20. | David | Terri | Frank |

There are 20 combinations in this activity. If you used the names of your own 6 friends you would have the same total of combinations.

## LET'S PARTY

Student Activity Sheet 2
Teacher Answer Sheet

John convinces his parents to let him invite 4 of his 6 best friends.

A. John's six best friends are:

Arnold
Betty
Cathy
David
Terri
Frank

B. Make a list of the possible combinations of 4 friends from the list of 6, that he could now invite. As you complete the chart, only write combinations that are different from those already listed.

| Combination # | Name | Name | Name | Name |
|---|---|---|---|---|
| 1. | Arnold | Betty | Cathy | David |
| 2. | Arnold | Betty | Cathy | Terri |
| 3. | Arnold | Betty | Cathy | Frank |
| 4. | Arnold | Betty | David | Terri |
| 5. | Arnold | Betty | David | Frank |
| 6. | Arnold | Betty | Terri | Frank |
| 7. | Arnold | Cathy | David | Terri |
| 8. | Arnold | Cathy | David | Frank |
| 9. | Arnold | Cathy | Terri | Frank |
| 10. | Arnold | David | Terri | Frank |
| 11. | Betty | Cathy | David | Terri |
| 12. | Betty | Cathy | David | Frank |
| 13. | Betty | Cathy | Terri | Frank |
| 14. | Betty | David | Terri | Frank |
| 15. | Cathy | David | Terri | Frank |
| 16. | | | | |
| 17. | | | | |
| 18. | | | | |
| 19. | | | | |
| 20. | | | | |
| 21. | | | | |
| 22. | | | | |
| 23. | | | | |
| 24. | | | | |
| 25. | | | | |

C. Count how many different combinations of your 6 friends could be invited.

How many do you get? 15

Is this more or less than the 20 combinations you get when you can only invite 3 friends over? less

# LET'S PARTY

## Student Activity Sheet 3
Teacher Answer Sheet

### FACTORIAL CHART

Using your calculator, when appropriate, complete the chart below to show all factorials from 1! to 11!

| Factorials | Products |
|---|---|
| 1! = 1 | 1 |
| 2! = 2 x 1 = | 2 |
| 3! = 3 x 2 x 1 = | 6 |
| 4! = 4 x 3 x 2 x 1 = | 24 |
| 5! = 5 x 4 x 3 x 2 x 1 = | 120 |
| 6! = 6 x 5 x 4 x 3 x 2 x 1 = | 720 |
| 7! = 7 x 6 x 5 x 4 x 3 x 2 x 1 = | 5040 |
| 8! = 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = | 40,320 |
| 9! = 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = | 362,880 |
| 10! = 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = | 3,628,800 |
| 11! = 11 x10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = | 39,916,800 |

What is the largest factorial you can compute on your calculator? 11!

Why? The answer to 12! would be too large to fit on the display.

## LET'S PARTY

### Student Activity Sheet 4
### Teacher Answer Sheet

With 10 friends find the number of combinations by choosing 1 at a time, 2 at a time, 3 at a time, etc. Use the combination formula and your answers from Student Activity Sheet 3 to complete these.

1) $\frac{10!}{1!(10-1)!} = \frac{10!}{1!\ 9!} = \frac{3628800}{1 \times 362880} = \frac{3628800}{362880} = 10$

2) $\frac{10!}{2!(10-2)!} = \frac{10!}{2!\ 8!} = \frac{3628800}{2 \times 40320} = \frac{3628800}{80640} = 45$

3) $\frac{10!}{3!(10-3)!} = \frac{10!}{3!\ 7!} = \frac{3628800}{6 \times 5040} = \frac{3628800}{30240} = 120$

4) $\frac{10!}{4!(10-4)!} = \frac{10!}{4!\ 6!} = \frac{3628800}{24 \times 720} = \frac{3628800}{17280} = 210$

5) $\frac{10!}{5!(10-5)!} = \frac{10!}{5!\ 5!} = \frac{3628800}{120 \times 120} = \frac{3628800}{14400} = 252$

6) $\frac{10!}{6!(10-6)!} = \frac{10!}{6!\ 4!} = \frac{3628800}{720 \times 24} = \frac{3628800}{17280} = 210$

7) $\frac{10!}{7!(10-7)!} = \frac{10!}{7!\ 3!} = \frac{3628800}{5040 \times 6} = \frac{3628800}{30240} = 120$

8) $\frac{10!}{8!(10-8)!} = \frac{10!}{8!\ 2!} = \frac{3628800}{40320 \times 2} = \frac{3628800}{80640} = 45$

9) $\frac{10!}{9!(10-9)!} = \frac{10!}{9!\ 1!} = \frac{3628800}{362880 \times 1} = \frac{3628800}{362880} = 10$

What do you observe about the answers?

The numbers get larger and then smaller. There are 2 of each answer except for the middle number.

Name ____________________

## LET'S PARTY

Student Activity Sheet 1

John had to decide which 3 of his six friends to invite to his house. He first makes a list of the possible combinations and decides which combination of friends he would most like to invite.

Note: To find combinations, form a list where the order you write the names does not matter. Combinations are subsets of the original set of names.

John has 6 friends: Arnold, Betty, Cathy, David, Terri, and Frank. John made an organized list of combinations using 3 of his friends. The combination of Arnold, David, and Frank is the same as David, Arnold, and Frank. The combination of Arnold, David, and Terri would be a different combination.

**Complete John's list by filling in the blanks:**

| Combination # | Name | Name | Name |
|---|---|---|---|
| 1. | Arnold | Betty | Cathy |
| 2. | Arnold | Betty | David |
| 3. | Arnold | Betty | Terri |
| 4. | Arnold | Betty | |
| 5. | Arnold | Cathy | |
| 6. | Arnold | Cathy | |
| 7. | Arnold | Cathy | |
| 8. | Arnold | David | |
| 9. | Arnold | David | |
| 10. | Arnold | Terri | |
| 11. | Betty | Cathy | |
| 12. | Betty | Cathy | |
| 13. | Betty | | |
| 14. | Betty | | |
| 15. | Betty | | |
| 16. | Betty | | |
| 17. | Cathy | | |
| 18. | | | |
| 19. | | | |
| 20. | | | |

There are 20 combinations in this activity. If you used the names of your own 6 friends you would have the same total number of combinations.

Name ____________________

## LET'S PARTY

### Student Activity Sheet 2

John convinces his parents to let him invite 4 of his 6 friends to his house.

A. John's six best friends are:

Arnold ____________________ David ____________________

Betty ____________________ Terri ____________________

Cathy ____________________ Frank ____________________

B. Make a list of the possible combinations of 4 friends from the list of 6, that he could now invite. As you complete the chart, only write combinations that are different from those already listed.

| Combination # | Name | Name | Name | Name |
|---|---|---|---|---|
| 1. | | | | |
| 2. | | | | |
| 3. | | | | |
| 4. | | | | |
| 5. | | | | |
| 6. | | | | |
| 7. | | | | |
| 8. | | | | |
| 9. | | | | |
| 10. | | | | |
| 11. | | | | |
| 12. | | | | |
| 13. | | | | |
| 14. | | | | |
| 15. | | | | |
| 16. | | | | |
| 17. | | | | |
| 18. | | | | |
| 19. | | | | |
| 20. | | | | |
| 21. | | | | |
| 22. | | | | |
| 23. | | | | |
| 24. | | | | |
| 25. | | | | |

C. Count how many different combinations of his 6 friends (taken 4 at a time) could be invited.
How many do you get? ____________
Is this more or less than the 20 combinations that John gets when he can only invite 3 friends over?

HAPPY BIRTHDAY!

Name ____________________

## LET'S PARTY
Student Activity Sheet 3

### FACTORIAL CHART

Using your calculator, when appropriate, compute the following factorials and complete the chart.

| Factorials | Products |
|---|---|
| 1! = 1 | 1 |
| 2! = 2 x 1 | 2 |
| 3! = 3 x 2 x 1 | 6 |
| 4! = 4 x 3 x 2 x 1 | 24 |
| 5! = 5 x 4 x 3 x 2 x 1 | |
| 6! = | |
| 7! = | |
| 8! = | |
| 9! = | |
| 10! = | |
| 11! = | |

What is the largest factorial you can compute on your calculator? ________

Why? ____________________________________________

____________________________________________

____________________________________________

____________________________________________

Name ____________________

## LET'S PARTY

### Student Activity Sheet 4

With 10 friends, find the number of combinations by choosing 1 at a time, 2 at a time, 3 at a time, etc. Use the combination formula and your answers from Student Activity Sheet 3 to complete these.

1) $\frac{10!}{1!(10-1)!} =$

2) $\frac{10!}{2!(10-2)!} =$

3) $\frac{10!}{3!(10-3)!} =$

4) $\frac{10!}{4!(10-4)!} =$

5) $\frac{10!}{5!(10-5)!} =$

6) $\frac{10!}{6!(10-6)!} =$

7) $\frac{10!}{7!(10-7)!} =$

8) $\frac{10!}{8!(10-8)!} =$

9) $\frac{10!}{9!(10-9)!} =$

What do you observe about the answers?

______________________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

Name ____________________

## LET'S PARTY
### Home Activity Sheet

A. Look up 7 television programs that air at different times on Saturday. Write them down.

____________________ ____________________

____________________ ____________________

____________________ ____________________

____________________

B. Assume you were only allowed to watch 3 of them. Using factorials, your chart from Student Activity Sheet 3, and your calculator. Compute how many different combinations of three programs you could watch.
(Write the formula below and show your computations)

________________________________________

________________________________________

________________________________________

________________________________________

________________________________________

Name ____________________

## LETS PARTY

Home Activity Sheet

C. Using a letter of your choice to abbreviate the name of each show, make a chart listing all possible combinations of exactly three shows that you could watch. Use this to check that the answer you got in part B using factorials was correct.

| Combination # | Names | Names | Names | Names |
|---|---|---|---|---|
| 1. | | | | |
| 2. | | | | |
| 3. | | | | |
| 4. | | | | |
| 5. | | | | |
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| 32. | | | | |
| 33. | | | | |
| 34. | | | | |
| 35. | | | | |
| 36. | | | | |
| 37. | | | | |
| 38. | | | | |

D. Did you get your answer to part B to match the number of combinations in part C?

________________

# THE CARNIVAL BOOTH

**GRADE:** 7 - 8

**STRAND:** Probability and Statistics

**SKILL:** Compute probabilities.

**MANAGEMENT**
**CLASS ORGANIZATION:** Pairs

**TIME FRAME:** Two or more math periods

**MATERIALS:** Calculators (student and overhead), tagboard, markers, rulers, coins

**VOCABULARY:** Probability, percent, theoretical probability, rectangle, square, triangle, parallelogram, circle, trapezoid, radius, base, height, length, width, pi

**PREREQUISITE SKILL:** Use formulas to compute area, convert fractions to percent

**LESSON**

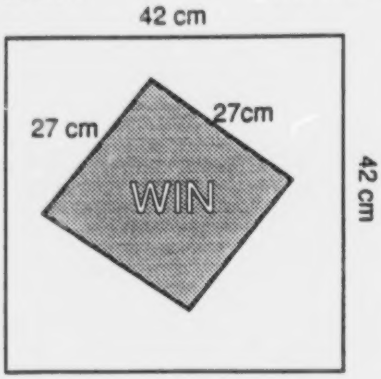
- **DIRECTED INSTRUCTION/GUIDED PRACTICE/INDEPENDENT PRACTICE:**
  The mathematical probability that something will or will not happen is called theoretical probability which is an important part of this lesson.
  Hand out and have students complete Student Activity Sheet 1 to review area formulas. Discuss.

  Tell the class: "Your school will have a carnival. Your committee is in charge of renting equipment. You will select a game boards for the Coin Toss booths. In these games, you toss a coin. If more than half the coin lands in the WIN area you win a prize. Your job is to select game boards where the probability of winning a prize is between 30% and 40%."

  Discuss the example below with the class. Use the Carnival Booth Transparency which follows these directions to help explain area formulas and the theoretical probability formula. Round answers to the nearest percent.

Board A

Area of rectangle = length x width = l x w



$$\frac{\text{Area of Win rectangle}}{\text{Area of Whole rectangle}} \times 100 = \underline{\qquad\quad}\%$$

Answer $\frac{729}{1764} \times 100 = 41\%$

Theoretical probability of winning = 41%

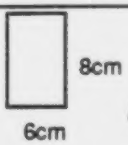
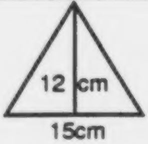
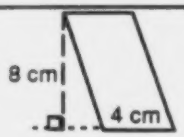
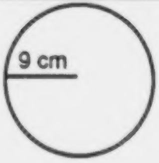
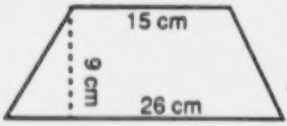
- Hand out and have students complete Student Activity Sheet 2.
- Discuss results.
- Hand out Student Activity Sheet 3 and materials for constructing game boards. Have students perform the experiment and complete the Student Activity Sheet.

## THE CARNIVAL BOOTH

Student Activity Sheet 1

Teacher Answer Sheet

To find the area of whole shapes, complete the following:

| | Area Formulas | |
|---|---|---|
| Area of a rectangle | length x width = l x w |  <br>AREA = l x w 6 x 8 = 48 sq. cm |
| Area of a triangle | $\frac{\text{base x height}}{2}$ |  <br>Area = $\frac{b \times h}{2}$<br>$\frac{15 \times 12}{2}$ = 90 sq. cm |
| Area of a parallelogram | base x height |  <br>Area = b x h<br>4 x 8 = 32 sq. cm |
| Area of a circle | π x r2 (use 3.14 for π) |  <br>Area = $\pi r^2$<br>3.14 x 9 x 9 = 254.34 sq. cm |
| Area of a trapezoid | $\frac{(b_1 + b_2) \times h}{2}$ |  <br>Area = $\frac{(b_1 + b_2) \times h}{2}$<br>$\frac{(15 + 26) \times 9}{2}$ = 184.5 sq. cm |

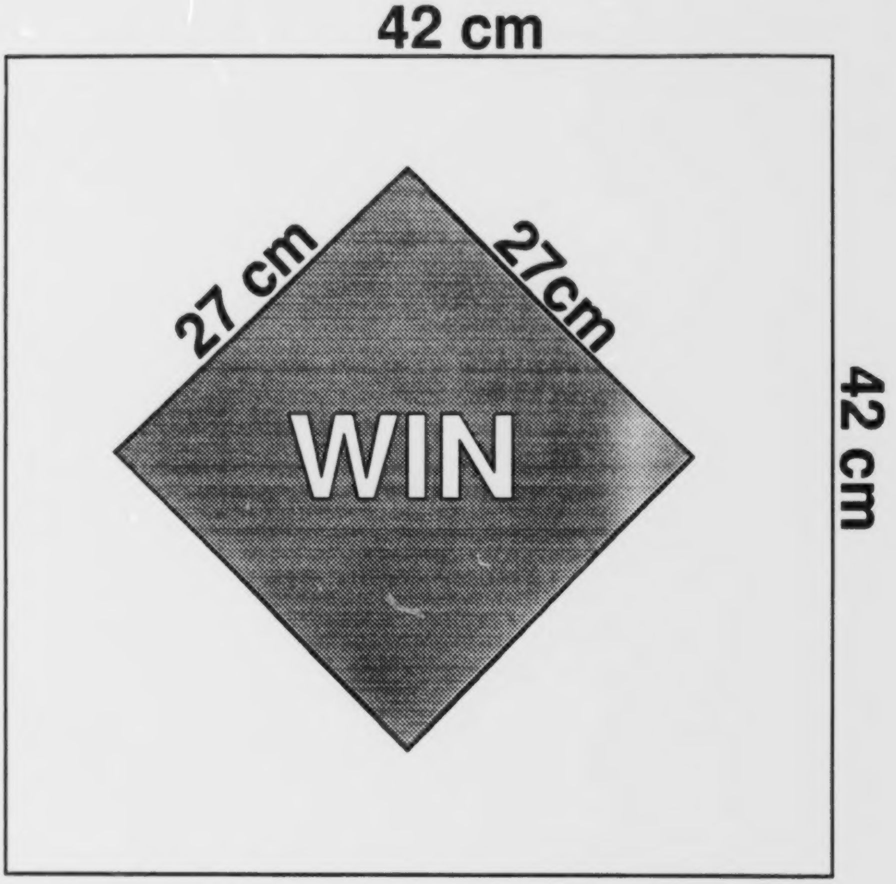
## THE CARNIVAL BOOTH - TRANSPARENCY - BOARD A

COMPUTING THEORETICAL PROBABILITY

$$\boxed{\text{THEORETICAL PROBABILITY OF WINNING} = \frac{\text{Area of Win}}{\text{Area of Whole}} \times 100}$$

Area of rectangle = length x width = l x w

Board A.



$$\frac{\text{Area of Win rectangle}}{\text{Area of Whole rectangle}} \times 100 = \_\_\_\_\_\_\_\_\_\_\%$$

# THE CARNIVAL BOOTH

Student Activity Sheet 2

Teacher Answer Sheet

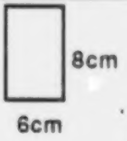
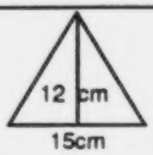
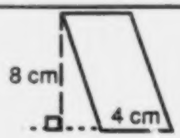
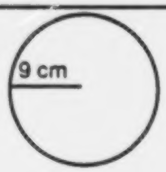
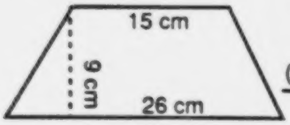
All answers are rounded to nearest whole number.
Note: Number directly preceding parentheses means multiplication. In formulas for the area of a triangle or trapezoid you can divide by 2 or multiply by $\frac{1}{2}$.

| PROBABILITY OF WINNING | USE THIS BOARD AT CARNIVAL? |
|---|---|
| BOARD A = $\frac{(27)^2}{(42)^2} = \times 100 = 41\%$ | no |
| BOARD B = $\frac{2(21)^2}{33 \times 60} \times 100 = 45\%$ | no |
| BOARD C = $\frac{35 \times 21}{(44)^2} = \frac{735}{1936} \times 100 = 38\%$ | yes |
| BOARD D = $\frac{2(12 \times 36)}{66 \times 36} = \frac{864}{2376} \times 100 = 36\%$ | yes |
| BOARD E = $\frac{2\left(\frac{(26 \times 22)}{2}\right)}{\frac{1}{2}(52 \times 42)} = \frac{572}{1092} \times 100 = 52\%$ | no |
| BOARD F = $\frac{3.14(12)^2}{\frac{1}{2}(65+38) \times 28} = \frac{452.16}{1442} \times 100 = 31\%$ | yes |
| BOARD G = $\frac{6(3.14)(7)^2}{75 \times 35} = = \frac{923.16}{2625} \times 100 = 35\%$ | yes |
| BOARD H = $\frac{4(14)^2}{\frac{1}{2}(77 \times 58)} = \frac{784}{2233} \times 100 = 35\%$ | yes |
| BOARD I = $\frac{\frac{1}{2}(42 \times 35)}{\frac{1}{2}(58 + 45)(35)} = \frac{735}{1802.5} \times 100 = 41\%$ | no |
| BOARD J = $\frac{3.14(16)^2}{3.14(28)^2} = \frac{803.84}{2461.76} \times 100 = 33\%$ | yes |

Name________________

## THE CARNIVAL BOOTH
### Student Activity Sheet 1

To find out the area of whole shapes, complete the following:

| | Area Formulas | | |
|---|---|---|---|
| Area of a rectangle | length x width = l x w |  | Area = l x w 6 x 8 = ______ sq. cm. |
| Area of a triangle | $\frac{\text{base x height}}{2}$ |  | Area = $\frac{b \times h}{2}$<br>$\frac{15 \times 12}{2}$ = ______ sq. cm |
| Area of a parallelogram | base x height |  | Area = b x h<br>4 x 8 = ______ sq. cm |
| Area of a circle | $\pi \times r^2$ (use 3.14 for $\pi$) |  | Area = $\pi r^2$<br>3.14 x 9 x 9 = ______ sq. cm |
| Area of a trapezoid | $\frac{(b_1+b_2) \times h}{2}$ |  | Area = $\frac{(b_1 + b_2) \times h}{2}$<br>$\frac{(15 + 26) \times 9}{2}$ = ______ sq.cm |

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Name________________

## THE CARNIVAL BOOTH
Student Activity Sheet 2

In this lesson you select game boards for the Coin Toss Booth. There are nine possible boards from which to choose.

Compute the Probability of winning for each board. Then indicate if you would rent the board. You want to rent boards with a 30% - 40% probability of winning.

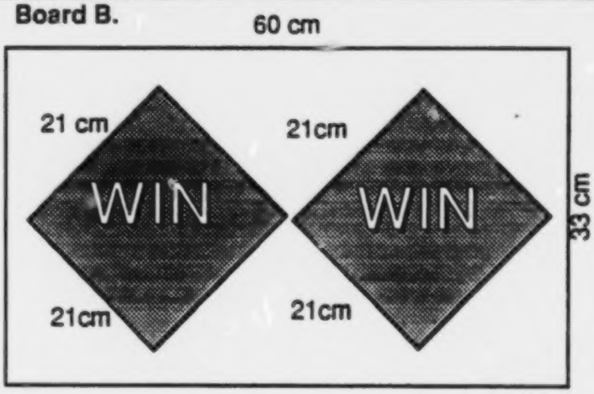
Note: To compute probability of winning use this formula:

$$\boxed{\text{THEORETICAL PROBABILITY OF WINNING} = \frac{\text{Area of Win}}{\text{Area of Whole}} \times 100}$$

(Round to the nearest whole number)

NOTE: Measurements should be followed although illustrations are not proportionally accurate.

**Board B.**

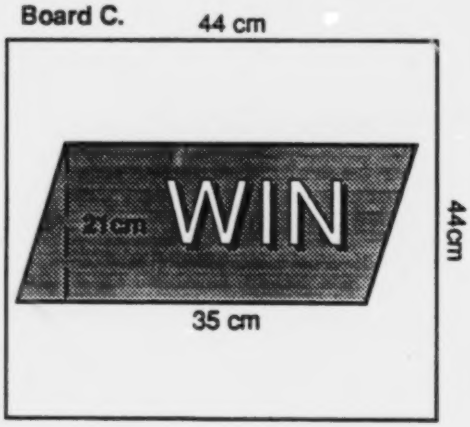


$\frac{\text{Area of both Win squares}}{\text{Area of Whole rectangle}} \times 100 =$ ______%

Probability of winning = __________%

Would you rent this board? ________

**Board C.**



$\frac{\text{Area of Win parallelogram}}{\text{Area of square}} \times 100 =$ ____

Probability of winning = __________%

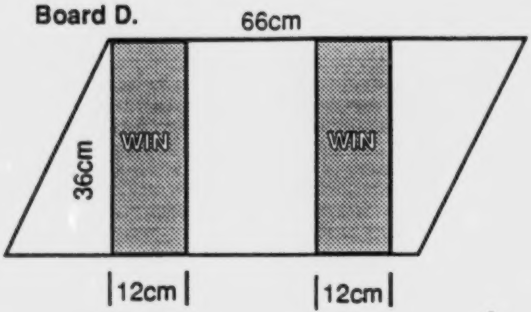
Would you rent this board? ________

Name________________

## THE CARNIVAL BOOTH

Student Activity Sheet 2 page 2

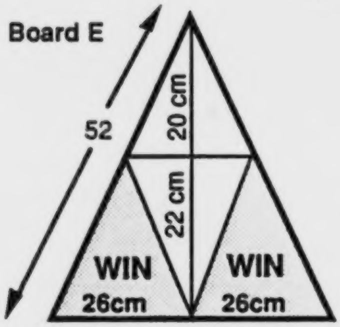
**Board D.**


66cm
36cm
WIN
WIN
12cm
12cm

Probability of winning = __________%

Would you rent this board? __________

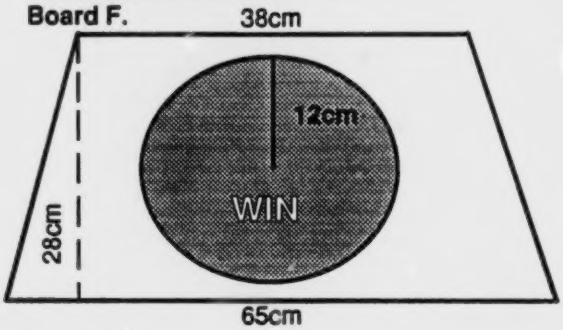
**Board E**


52
20 cm
22 cm
WIN
WIN
26cm
26cm

Probability of winning = ______ %

Would you rent this board? __________

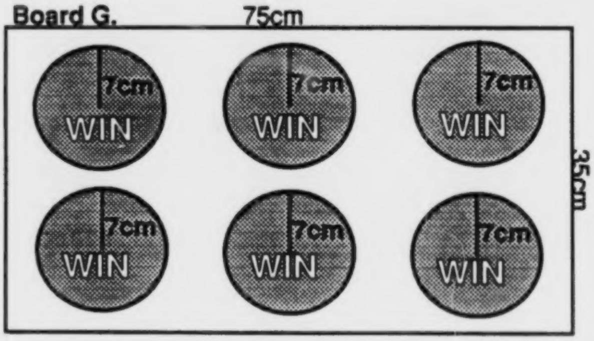
**Board F.**


38cm
12cm
WIN
28cm
65cm

Probability of winning = __________%

Would you rent this board? __________

**Board G.**


75cm
7cm
WIN
7cm
WIN
7cm
WIN
7cm
WIN
7cm
WIN
7cm
WIN
35cm

Probability of winning = __________%

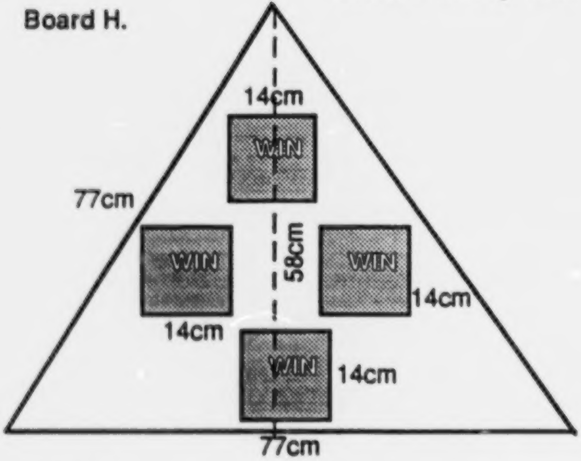
Would you rent this board? __________

Name_________________

## THE CARNIVAL BOOTH

Student Activity Sheet 2 page 3

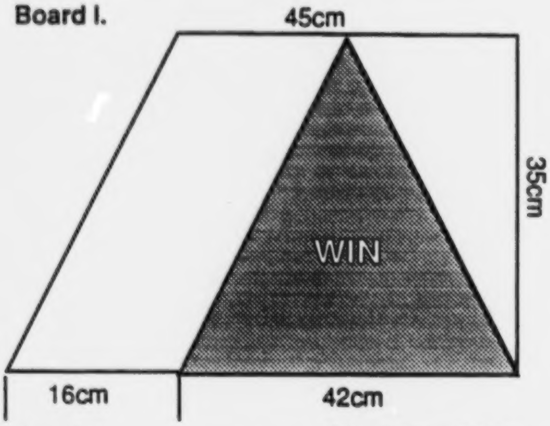
**Board H.**


14cm
WIN
77cm
WIN
58cm
WIN
14cm
14cm
WIN
14cm
77cm

Probability of winning = __________%

Would you rent this board? ________

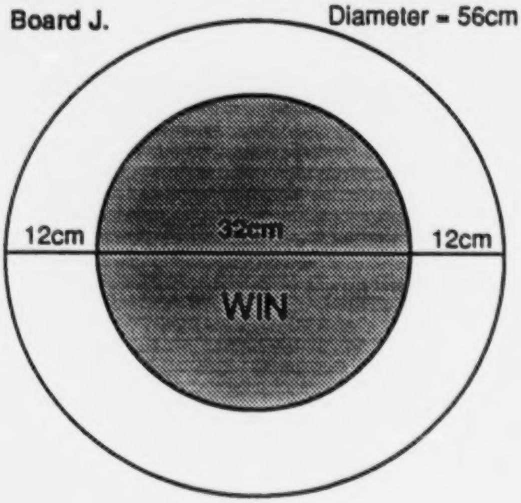
**Board I.**


45cm
35cm
WIN
16cm
42cm

Probability of winning = ______ %

Would you rent this board? ________

**Board J.**


Diameter = 56cm
12cm
32cm
12cm
WIN

Probability of winning = __________%

Would you rent this board? ________

Name____________________

## THE CARNIVAL BOOTH

### Student Activity Sheet 3

Select one of the nine game boards from Student Activity Sheet 2. Construct the game board on tagboard. Note: Measurements should be followed when constructing game boards although illustrations are not proportionately accurate. Shade in the Win area. Place the game board on the floor and stand next to the board. Do an experiment to see if the results come close to the theoretical probability.

COMPUTING EXPERIMENTAL PROBABILITY

Gameboard selected ______________

Drop a coin from about 1.5 meters above the board. Repeat the activity 50 times, keeping track of the number of times the coin (more than half of the coin) lands in the WIN area.

| TALLY | |
|---|---|
| WIN | LOSE |
| | |

EXPERIMENTAL PROBABILITY IN PERCENT

$$\frac{\text{NUMBER IN WIN AREA}}{\text{50 TRIALS}} \times 100 = \frac{\_\_\_\_\_\_\_\_}{50} \times 100 = \_\_\_\_\ \%$$

What was the experimental probability for your game board?______________

COMPUTING THEORETICAL PROBABILITY

$$\text{THEORETICAL PROBABILITY OF WINNING} = \frac{\text{Area of Win}}{\text{Area of Whole}} \times 100$$

In the work space below, find the theoretical probability for the game board you chose.

What was the theoretical probability for your game board? ________________

What was the experimental probability for your game board? ______________

Were you close to the theoretical probability? __________________________

Do you think you would get the same results if you did the experiment again?

__________________

# NUMBER GENERATION

**GRADE:** 7 - 8

**STRAND:** Logic

**SKILL:** Record data and make observations about results.

## MANAGEMENT

**CLASS ORGANIZATION:** Pairs

**TIME FRAME:** One to two math periods

**MATERIALS:** Calculator with 8 digit display. (Data may differ with scientific calculator)

**VOCABULARY:** Digits, tally, mean, mode, median

## LESSON

- **DIRECTED INSTRUCTION/ GUIDED PRACTICE:**

Note to Teacher: This lesson uses the square root key, $\boxed{\sqrt{\ }}$, to generate sets of digits for exploration. A knowledge of the concept of square root is not necessary for successful completion of the lesson.
This lesson is concerned only with the digit furthest to the right on the calculator display. We will start with some number and repeatedly press $\boxed{\sqrt{\ }}$, keeping track of what digit appears at the far right of the display.

For example, start with 300001. (This choice is arbitrary.) Press $\boxed{\sqrt{\ }}$, display shows 547.72347. Write just the 7 at the far right of the display. Press $\boxed{\sqrt{\ }}$ again, display shows 23.403492, but write just the last digit, 2. Continue generating a sequence of digits that looks like this:

Start
300001

Sequence
7, 2, 5, 7, 6, 3, 8, 4, 7, 2, 9, 6, 6, 7, 9, 4, 1, 8, 9, 9, 9, 9, 4, 6, 2

One more press and the display goes from 1.0000002 to 1. From now on the display won't change no matter how many times you press $\boxed{\sqrt{\ }}$. Keep track of these digits without counting the digit in the one's column of the starting number or of the final number 1.
Ask the class what they notice about these digits. Possible observations include these:

- There are 25 digits on the list.
- Every digit except 0 occurs at least once.
- The digit 9 occurs most often (6 times) while 1, 3, and 5 occur only once.

- There doesn't seem to be any pattern to the digits.
- Although it's not the focus of this lesson, students might comment on all the numbers displayed, noticing that the numbers decrease rapidly at first and then approach 1 more slowly.
- After just a few presses of √ the numbers are between 1 and 2.

Note: The list above makes it easy to see in what order the digits appear, but it's hard to see how many of each digit there are. Suggest this format, in which the digits are tallied as they appear:

| | |
|---|---|
| 0 | |
| 1 | \| |
| 2 | \|\|\| |
| 3 | \| |
| 4 | \|\|\| |
| 5 | \| |
| 6 | \|\|\|\| |
| 7 | \|\|\|\| |
| 8 | \|\| |
| 9 | 𝍸 \| |

Be sure that everyone is familiar with the 𝍸 notation to indicate five occurrences of the item being tallied.

Ask, "Do you think we'll get a similar bunch of digits if we use a different starting number?" There's really no way to tell at this point, so try another starting number. Use 400001. Again, this is an arbitrary choice. Either as a whole class or in pairs, tally the digits as before. Results should then look like this:

| | 300001 | 400001 |
|---|---|---|
| 0 | | |
| 1 | \| | \|\| |
| 2 | \|\|\| | 𝍸 |
| 3 | \| | \|\| |
| 4 | \|\|\| | \|\| |
| 5 | \| | 𝍸 |
| 6 | \|\|\|\| | \|\|\| |
| 7 | \|\|\|\| | \|\|\| |
| 8 | \|\| | \|\| |
| 9 | 𝍸 \| | \| |
| Total Number of Presses | 25 | 25 |

Allow for discussion. How are these sets of numbers alike and different?

- They're alike in that all digits except 0 occur in both.
- In the case of most digits, though, the number of times they occur is different. Only the 8 occurs the same number of times.
- In both cases it took 25 presses before reaching 1.

If your class is familiar with mean, median, and mode, discuss these. "Without computing, can you tell which set has the bigger mean?" (The first set, since it has more of the larger digits. Actually the mean of the 25 digits in the first column is 5.96; the mean of the second column is 4.6). Note: the median is obtained by counting tally marks until you reach the 13th one-(the middle one)-the median of the first first column is 6; the median of the second is 5. The first column has one mode, 9; the second has two, 2 and 5. Note: the mode is the number that appears most often.

- A Student Activity Sheet is provided for student exploration of the questions below.
- Is there a starting number for which some digit (other than zero) does not appear?
- "Is there a starting number that would give more than six of some digit?"
- "Are there starting numbers that would give exactly the same tallies as 400001 or 300001 did?"
- "Do you think that the tallies for 300001 and 400001 are typical of what we would get by starting with other numbers?"
- "Are there starting numbers that would take 26 presses to reach 1?"

Only through further experimentation can students find answers to these questions. In fact, one aim of the lesson is to get students to ask questions like these and to generate more data in exploring possible answers.

- **INDEPENDENT PRACTICE:**

Distribute Student Activity Sheet 1. Assign one starting number to each pair of students and ask them to **tally the digits** that result. At the end of the lesson compile the class results (see completed worksheet) and discuss. You can use the last column to record the total number of times each digit appears. Students might comment on some of the following points:

- The digit 0 never appears in the tallies (because the calculator displays decimals as far as possible to the right, and unnecessary 0's are dropped).
- 700001 and the larger starting numbers take 26 presses of $\boxed{\surd}$ to reach 1. (This leads to the natural question "If 600001 takes 25 presses and 700001 takes 26, what's the smallest starting number that requires 26?")
- Our first starting number, 300,001, now appears to be not very typical, when compared with the whole set of data. The occurrence of six 9's seems very unusual, especially since 9 is the least frequently obtained digit overall.
- The digit 2 occurs considerably more often than any other digit.

- Sometimes not every digit appears (9 doesn't occur when we start with 700001 or 1000001).
- No digit appears more than six times with a single starting number. All of these observations can lead to further conjectures, which in turn can lead to further investigation and data gathering.
  For example: the starting number, 300001, is not very typical of the whole set of ten starting numbers on the Student Activity Sheet. Are these ten starting numbers fairly typical of all the whole numbers? For example, is 2 really the most frequent digit and 9 the least frequent?
- Are there starting numbers for which two digits (other than zero) do not appear?
- If you choose a starting number at random how likely is it that no digit will appear more than four times?

- **EXTENSION:**
  - Hand out Student Activity Sheet.
  - Do the same investigation with ten numbers that are closer together.
  - Attached are the tallies for 600001, 600002, 600003, . . ., 600010. Note: the patterns of digits are more similar to each other than the first set of ten. Why? They suggest, also that 6, not 2, might be the most frequently occurring digit.
  - Hand out Student Activity Sheet 2 for additional student exploration.

## NUMBER GENERATION

### Student Activity Sheet 1

### Teacher Answer Sheet

| DIGIT | 100001 | 200001 | 300001 | 400001 | 500001 | 600001 |
|---|---|---|---|---|---|---|
| 0 | | | | | | |
| 1 | \| | \|\|\|\| | \| | \|\| | \|\|\|\| | \|\| |
| 2 | \|\|\|\| | \|\|\| | \|\|\| | 卌 | 卌 | \|\|\|\| |
| 3 | \|\|\|\| | \|\|\|\| | \| | \|\| | \| | \|\|\| |
| 4 | \|\| | \|\|\| | \|\|\| | \|\| | \|\|\| | 卌 |
| 5 | \| | \|\| | \| | 卌 | \|\| | \|\|\| |
| 6 | \|\|\| | \|\| | \|\|\|\| | \|\|\| | \|\| | \|\|\|\| |
| 7 | \| | \|\|\|\| | \|\|\|\| | \|\|\| | \| | \| |
| 8 | 卌 \| | \| | \|\| | \|\| | \|\|\| | \|\| |
| 9 | \|\|\| | \|\| | 卌 \| | \| | \|\|\|\| | \| |
| | 25 | 25 | 25 | 25 | 25 | 25 |

| DIGIT | 700001 | 800001 | 900001 | 1000001 | | |
|---|---|---|---|---|---|---|
| 0 | | | | | | |
| 1 | \|\|\|\| | \|\|\| | \|\|\|\| | \|\| | | 27 |
| 2 | 卌 | \| | \|\|\|\| | \|\| | | 36 |
| 3 | \|\|\|\| | \|\|\|\| | \|\| | \|\|\|\| | | 29 |
| 4 | \| | \| | \|\| | \|\| | | 24 |
| 5 | \|\| | \|\|\|\| | \|\|\|\| | 卌 \| | | 30 |
| 6 | \|\|\| | \|\|\| | \|\| | \|\| | | 28 |
| 7 | 卌 \| | \|\| | \|\|\| | 卌 | | 30 |
| 8 | \| | \|\|\|\| | \|\|\| | \|\|\| | | 27 |
| 9 | | \|\|\|\| | \|\| | | | 23 |
| | 26 | 26 | 26 | 26 | | 254 |

## NUMBER GENERATION

### Student Activity Sheet 3

### Teacher Answer Sheet

| DIGIT | 600001 | 600002 | 600003 | 600004 | 600005 | 600006 |
|---|---|---|---|---|---|---|
| 0 | | | | | | |
| 1 | 𝍷𝍷 | | 𝍷 | | | 𝍷𝍷 |
| 2 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 |
| 3 | 𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍸 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷 | 𝍷𝍷𝍷 |
| 4 | 𝍸 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷 | 𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 |
| 5 | 𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍸 𝍷𝍷 | 𝍸 𝍷 | 𝍷𝍷𝍷 |
| 6 | 𝍷𝍷𝍷𝍷 | 𝍸 𝍷𝍷 | 𝍸 𝍷 | 𝍸 | 𝍷𝍷𝍷 | 𝍸 𝍷𝍷 |
| 7 | 𝍷 | 𝍷 | 𝍷 | 𝍷 | 𝍷𝍷𝍷 | |
| 8 | 𝍷𝍷 | | | | | 𝍷 |
| 9 | 𝍷 | 𝍷 | 𝍷 | 𝍷 | 𝍷𝍷 | 𝍷 |
| | 25 | 25 | 25 | 25 | 25 | 25 |

| DIGIT | 600007 | 600008 | 600009 | 600010 | | |
|---|---|---|---|---|---|---|
| 0 | | | | | | |
| 1 | 𝍷 | 𝍷 | 𝍷𝍷 | 𝍷𝍷 | | 11 |
| 2 | 𝍸 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍸 𝍷 | | 43 |
| 3 | 𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷 | 𝍷𝍷𝍷 | | 35 |
| 4 | 𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷 | | 37 |
| 5 | 𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | | 41 |
| 6 | 𝍸 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | 𝍷𝍷𝍷𝍷 | | 49 |
| 7 | 𝍷𝍷 | 𝍷 | 𝍷𝍷𝍷 | 𝍷 | | 14 |
| 8 | 𝍷 | 𝍷 | | | | 5 |
| 9 | 𝍷𝍷 | 𝍷𝍷 | 𝍷𝍷 | 𝍷𝍷 | | 15 |
| | 25 | 25 | 25 | 25 | | 254 |

Name ____________________

## NUMBER GENERATION

Student Activity Sheet 1

| | STARTING NUMBERS | | | | |
|---|---|---|---|---|---|
| DIGIT | 100001 | 200001 | 300001 | 400001 | 500001 |
| 0 | | | | | |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | | | | | |
| total # of presses | | | | | |

| | STARTING NUMBERS | | | | |
|---|---|---|---|---|---|
| DIGIT | 600001 | 700001 | 800001 | 900001 | 1000001 |
| 0 | | | | | |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | | | | | |
| total # of presses | | | | | |

Name ____________________

## NUMBER GENERATION

Student Activity Sheet 2

| | STARTING NUMBERS | | | | |
|---|---|---|---|---|---|
| DIGIT | | | | | |
| 0<br>1<br>2<br>3<br>4 | | | | | |
| 5<br>6<br>7<br>8<br>9 | | | | | |
| total # of presses | | | | | |

| | STARTING NUMBERS | | | | |
|---|---|---|---|---|---|
| DIGIT | | | | | |
| 0<br>1<br>2<br>3<br>4 | | | | | |
| 5<br>6<br>7<br>8<br>9 | | | | | |
| total # of presses | | | | | |

Name ____________________

## NUMBER GENERATION

### Student Activity Sheet 3

| | STARTING NUMBERS | | | | |
|---|---|---|---|---|---|
| DIGIT | 600001 | 600002 | 600003 | 600004 | 600005 |
| 0 | | | | | |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | | | | | |
| total # of presses | | | | | |

| | STARTING NUMBERS | | | | |
|---|---|---|---|---|---|
| DIGIT | 600007 | 600008 | 600009 | 600010 | |
| 0 | | | | | |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | | | | | |
| total # of presses | | | | | |

## THIS LESSON RULES!

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Logic |
| **SKILL:** | Classification, logical reasoning, rounding, using function rules. |
| **MANAGEMENT**<br>**CLASS ORGANIZATION:** | Whole group, individual or pairs |
| **TIME FRAME:** | One or two math periods |
| **MATERIALS:** | Calculator, Data Recording Sheet |
| **VOCABULARY:** | Odd, even |
| **PREREQUISITE SKILL:** | Round numbers, order decimals, follow a defined rule, classification |

**LESSON**

To the teacher: This lesson is different from any you may have seen before and certainly different from any your students have experienced. It is one of the CAMP-LA Lessons on original mathematical research at the student's level. As a teacher -- stress to the students that they are taking on the role of "research mathematicians."

- **DIRECTED INSTRUCTION:**

1. "Here is a rule that can be applied to numbers. We'll call it Function Rule F." On the board write

**FUNCTION RULE F**
Multiply the number by 7
and then round down* to
the nearest whole number

*(Rounding down is also called "truncating" or "chopping off" the fractional part.)

As a class, apply Rule F to several starting numbers to see what happens. For example, if you start with 5.4 you first multiply by 7, getting 37.8. Rounding down to the nearest whole number gives 37. So, Rule F, applied to 5.4, gives 37. We can write:

$$5.4 \xrightarrow{F} 37$$

You can read it as: "5.4 is changed by Rule F to 37" or "Rule F sends 5.4 to 37." Try several other examples by asking your students to supply the missing numbers. Use examples such as these:

| | (answer) |
|---|---|
| $100 \xrightarrow{F} \underline{\ ?\ }$ | (700, no rounding needed) |
| $100.1 \xrightarrow{F} \underline{\ ?\ }$ | (700.7 → 700) |
| $100.2 \xrightarrow{F} \underline{\ ?\ }$ | (701.4 → 701) |

F

0.01 → ___?___ (0.07 → 0)
F
0.86 → ___?___ (6.02 → 6)

In discussing problems like the next three, be sure to get more than one starting number, but don't expect anyone to describe all possible starting numbers that work.
Begin with a number, apply Function Rule F, and get 35.

1

FUNCTION RULE F

___?___ → 35

Ask, "What could the starting number be?" Elicit student input. The function rule changes 5 into 35 and any number from 5 up to but not including $5\frac{1}{7}$ or 5.142857 ... into 35.

ANSWER KEY

FUNCTION RULE F

___?___ → 35

Any number between 5 and 5.142857 are possible numbers that work.

The range of products that will round down to 35 is 35 to 35.999999. The inverse of the function rule, (divide by 7) is applied above to find the range of missing factors.

2

FUNCTION RULE F

___?___ → 24

Ask, "What could the starting numbers be?" Solution: Divide 24 and 24.999999 by 7, that is, work backwards.

ANSWER KEY

FUNCTION RULE F

___?___ → 24

(3.4285714...
to
3.5714284...)

3

FUNCTION RULE F

_?_ → 44.1

Ask, "What could the starting numbers be?"
ANSWER KEY

FUNCTION RULE F

_?_ → 44.1

(no solution)

(Try any numbers as factors that students suggest, without comment, until they realize that no number will work since the result of applying Function Rule F must be a whole number.)

Function Rule F provides a way to separate all numbers into two groups - the ones that result in even numbers after applying Function Rule F and the numbers that result in odd numbers after applying Function Rule F:

| RESULT IN EVEN NUMBER | RESULT IN ODD NUMBER |
|---|---|
| | |

| RESULT IN EVEN NUMBER | RESULT IN ODD NUMBER |
|---|---|
| 100 (700) | 5.4 (37) |
| 100.1 (700) | 100.2 (701) |
| 3.49 (24) | 5.0 (35) |
| 0.01 (0) | 5.1 (35) |
| 0.86 (6) | $4\frac{1}{2}$ (31) |
| 5.5 (38) | |
| 3.52 (24) | |
| $3\frac{1}{2}$ (24) | |

Draw a chart on the board as above. Put the starting numbers in the proper columns. Your chart might look like the one on the right. The results of applying Function Rule F are in parentheses.

Ask the students what they notice. A few observations might be:
- If a starting number is an even whole number, it will go in the EVEN column (because 7 times any even number is even).
- If a starting number is an odd whole number it will go in the ODD column (because 7 times any odd number is odd).
- If a number is a whole number plus $\frac{1}{2}$, then it will go in the column other than the one for the whole number by itself (e.g. $4\frac{1}{2}$ goes in the odd column because $4\frac{1}{2} \rightarrow 31$, whereas 4 goes in the even column; $3\frac{1}{2}$ goes in the even column because $3\frac{1}{2} \rightarrow 24$, whereas 3 goes in the odd column).

- Check any observations students notice to verify accuracy.

2. Now introduce Rule G

**FUNCTION RULE G**
Multiply the number by 8
and then round down to the
nearest whole number.

Notice that the only difference between Rules G and F is that G multiplies by 8 (rather than 7) before rounding down.
Apply Rule G to some of the numbers you have already used as starting numbers. For example:

$5.4 \xrightarrow{G} \underline{\quad 43}$

$100 \xrightarrow{G} \underline{\quad ?}$ (800)

$100.1 \xrightarrow{G} \underline{\quad ?}$ (800)

$100.2 \xrightarrow{G} \underline{\quad ?}$ (801)

$5 \xrightarrow{G} \underline{\quad ?}$ (40)

$\underline{\quad ?\quad} \xrightarrow{G} \underline{\quad 24}$

(Any number from 3 up to, but not including, $3\frac{1}{8}$ or 3.125. On the calculator, the largest number that works is 3.1249999).

$\underline{\quad ?\quad} \xrightarrow{G}$ 35

(Any number from $4\frac{3}{8}$ or 4.375 up to, but not including, $4\frac{1}{2}$ or 4.5).

3. Now draw a chart that allows us to classify a number according to both rules simultaneously. Label the four cells A, B, C and D for convenience.

RESULT OF USING RULE F

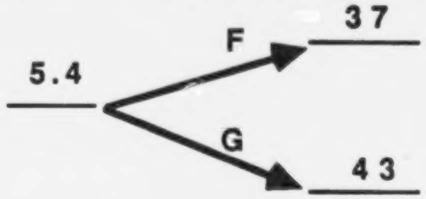
| | | EVEN | ODD |
|---|---|---|---|
| RESULT OF USING RULE G | EVEN | A. | B. |
| | ODD | C. | D. |

Hand out Student Activity Sheet 1 and the Data Recording Sheet.

To place a number on this chart you must use both rules F and G on your starting number. For example, which cell does 5.4 go in? First apply F to 5.4. Write

$$5.4 \xrightarrow{F} 37$$

37 is odd, so the starting number 5.4 must go in the "ODD" column (cell B or D). What does Rule G do to 5.4? Continue your diagram like this:



43 is also odd, so 5.4 must go in the "ODD" row (cell C or D). Conclude that 5.4 must go in cell D and write it in.

RESULT OF USING RULE F

| | | EVEN | ODD |
|---|---|---|---|
| RESULT OF USING RULE G | EVEN | A. | B. |
| | ODD | C. | D.<br>5.4 |

Students should record these results on their own chart and Data Recording Sheet. Similarly, they put other starting numbers you have already used into their proper cells.

- **INDEPENDENT PRACTICE:**

Encourage students to use their Data Recording Sheets to find other numbers that would go in each of the cells.
When the students have had a chance to put several numbers in the chart, discuss class observations of results. As part of this discussion ask questions like:

- What are some numbers between 0 and 0.2 that would go in cell A? (Anything between 0 and 0.1249999.)

- What are some numbers between 0 and 0.2 that would go in cell C? (Anything between $\frac{1}{8}$ and $\frac{1}{7}$ i.e between 0.125 and 0.1428571.)

- What are some numbers between 0 and 0.2 that would go in cell B? (There aren't any.)

- Where do the whole numbers go? (Rule G, applied to any whole number, results in an even number, so all whole numbers must go in cells A or B. We already know that the even whole numbers must go in A or C, so in fact they must go in A. Similarly, the odd whole numbers must go in B.)

Extensions:

1. Suppose you used the following 100 starting numbers:
   0.00, 0.01, 0.02, 0.03, ..., 0.98, 0.99.

How do you think they would be distributed among the four cells? Would each cell get 25 of them? Look for efficient ways to decide where the 100 numbers go. (The chart below shows, surprisingly, that the 100 numbers are not distributed equally among the cells.)

RESULT OF USING RULE F

| | | EVEN | ODD |
|---|---|---|---|
| RESULT OF USING RULE G | Even | A. .00 .29 .58 .86<br>.01 .3 .59 .87<br>.02 .31 .6<br>.03 .32 .61<br>.04 .33 .62<br>.05 .34<br>.06 .35<br>.07 .36<br>.08 .37<br>.09<br>.1<br>.11<br>.12<br>(29 numbers) | B. .25 .5 .75<br>.26 .51 .76<br>.27 .52 .77<br>.28 .53 .78<br>.54 .79<br>.55 .8<br>.56 .81<br>.57 .82<br>.83<br>.84<br>.85<br>(23 numbers) |
| | Odd | C. .13 .38 .63 .88<br>.14 .39 .64 .89<br>.4 .65 .9<br>.41 .66 .91<br>.42 .67 .92<br>.68 .93<br>.69 .94<br>.7 .95<br>.71 .96<br>.97<br>.98<br>.99<br>(28 numbers) | D. .15 .43 .72<br>.16 .44 .73<br>.17 .45 .74<br>.18 .46<br>.19 .47<br>.2 .48<br>.21 .49<br>.22<br>.23<br>.24<br>(20 numbers) |

Students who get this far should be encouraged to continue by looking at these 100 numbers: 1.00, 1.01, 1.02, ... 1.98,1.99. They will find that the full set of 200 numbers are more evenly distributed among the cells.)

2. Drop Rule G and replace it with Rule H. Hand out Student Activity Sheet 2.

**RULE H**
Multiply the number by 9
and then round down to
the nearest whole number.

Students explore how this rule change affects the classification of starting numbers. More generally, students can explore what happens when the rules are changed to multiply by other numbers before rounding down.

Name________________________

## THIS LESSON RULES!

Student Activity Sheet 1

**RULE F**
Multiply the number by 7
and then round down to
the nearest whole number

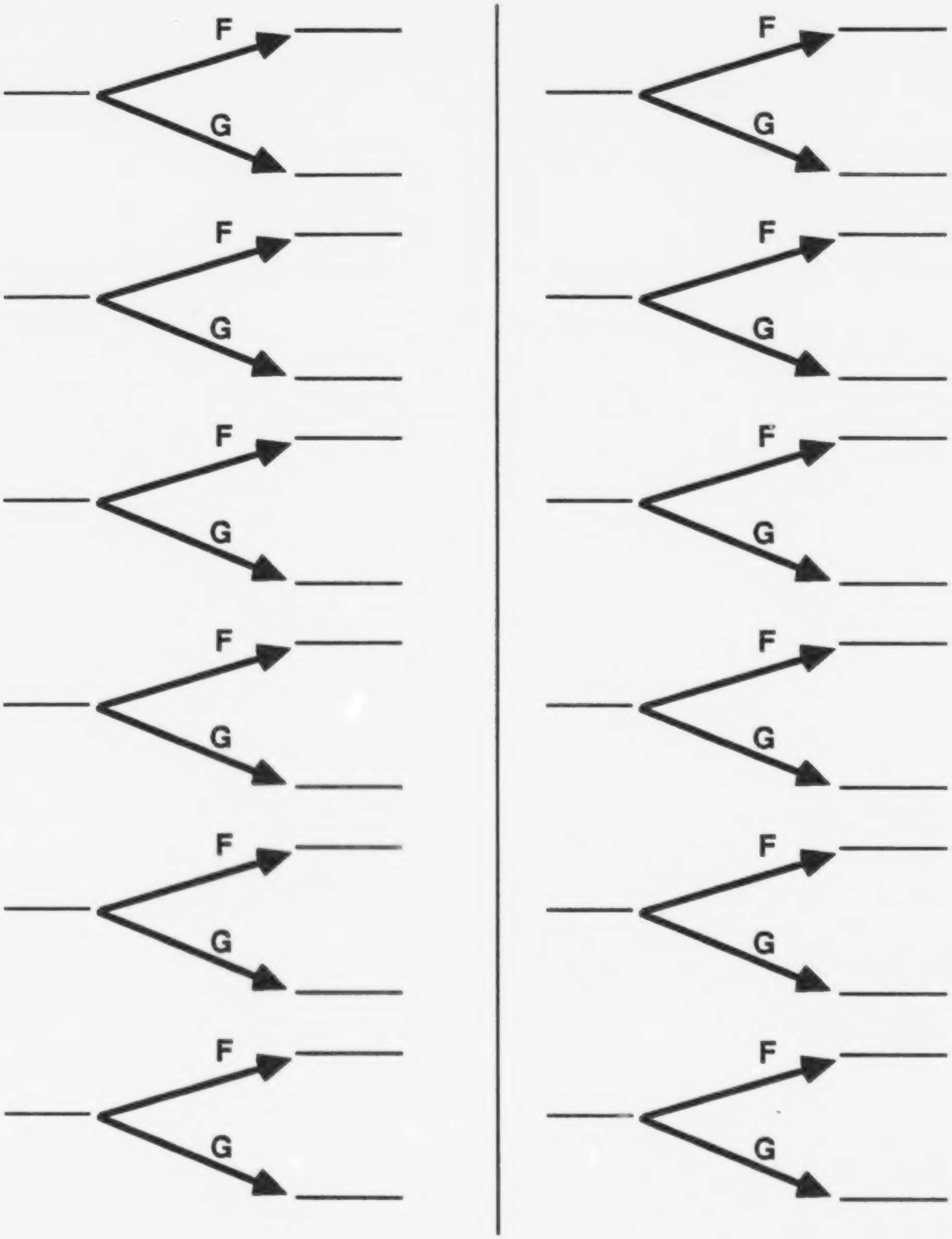
**RULE G**
Multiply the number by 8
and then round down to
the nearest whole number

|  |  | RESULT OF USING RULE F |  |
|---|---|---|---|
|  |  | EVEN | ODD |
| RESULT OF USING RULE G | EVEN | A. | B. |
|  | ODD | C. | D. |

Name ______________________________

**THIS LESSON RULES!**

Data Recording Sheet


F
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G

Name______________________

## THIS LESSON RULES!

Student Activity Sheet 2

**RULE F**
Multiply the number by 7
and then round down to
the nearest whole number

**RULE H**
Multiply the number by 9
and then round down to
the nearest whole number

| | | RESULT OF USING RULE F | |
|---|---|---|---|
| | | EVEN | ODD |
| RESULT OF USING RULE H | EVEN | A. | B. |
| | ODD | C. | D. |

0123456789

# CHAPTER 2 ASSESSMENT:
## LOGIC, PROBABILITY AND STATISTICS

1. You are on a television game show. Your challenge is to pick exactly 19 bills that are worth a total of $500 from stacks of $20 bills and $50 bills. How many of each bill would you select?

   Explain how you arrived at your answer.

   Student response: 15 - $20 bills
   4 - $50 bills

   Students should use a chart or other organized method of solution.

2. 1492 - - Columbus sailed the ocean blue. Make a number sentence that is true. Use the digits 1, 4, 9, 2 once and only once to equal the numbers 1 to 20. No other digits may be used. The four operation signs, x, -, +, and ÷, and also square root and exponents may be used as often as you wish.

   Student responses will vary. One solution for each number from 1 to 20 is shown.

| | |
|---|---|
| 1 $= (2-1) \times (\sqrt{9} - \sqrt{4})$ | 11 $= 9 + 4 - (2 \times 1)$ |
| 2 $= 9 - (1 + 4 + 2)$ | 12 $= 9 + 4 - 2 + 1$ |
| 3 $= \frac{9 - (4 + 2)}{1}$ | 13 $= 9 \times 2 - (4 + 1)$ |
| 4 $= (9 - 2) - (4 - 1)$ | 14 $= (9 \times 2) - (4 \times 1)$ |
| 5 $= 24 - 19$ | 15 $= 29 - 14$ |
| 6 $= (4 + 2) \times 1 \times \sqrt{9}$ | 16 $= 9 + 4 + 2 + 1$ |
| 7 $= \frac{1 + 4 + 9}{2}$ | 17 $= 4 \times 2 + 9 \times 1$ |
| 8 $= 9 - (4 + 2) + 1$ | 18 $= 4 \times 2 + 9 + 1$ |
| 9 $= \frac{1 + 9}{2} + 4$ | 19 $= 4 \times 9 + 2 + 1$ |
| 10 $= (9 - 4) \times 2 \times 1$ | 20 $= 9 \times 2 \times 1 + \sqrt{4}$ |

3. This week your class is publishing a newspaper. You were assigned the role of weatherperson. Your task is to measure and record the weather three times a day for five days. Your record of the morning, noon, and night temperatures are on the chart below. Describe the average temperature. Would you use mean, median, mode, or your own method of finding an average? Does your answer accurately describe the weather? Should you use all the data? Discuss how you arrived at your conclusions.

**Temperature Readings In Degrees Fahrenheit**

| | Day 1 | Day 2 | Day 3 | Day 4 | Day 5 |
|---|---|---|---|---|---|
| 7:00 am | 57 | 59 | 61 | 61 | 72 |
| 12:00 noon | 70 | 72 | 71 | 73 | 92 |
| 7:00 pm | 62 | 61 | 60 | 59 | 75 |

Student responses will vary. If all data are used, the median is 62. (62 is the number in the middle when all data are listed from smallest to largest.) The mode is 61 because it occurs most often. The mean is 67 because 1009 ÷ 15 = 67. Students may feel it is best to use only one time of day. They may question the appropriateness of the times of the day that were chosen.

3. Write a problem that requires collecting data and the use of mean, median, and mode.

Student responses may vary.

4. CAMP-LA Ice Cream store has 37 different flavors to offer. Discuss how many different ways you can make a two-scoop ice cream cone. Chocolate on top of vanilla is considered different than vanilla on top of chocolate.

Student response: 1369. There are 37 choices for the first scoop and 37 choices for the second scoop. There are 37 x 37 = 1369 total possibilities.

5. Choose five different recording artists. Design a survey to determine the percent and number of students that prefer each artist. Display the results using charts and circle graphs. Interpret your findings and compare them with other students' results.

Student response will vary.

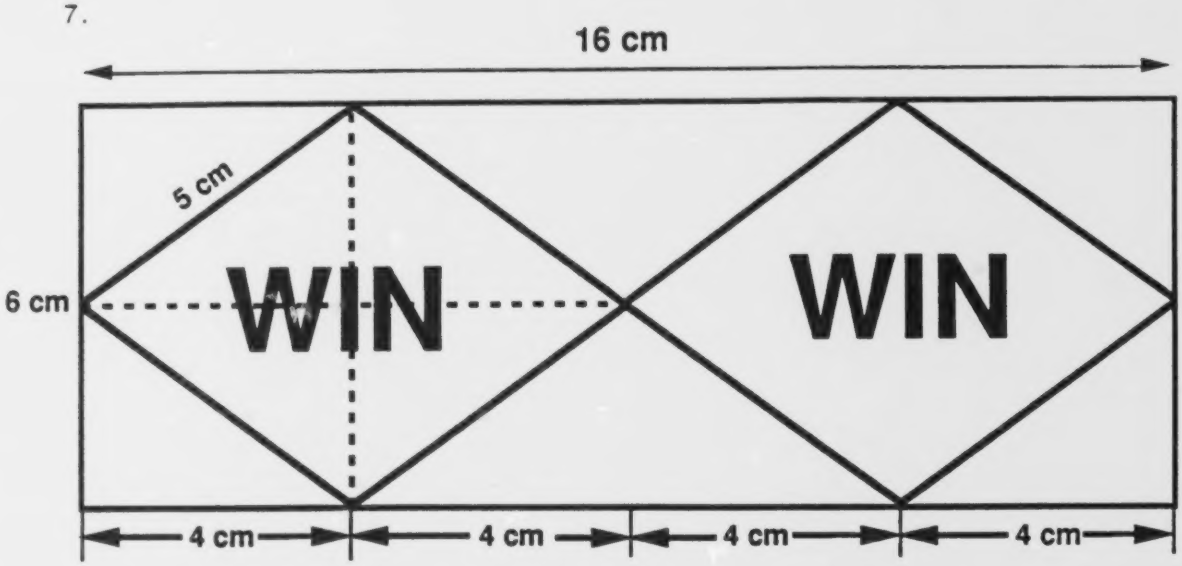
6. You have been invited for lunch at a restaurant that has a salad bar with 7 items. This restaurant has some unusual restrictions on what you can choose at their salad bar. There is a sign posted that states you may take no more than five different items. Suppose you decide to take the maximum number of items (five). How many different salad combinations could you choose? Show the method you used to get your answer.

Student responses should either include an organized chart or use the combination rule from the lesson "Lets Party."

$$\frac{7!}{5! \times 2!} = \frac{5040}{120 \times 2} = 21$$

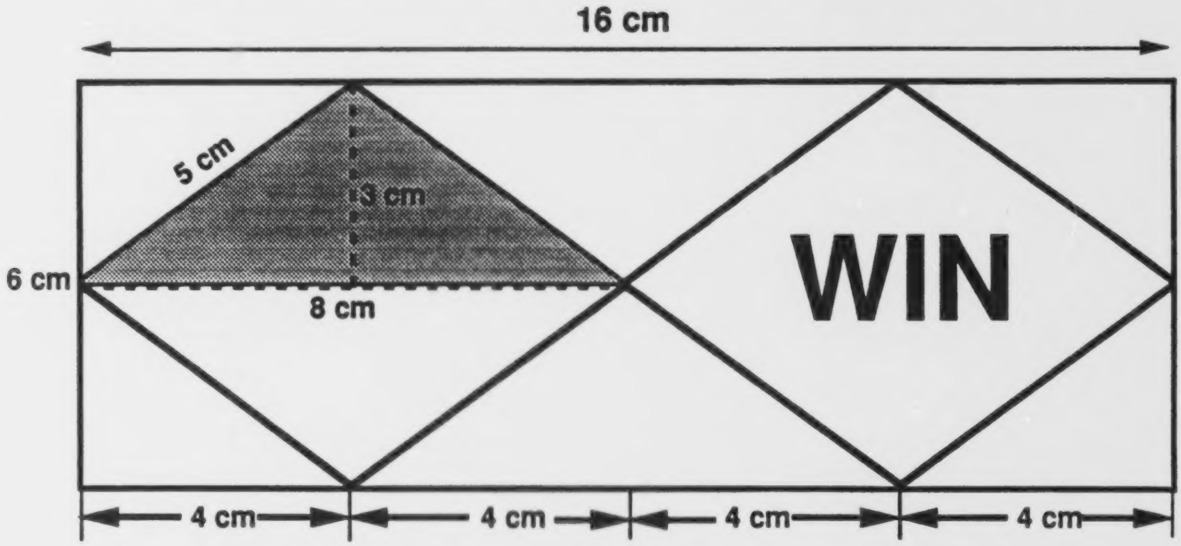
152

7.


16 cm
5 cm
6 cm
WIN
WIN
4 cm
4 cm
4 cm
4 cm

Discuss the probability of winning with this game board. A win occurs when more then $\frac{1}{2}$ of a dropped coin lands in the win area. (Hint: Divide shapes into triangles to compute areas.)

Student responses should show how they computed the WIN area and the total area.


16 cm
5 cm
3 cm
6 cm
8 cm
WIN
4 cm
4 cm
4 cm
4 cm

The shaded triangle has base 8 and height 3 for an area of $.5 \times 8 \times 3 = 12$ $\text{cm}^2$. The WIN area is made up of 4 triangles of that size. Its area is $4 \times 12 = 48$ $\text{cm}^2$. The entire board is a rectangle with area $6 \times 16 = 96$ $\text{cm}^2$ the probability of winning is $\frac{48}{96} \times 100 = 50\%$.

8. Function Rule E: Multiply a number by 6 and round down to the nearest whole number.

a. Find some numbers, which will be changed by using the Function Rule E, into the whole number 36.

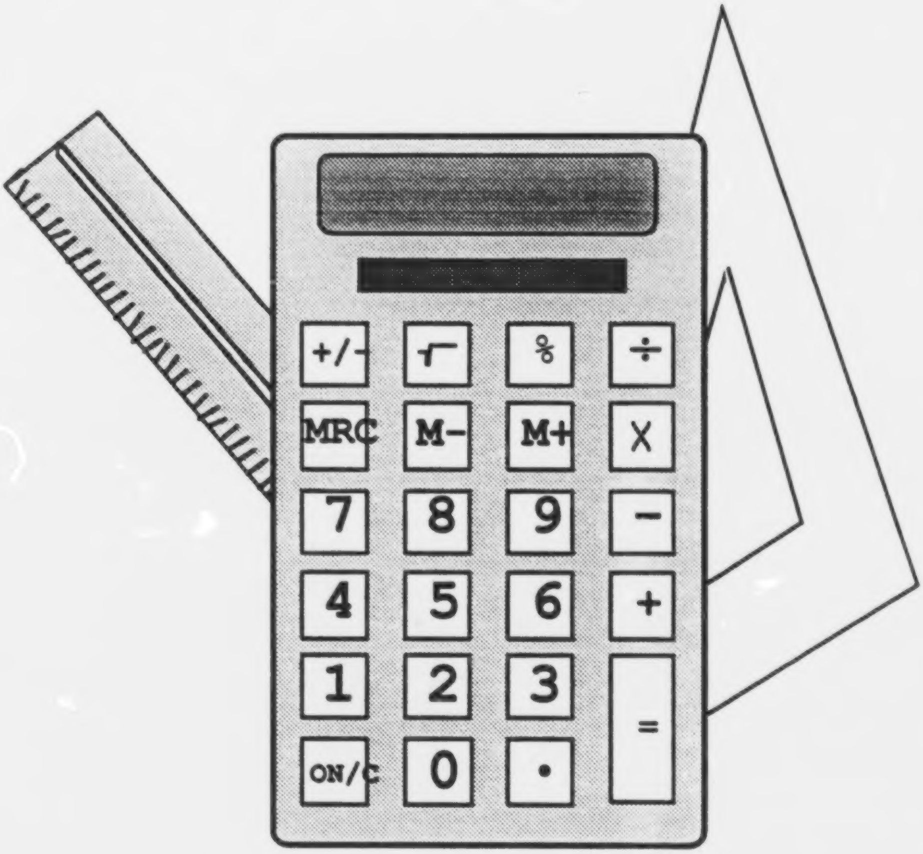
b. What is the range of numbers that Rule E will change into the whole number 36? Discuss the method you used to find the result.

Student responses will vary. The numbers must be less than 6.16666... and greater than or equal to 6. The range can be found from the results of two division problems.

$$37 \div 6 = 6.16666 \ldots$$
$$36 \div 6 = 6$$

# CAMP-LA

**Calculators and Mathematics Project, Los Angeles**


+/-
√
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5
6
+
1
2
3
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.

## Grades 7 - 8

# CHAPTER 3
## MEASUREMENT/ GEOMETRY

# HOW DO YOU SPEND YOUR TIME?

**GRADE:** 7 - 8

**STRAND:** Measurement

**SKILL:** Measure time in hours and minutes. Regroup when adding and subtracting hours and minutes.

**MANAGEMENT:**

**CLASS ORGANIZATION:** Pairs

**TIME FRAME:** Two or three math periods

**MATERIALS:** Calculator, protractor, school year calendar

**VOCABULARY:** Hour, minute, a.m., p.m., leisure time, sibling

**PREREQUISITE SKILL:** Experience with fractions, decimals, and percents; measure angles and construct circle graphs; change fractions to decimals

## LESSON

- **DIRECTED INSTRUCTION:**

The purpose of this lesson is to make students aware of how they spend their time.

1. Distribute Student Activity Sheet 1, which is a sample of a student's school day. Begin by computing the number of hours in class each day at Hamada Elementary School. Class starts at 8:00 a.m. and ends at 2:45 p.m. This school has a 30 minute lunch and a 15 minute recess. How many hours are spent in class?

   8:00 a.m. to 2:45 p.m. = 6 hours 45 minutes
   Subtract 45 minutes for lunch and recess

   Total amount of class time = 6 hours

2. With a partner, students complete the first column to show 24 hours of a student's school day.

3. Use the information in the Hours Spent column to complete the remainder of the chart. Note: If you spend 6 hours in class, then the fraction of the school day is 6/24. The decimal equivalent is 6 ÷ 24 = 0.25 which is 25%. The number of degrees in the circle graph for class time is .25 x 360 degrees = 90 degrees.

4. Use a protractor with the information on Student Activity Sheet 1 to complete the circle graph.

- **GUIDED PRACTICE:**
  - Hand out and have students complete Student Activity Sheet 2. Discuss results.
  - Before students complete Student Activity Sheet 3, discuss:

    1. You are in school 178 days a year.

       - How many hours have you spent in school compared to the number of hours in a year?

       - How many hours a year do you sleep? Eat?

       - What leisure activities occupy your time each year?

    2. What percent of the year is made up of weekends? School holidays?

- **INDEPENDENT PRACTICE:**
  Complete Student Activity Sheet 3. Discuss results.

- **EVALUATION:**
  Observation of responses on Student Activity Sheets.

- **HOME ACTIVITY:**
  Students complete Home Activity Sheets 1 and 2.

Name ____________________

## HOW ONE HAMADA ELEMENTARY SCHOOL STUDENT SPENDS HIS TIME

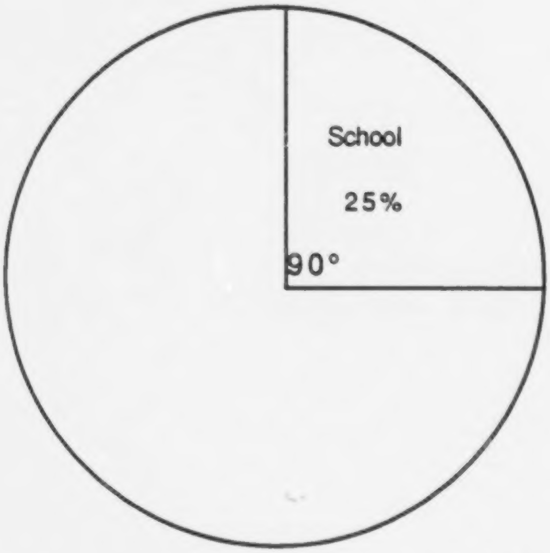
Student Activity Sheet 1

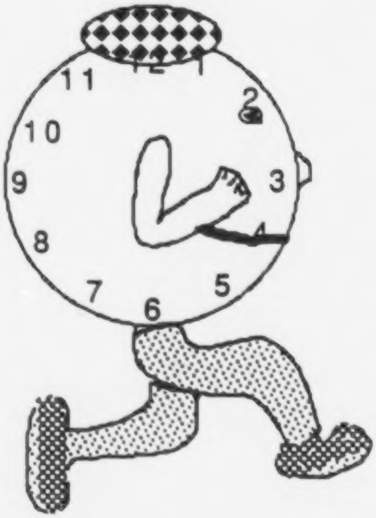
Circle graphs are used to picture percents.

Directions: Complete the chart and the circle graph to find how a student might spend his day. (Round off numbers when appropriate.)

| **Activity** | **Hours Spent** | **Fraction** | **Decimal** | **Decimal x 100 = Percent** | **Decimal x 360° = degrees for graph below** |
|---|---|---|---|---|---|
| Class Time | 6 | 6/24 | 0.25 | 25% | 90° |
| Eating | | | | | |
| Sleeping | 9 | | 0.375 | 37.5% | |
| Doing Homework | | | | | |
| Leisure Time | | | | | |
| **Total** | **24** | **1** | **1.0** | **100%** | **360°** |

Complete the circle graph using a protractor.





Name ______________

## HOW DO YOU SPEND YOUR TIME?

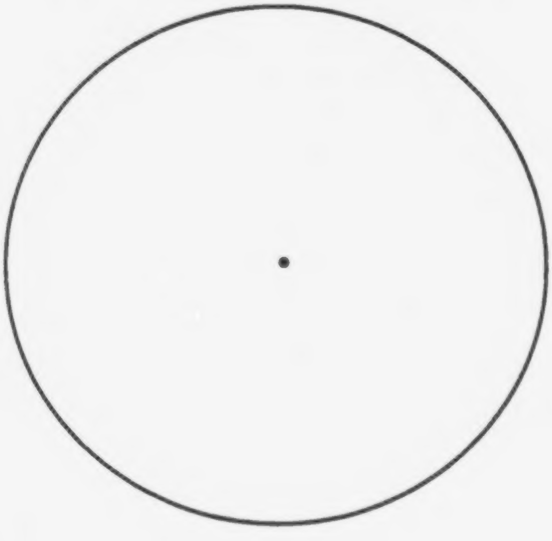
Student Activity Sheet 2

Circle graphs are used to picture percents.

Directions: Complete the chart and the circle graph to find how you spend your day. (Round off numbers when appropriate.)

| Activity | Hours Spent | Fraction | Decimal | Decimal x 100 = Percent | Decimal x 360° = degrees for graph below |
|---|---|---|---|---|---|
| School | | | | | |
| Eating | | | | | |
| Sleeping | | | | | |
| Doing Homework | | | | | |
| Leisure Time | | | | | |
| **Total** | | | | | |

Complete the circle graph using a protractor.



11
12
1
2
10
9
3
8
4
7
6
5

Name ____________________

## HOW DO YOU SPEND YOUR TIME?
Student Activity Sheet 3

Circle graphs are used to picture percents.

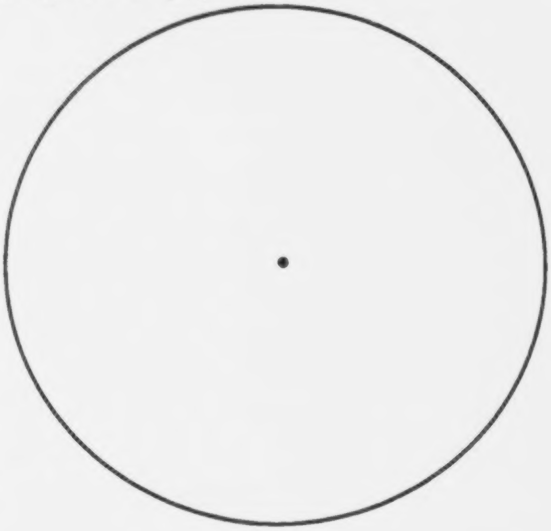
Hours in a day: ____________________
Hours in a year: ____________________
(365 days x 24 hours)

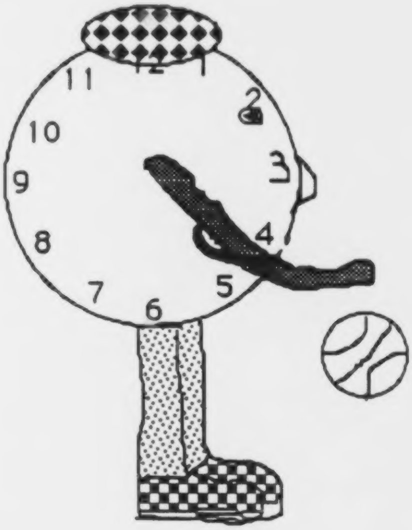
Directions: Use the total number of hours to complete the chart below to find **what percent of your year is leisure time.**

Predict first and record ➔ ☐ %

| Activity | Hours | Fraction | Decimal | Percent | Degrees |
|---|---|---|---|---|---|
| Hours in school:<br>_____ days x 6 hours | | | | | |
| Hours for personal needs:<br>(365 x _____ hours each day)<br>[Eating, sleeping,homework] | | | | | |
| Remaining leisure time: | | | | | |
| **Total** | 24 | 1 | 1.0 | 100% | 360° |

Complete the graph.





Name____________________

## HOW DO YOU SPEND YOUR TIME?

### Home Activity Sheet 1

Ask a parent, sister, or brother about their activities, and then complete the charts. If you do not have any brothers or sisters ask another student who is not in your class.

Directions: Complete the chart and circle graph to **find what percent of their day is leisure time.**

**Parent**

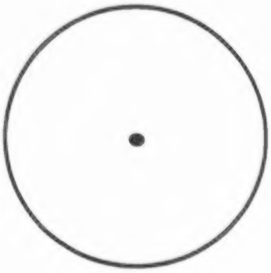
| Activity | Hours | Fraction | Decimal | Percent | Degrees |
|---|---|---|---|---|---|
| Work or School | | | | | |
| Eating | | | | | |
| Sleeping | | | | | |
| Leisure time | | | | | |
| Total | | | | 100% | 360° |

**Sibling or Friend**

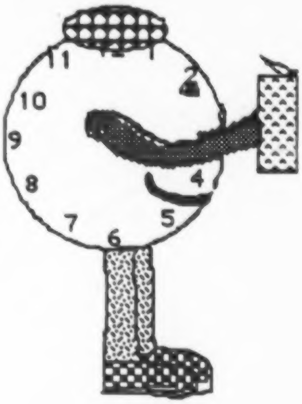
| Activity | Hours | Fraction | Decimal | Percent | Degrees |
|---|---|---|---|---|---|
| Work or School | | | | | |
| Eating | | | | | |
| Sleeping | | | | | |
| Leisure time | | | | | |
| Total | | | | 100% | 360° |

Complete the circle graphs

Parent



Sibling



Name____________________

## HOW DO YOU SPEND YOUR TIME?

Home Activity Sheet 2

Hours in a day: ____________________

Hours in a year: ____________________

(365 days x 24 hours)

Directions: Use the total number of hours to complete the chart below to find **what percent of your year is leisure time.**

**Parent**

| Activity | Hours | Fraction | Decimal | Percent | Degrees |
|---|---|---|---|---|---|
| Hours at work or school:<br>______ days x______ hours | | | | | |
| Hours for personal needs:<br>(365 x _____ hours each day)<br>[Eating, sleeping,etc.] | | | | | |
| Remaining leisure time: | | | | | |
| **Total** | | | | 100% | 360° |

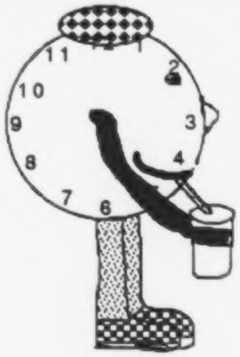
**Sibling or Friend**

| Activity | Hours | Fraction | Decimal | Percent | Degrees |
|---|---|---|---|---|---|
| Hours at work or school:<br>______ days x______ hours | | | | | |
| Hours for personal needs:<br>(365 x _____ hours each day)<br>[Eating, sleeping,etc.] | | | | | |
| Remaining leisure time: | | | | | |
| **Total** | | | | 100% | 360° |

Complete the circle graph

Parent

Sibling



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# I HAVE, WHO HAS?

**GRADE:** 7 - 8

**STRAND:** Geometry

**SKILL:** Identify geometric shapes, find perimeters.

## MANAGEMENT

**CLASS ORGANIZATION:** Whole class

**TIME FRAME:** One math period

**MATERIALS:** Calculator, transparency for directed lesson "I Have, Who Has?" Master for game cards (cut prior to lesson)

**VOCABULARY:** Perimeter, regular polygon, square, rhombus, rectangle, parallelogram, trapezoid, quadrilateral, regular pentagon, regular hexagon, regular octagon, regular decagon

**PREREQUISITE SKILL:** Find perimeter, differentiate between polygons

## LESSON

- **DIRECTED INSTRUCTION:**
  - (Use transparency of Polygons) Review vocabulary and characteristics of the following polygons: pentagon, hexagon, octagon, decagon, rhombus, parallelogram and triangle (scalene, isosceles and equilateral.) Students can draw their own shapers to practice vocabulary use of names of polygons.
  - Students practice finding perimeters, and finding the length of an unknown side when the perimeter is given.

- **GUIDED PRACTICE:**
  1. Shuffle all cards. Hand out the entire set of 36 "I Have, Who Has?" game cards. Some students may have more than 1 card.
  2. Choose a student to start the game by reading the "Who has?" question on the game card. Example: Who has the perimeter of a regular hexagon whose sides measure 4.8 units?
  3. All students compute the answer using mental math or their calculator.
  4. If no one computes the perimeter correctly, assist the student who asked the question to give some hints as to the characteristics of a regular hexagon so they can successfully compute the perimeter.
  5. The student, who has the card with the correct answer, reads the entire card. "I have 28.8 units. Who has the length of a rectangle with a perimeter of 8.3 units and a width of 1.7 units?"

- **INDEPENDENT PRACTICE:**

  Continue the game until all cards are read. The student who was chosen to start will be the last to speak and will read the "I Have " statement only.

  **Options** - Choose a student to use a stopwatch to time the game. Trade cards and do the same game a second time using another student to begin.

Another option would be to cut the cards into 2 parts - one containing the "I Have" and one containing the "Who Has?" Give each student 1 of each. Students will be motivated to improve their previous response time and will become faster problem solvers.

**Caution** - Teacher needs to follow along on Teacher Master Sheet to make sure correct answers are being given and to prompt when necessary. The responses are in order.

- **EVALUATION:**
  - Successful completion of the "I Have, Who Has" game.
- **HOME ACTIVITY:**

  Each student writes a five question "I Have, Who Has?" game.

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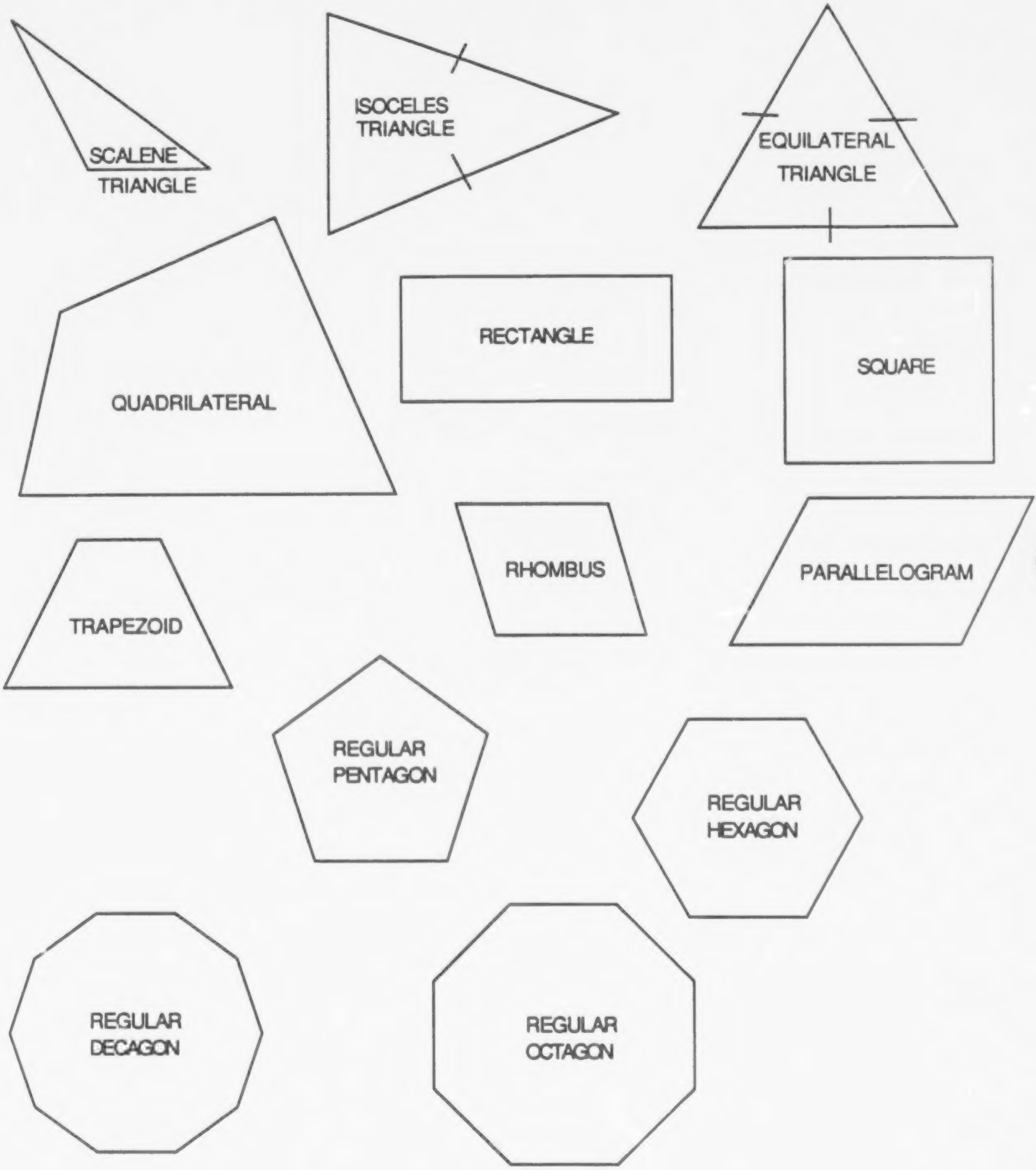
## I HAVE, WHO HAS?

Transparency


SCALENE TRIANGLE
ISOCELES TRIANGLE
EQUILATERAL TRIANGLE
QUADRILATERAL
RECTANGLE
SQUARE
TRAPEZOID
RHOMBUS
PARALLELOGRAM
REGULAR PENTAGON
REGULAR HEXAGON
REGULAR DECAGON
REGULAR OCTAGON

## I HAVE, WHO HAS?

Teacher Master Copy

I have 34 units.
Who has the perimeter of a square whose sides measure 4.1 units?

I have 16.4 units.
Who has the perimeter of a equilateral triangle whose sides measure 3.5 units?

I have 10.5 units.
Who has the perimeter of a rectangle with a width of 2.8 units and a length of 4.9 units?

I have 15.4 units.
Who has the perimeter of a regular octagon whose sides measure 2.25 units?

I have 18 units.
Who has the perimeter of a regular hexagon whose sides measure 3.8 units?

I have 22.8 units.
Who has the perimeter of a rectangle with a length of 13.6 units and a width of 2.75 units?

I have 32.7 units.
Who has the perimeter of a rhombus whose sides measure 5.125 units?

I have 20.5 units.
Who has the perimeter of a quadrilateral whose sides measure 10, 9.4, 6.7, and 8.8 units?

I have 34 .9 units.
Who has the perimeter of an isosceles triangle two of whose sides measure 7.65 units and whose third side measures 1.7 units?

I have 17 units.
Who has the perimeter of a rectangle with a width of 5 units and a length of twice the width?

I have 30 units.
Who has the perimeter of a scalene triangle whose sides measure 1.5, 2, and 2.5 units?

I have 6 units.
Who has the length of a side of a square whose perimeter is 25 units?

I have 6.25 units.
Who has the perimeter of a regular pentagon whose sides measure 3.2 units?

I have 16 units.
Who has the length of one side of an isosceles triangle with a perimeter of 20 units. The other two sides measure 8 units each?

I have 4 units.
Who has the perimeter of a parallelogram whose sides measure 3.8 and 9.6 units?

I have 26.8 units.
Who has the width of a rectangle with a perimeter of 30.2 units and a length of 9.5 units?

I have 5.6 units.
Who has the perimeter of a regular decagon whose sides measure 2.5 units?

I have 25 units.
Who has the perimeter of a quadrilateral whose sides measure 1.2, 3, 5.4 and 1.25 units?

I have 10.85 units.
Who has the length of a side of a square with a perimeter of 125 units?

I have 31.25 units.
Who has the length of a side of a regular pentagon with a perimeter of 35.5 units?

I have 7.1 units.
Who has the perimeter of a rectangle with a length of 2.9 units and a width of 5.8 units?

I have 17.4 units.
Who has the length of a side of a regular octagon with a perimeter of 98 units?

I have 12.25 units.
Who has the perimeter of a quadrilateral whose sides measure 2.3, 1.8, 5.7, and 3.4 units?

I have 13.2 units.
Who has the perimeter of a regular hexagon whose sides measure 4.8 units?

I have 28.8 units.
Who has the length of a rectangle with a perimeter of 8.3 units and a width of 1.7 units?

I have 2.45 units.
Who has the perimeter of a regular pentagon whose sides measure 3.7 units?

I have 18.5 units.
Who has the side of a regular decagon with a perimeter of 98.42 units?

I have 9.842 units.
Who has the perimeter of a equilateral triangle whose sides measure 5.3 units?

I have 15.9 units.
Who has the length of a side of a regular hexagon with a perimeter of 39.6 units?

I have 6.6 units.
Who has the perimeter of a rectangle with a length of 8.4 units and a width of 4.2 units?

I have 25.2 units.
Who has the length of the third side of an isosceles triangle with a perimeter of 34 if the two other sides measure 12.4 units each?

I have 9.2 units.
Who has the perimeter of a quadrilateral whose sides measure 3.6, 5.8, 6.5 and 4.8 units?

I have 20.7 units.
Who has the perimeter of a rhombus whose sides measure 7.25 units?

I have 29 units.
Who has the perimeter of a rectangle with a length of 5.8 units and a width of 4.7 units?

I have 21 units.
Who has the length of a side of a regular pentagon whose perimeter is 19.5 units?

I have 3.9 units.
Who has the perimeter of a scalene triangle whose sides measure 12.2, 14.6 and 7.2 units?

I have 7.1 units

Who has the perimeter of a rectangle with a length of 2.9 units and a width of 5.8 units?

I have 2.45 units.

Who has the perimeter of a regular pentagon whose sides measure 3.7 units?

I have 17.4 units.

Who has the length of a side of a regular octagon with a perimeter of 98 units?

I have 18.5 units.

Who has the side of a regular decagon with a perimeter of 98.42 units?

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Who has the perimeter of a quadrilateral whose sides measure 2.3 units, 1.8 units, 5.7 units and 3.4 units?

I have 9.842 units.

Who has the perimeter of an equilateral triangle whose sides measure 5.3 units?

I have 13.2 units.

Who has the perimeter of a regular hexagon whose sides measure 4.8 units?

I have 15.9 units.

Who has the length of a side of a regular hexagon with a perimeter of 39.6 units?

I have 28.8 units.

Who has the length of a rectangle with a perimeter of 8.3 units and a width of 1.7 units?

I have 6.6 units.

Who has the perimeter of a rectangle with a length of 8.4 units and a width of 4.2 units?

| | |
|---|---|
| I have 26.8 units.<br>Who has the width of a rectangle with a perimeter of 30.2 units and a length of 9.5 units? | I have 30 units.<br>Who has the perimeter of a scalene triangle whose sides measure 1.5 units, 2 units and 2.5 units? |
| I have 5.6 units.<br>Who has the perimeter of a regular decagon whose sides measure 2.5 units? | I have 6 units.<br>Who has the length of a side of a square whose perimeter is 25 units? |
| I have 25 units.<br>Who has the perimeter of a quadrilateral whose sides measure 1.2 units, 3 units, 5.4 units and 1.25 units? | I have 6.25 units.<br>Who has the perimeter of a regular pentagon whose sides measure 3.2 units? |
| I have 10.85 units.<br>Who has the length of a side of a square with a perimeter of 125 units? | I have 16 units.<br>Who has the length of one side of an isosceles triangle with a perimeter of 20 units. The other two sides measure 8 units each? |
| I have 31.25 units.<br>Who has the length of a side of a regular pentagon with a perimeter of 35.5 units? | I have 4 units.<br>Who has the perimeter of a parallelogram whose sides measure 3.8 and 9.6 units? |

I have 34 units.

Who has the perimeter of a square whose sides measure 4.1 units?

I have 22.8 units?

Who has the perimeter of a rectangle with a length of 13.6 units and a width of 2.75 units?

I have 16.4 units.

Who has the perimeter of an equilateral triangle whose sides measure 3.5 units?

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Who has the perimeter of a regular octagon whose sides measure 2.25 units?

I have 34.9 units.

Who has the perimeter of an isosceles triangle, two of whose sides measure 7.65 units, and whose third side measures 1.7 units?

I have 18 units.

Who has the perimeter of a regular hexagon whose sides measure 3.8 units?

I have 17 units.

Who has the perimeter of a rectangle with a width of 5 units and a length of twice the width?

I have 25.2 units.

Who has the length of the third side of an isosceles triangle with a perimeter of 34 if the other 2 sides measure 12.4 units each?

---

I have 9.2 units

Who has the perimeter of a quadrilateral whose sides measure 3.6 units, 5.8 units, 6.5 units, and 4.8 units?

---

I have 20.7 units.

Who has the perimeter of a rhombus whose sides measure 7.25 units?

---

I have 29 units.

Who has the perimeter of a rectangle with a length of 5.8 units and a width of 4.7 units?

---

I have 21 units.

Who has the length of a side of a regular pentagon whose perimeter is 19.5 units?

---

I have 3.9 units.

Who has the perimeter of a scalene triangle whose sides measure 12.2 units, 14.6 units and 7.2 units?

## IT'S ALL IN HOW YOU LOOK AT IT

| | |
|---|---|
| GRADE: | 7 - 8 |
| STRAND: | Measurement/Geometry |
| SKILL: | Find areas of triangles |

### MANAGEMENT

| | |
|---|---|
| CLASS ORGANIZATION: | Whole class, pairs |
| TIME FRAME: | Two math periods |
| MATERIALS: | Calculator, Transparency of Student Activity Sheet 1, metric rulers |
| VOCABULARY: | Height, base, vertex, perpendicular, line segment |
| PREREQUISITE SKILL: | Measurement skills |

### LESSON

This lesson shows that any of the 3 sides of a triangle can be thought of as the base when using the formula "A = .5 x base x height" to find the area of a triangle.

Note: .5 is used instead of $\frac{1}{2}$ because it is easier to use with the calculator.

Throughout this lesson the word **height** will refer to both a triangle's altitude and the measure of the altitude.

- **DIRECTED INSTRUCTION:**
  - Hand out Student Activity Sheet 1.
  - Review listed vocabulary.
  - Tell the class:
    - Solid lines form the triangle ABC
    - Dashed lines are either extensions of sides or heights of the triangle.
  - Students turn their paper so that $\overline{AC}$ is parallel to the edge of their desks. For now, $\overline{AC}$ is the base.
  - Tell the class:
    - The height is drawn from the vertex opposite a base to the line containing the base, so that it is perpendicular to the base.
    - In some cases, the height will touch the base, and in other instances, it touches an extension of the base.
  - Students answer question 1.
  - Discuss why $\overline{BD}$ is the height.
  - Students follow a similar process for $\overline{AB}$ and $\overline{BC}$ to answer question 2 and 3.
  - Discuss results. Do the first line of the chart with the class then students complete the chart on the bottom of their activity sheet.
  - Ask students if their 3 answers for the area of the triangle ABC are exactly the same.
  - Discuss that all measurements involve approximations, so that the three area measurements should be close to each other but will not necessarily come out exactly the same.

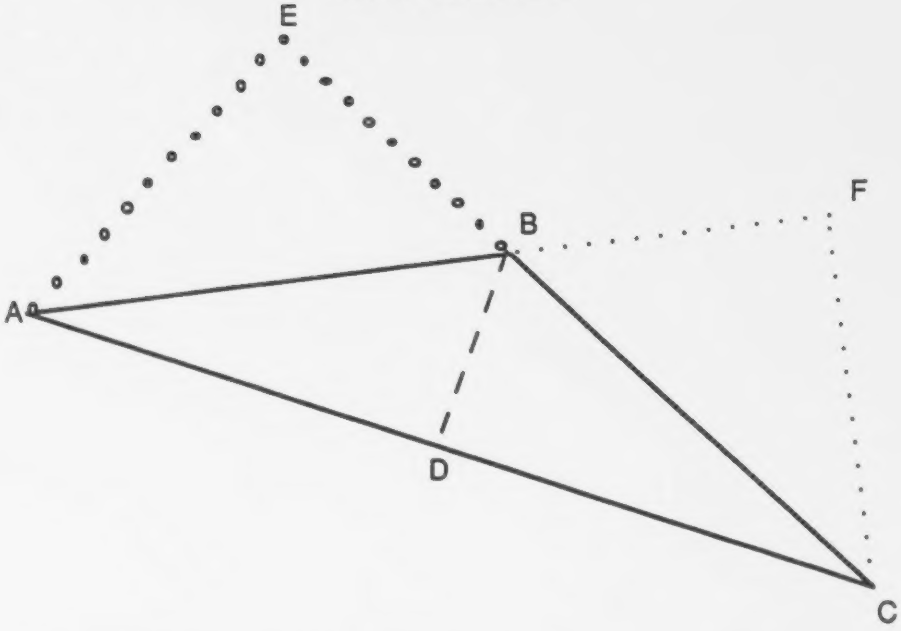
- **GUIDED PRACTICE:**
  Hand out Student Activity Sheet 2. Students complete the chart for triangle ACE. Discuss student results.

- **INDEPENDENT PRACTICE:**
  Students complete the chart for triangles HKG and NPM. Discuss the fact that the height may be the same as a side of the triangle. Hand out Student Activity Sheet 3 parts 1 and 2. Students complete the Activity Sheet, then discuss results.

- **EVALUATION:**
  Observe responses on Student Activity Sheets.

# IT'S ALL IN HOW YOU LOOK AT IT

Student Activity Sheet 1
Teacher Answer Sheet



- The height is drawn from the vertex opposite a base to the line containing the base, and perpendicular to the base.
- In some cases, the height will touch the base and in other instances, it touches an extension of the base.

Look at ΔABC

1. If you use $\overline{AC}$ as the base, the height would be line segment $\overline{DB}$

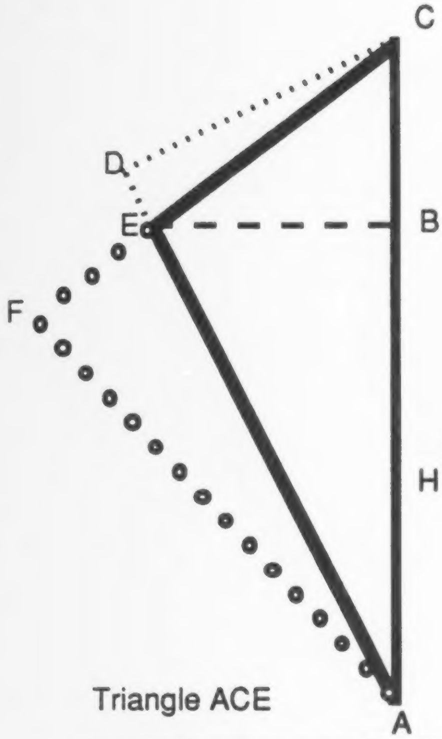
2. If you use $\overline{AB}$ as the base, the height would be line segment $\overline{CF}$

3. If you use $\overline{BC}$ as the base, the height would be line segment $\overline{AE}$

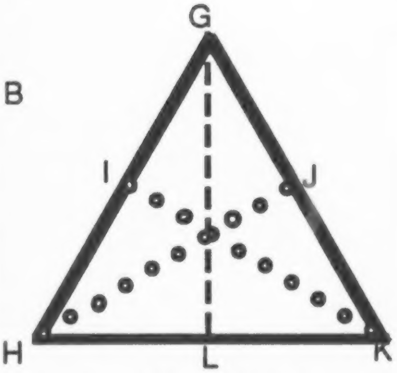
4. **Area of triangle = .5 x base x height**

| Base | Vertex opposite the base | Height | Measure of Base round to the nearest tenth of a cm | Measure of Height round to the nearest tenth of a cm | .5 x B x H | ≈ Area round to the nearest tenth of a cm$^2$ |
|---|---|---|---|---|---|---|
| $\overline{AC}$ | B | $\overline{BD}$ | 13.8 | 3.1 | 21.39 | 21.4 |
| $\overline{BC}$ | A | $\overline{AE}$ | 7.7 | 5.7 | 21.945 | 21.9 |
| $\overline{AB}$ | C | $\overline{CF}$ | 7.5 | 5.8 | 21.75 | 21.8 |

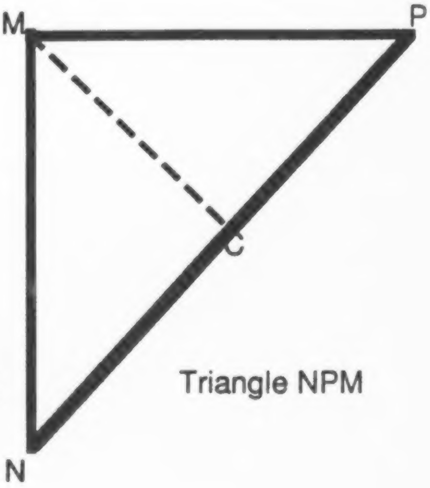
# IT'S ALL IN HOW YOU LOOK AT IT
## Student Activity Sheet 2
## Teacher Answer Sheet



Triangle ACE



Triangle HKG



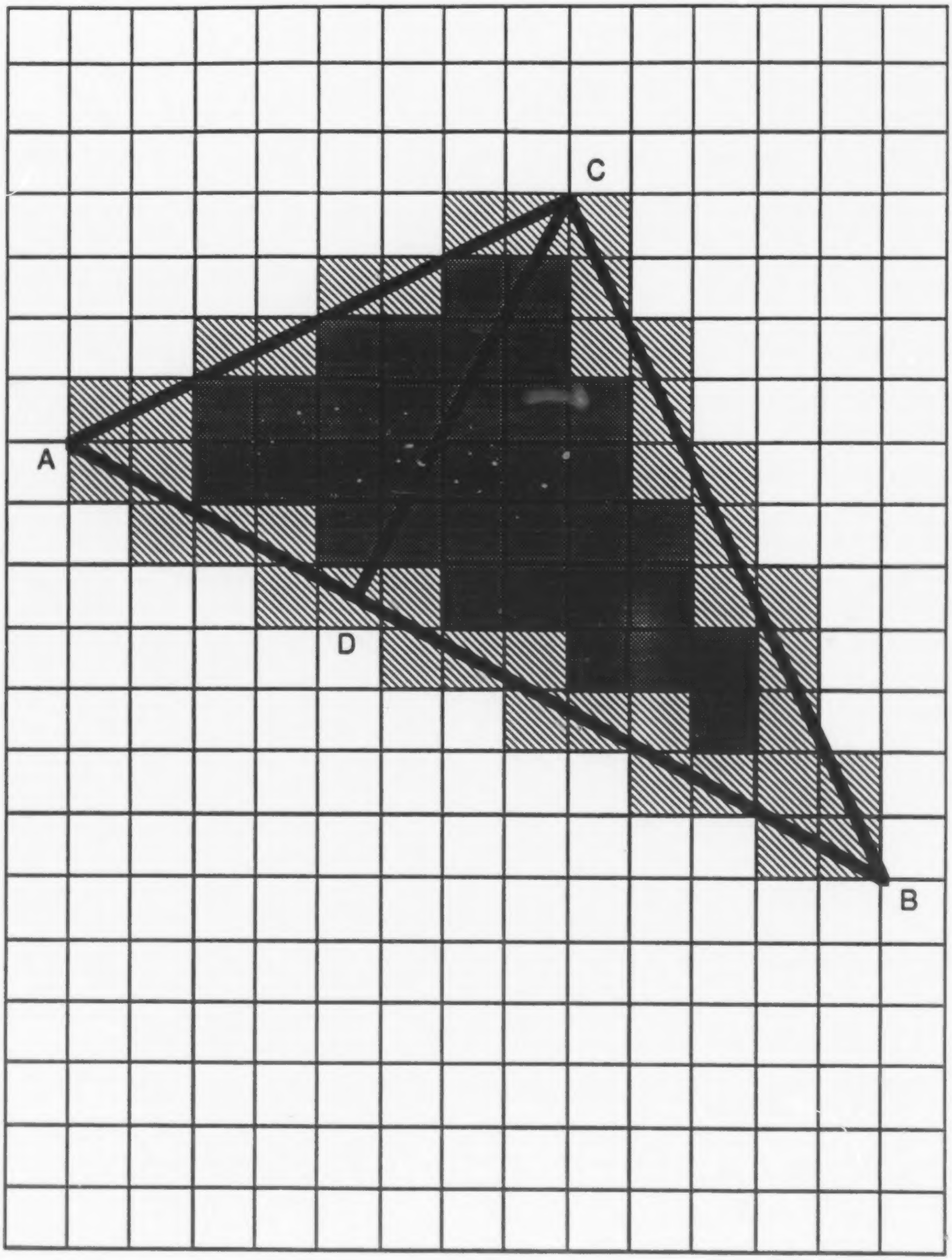
Triangle NPM

**Area of triangle = .5 x base x height**

| Triangle | Base | Height | Measure of Base round to the nearest tenth of a cm | Measure of Height round to the nearest tenth of a cm | .5 x B x H | ≈ Area round to the nearest tenth of a $cm^2$ |
|---|---|---|---|---|---|---|
| ACE | $\overline{AC}$ | $\overline{BE}$ | 9.7 | 3.5 | 16.975 | 17.0 |
| ACE | $\overline{EC}$ | $\overline{AF}$ | 4.5 | 7.8 | 17.55 | 17.6 |
| ACE | $\overline{AE}$ | $\overline{DC}$ | 7.8 | 4.5 | 17.55 | 17.6 |
| HKG | $\overline{HK}$ | $\overline{GL}$ | 5.0 | 4.4 | 11 | 11.0 |
| HKG | $\overline{GK}$ | $\overline{HJ}$ | 5.0 | 4.4 | 11 | 11.0 |
| HKG | $\overline{GH}$ | $\overline{KI}$ | 5.0 | 4.4 | 11 | 11.0 |
| NPM | $\overline{PM}$ | $\overline{MN}$ | 5.5 | 6.0 | 16.5 | 16.5 |
| NPM | $\overline{MN}$ | $\overline{MP}$ | 6.0 | 5.5 | 16.5 | 16.5 |
| NPM | $\overline{PN}$ | $\overline{MC}$ | 8.2 | 4.0 | 16.4 | 16.4 |

# IT'S ALL IN HOW YOU LOOK AT IT

Teacher Answer Sheet 3

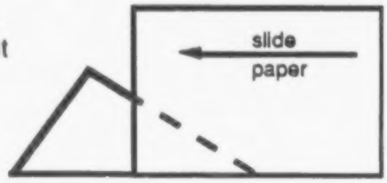


## IT'S ALL IN HOW YOU LOOK AT IT

Student Activity Sheet 4
Teacher Answer Sheet

1. Estimate the area of triangle ΔABC. ___answer will vary___ sq. cm
2. Lightly shade in and then count the number of squares which are completely inside of ΔABC. There are ___34___ squares. Is this greater than or smaller than the area of the triangle? ___smaller___
3. If the result from question 2 changes your estimate for the area of ΔABC, write your new estimate. ___answer will vary___ sq. cm
4. Lightly shade in all squares that have any part of them inside ΔABC. Count the total number of squares you have shaded so far in this lesson (All squares totally or partly in the triangle). There are ___74___ squares. Is this greater than or smaller than the area of the triangle? ___greater___
5. If the result from question 4 changes your estimate for the area of ΔABC, write your new estimate. ___answer will vary___ sq. cm
6. Look at triangle ΔABC. Choose one side to be a base. Use a dash line for the height. (Hint: slide a piece of paper along the base you have chosen until it lines up with the opposite vertex. Use the edge of the paper as a guide for dashing in the height.)

| **Measure of Base** | **Measure of Height** | **Area of Triangle ABC**<br>.5 x base x height |
|---|---|---|
| Possible answer: | | |
| $\overline{AB}$ 15 cm | $\overline{CD}$ 7.2 cm | 54 cm$^2$ |
| $\overline{AC}$ 9 cm | $\overline{BC}$ 12 cm | 54 cm$^2$ |
| $\overline{BC}$ 12 cm | $\overline{AC}$ 9 cm | 54 cm$^2$ |
| ______ | ______ | ______ |



7. Which was closer to the area of ΔABC, your answer to question 1, 3, or 5? ______

______

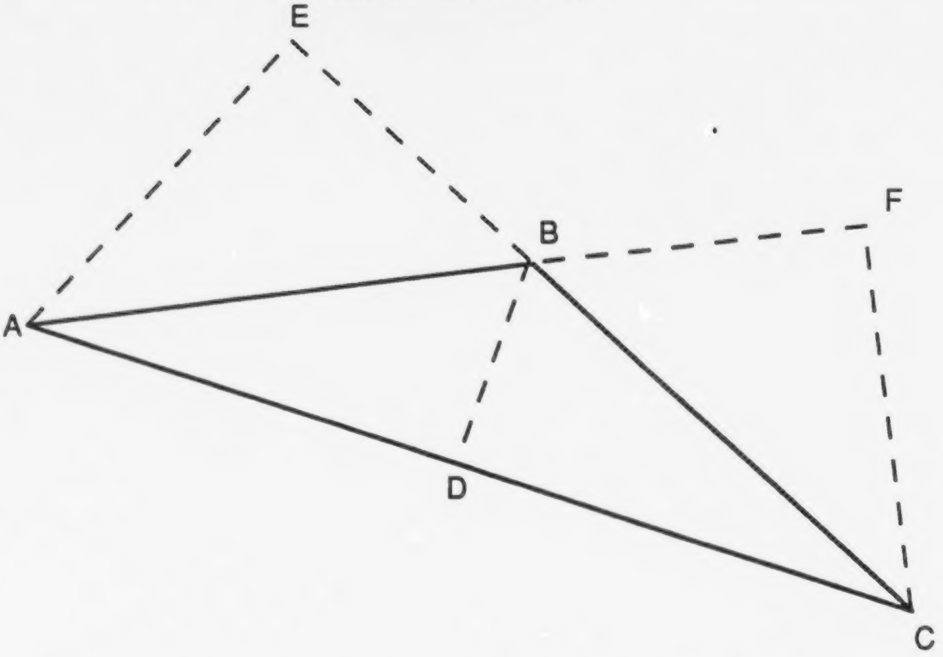
Why? ______________________________

______________________________

Name ____________________

## IT'S ALL IN HOW YOU LOOK AT IT

Student Activity Sheet 1



- The height is drawn from the vertex opposite a base to the line containing the base, and perpendicular to the base.
- In some cases, the height will touch the base and in other instances, it touches an extension of the base.

Look at $\triangle ABC$

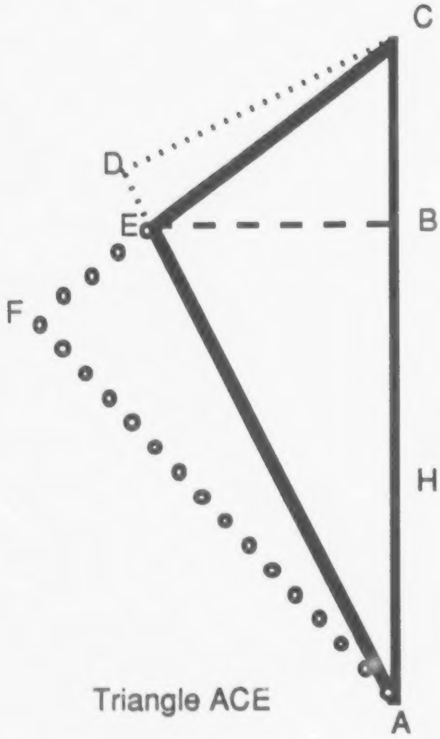
1. If you use $\overline{AC}$ as the base, the height would be line segment ____________________
2. If you use $\overline{AB}$ as the base, the height would be line segment ____________________
3. If you use $\overline{BC}$ as the base, the height would be line segment ____________________
4. **Area of triangle = .5 x base x height**

| Base | Vertex opposite the base | Height | Measure of Base round to the nearest tenth of a cm | Measure of Height round to the nearest tenth of a cm | .5 x B x H | ≈ Area round to the nearest tenth of a cm$^2$ |
|---|---|---|---|---|---|---|
| $\overline{AC}$ | | | | | | |
| $\overline{BC}$ | | | | | | |
| $\overline{AB}$ | | | | | | |

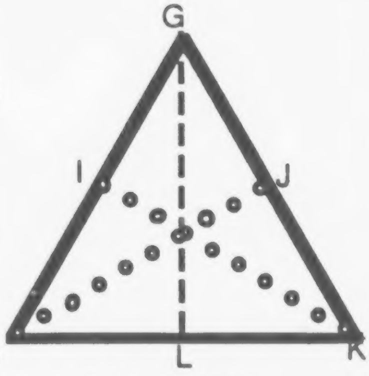
Name ____________________

# IT'S ALL IN HOW YOU LOOK AT IT

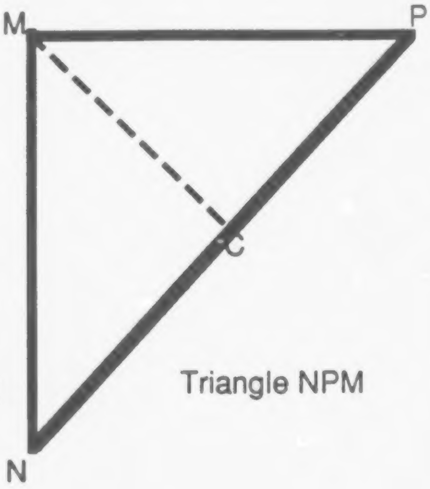
Student Activity Sheet 2



Triangle ACE



Triangle HKG



Triangle NPM

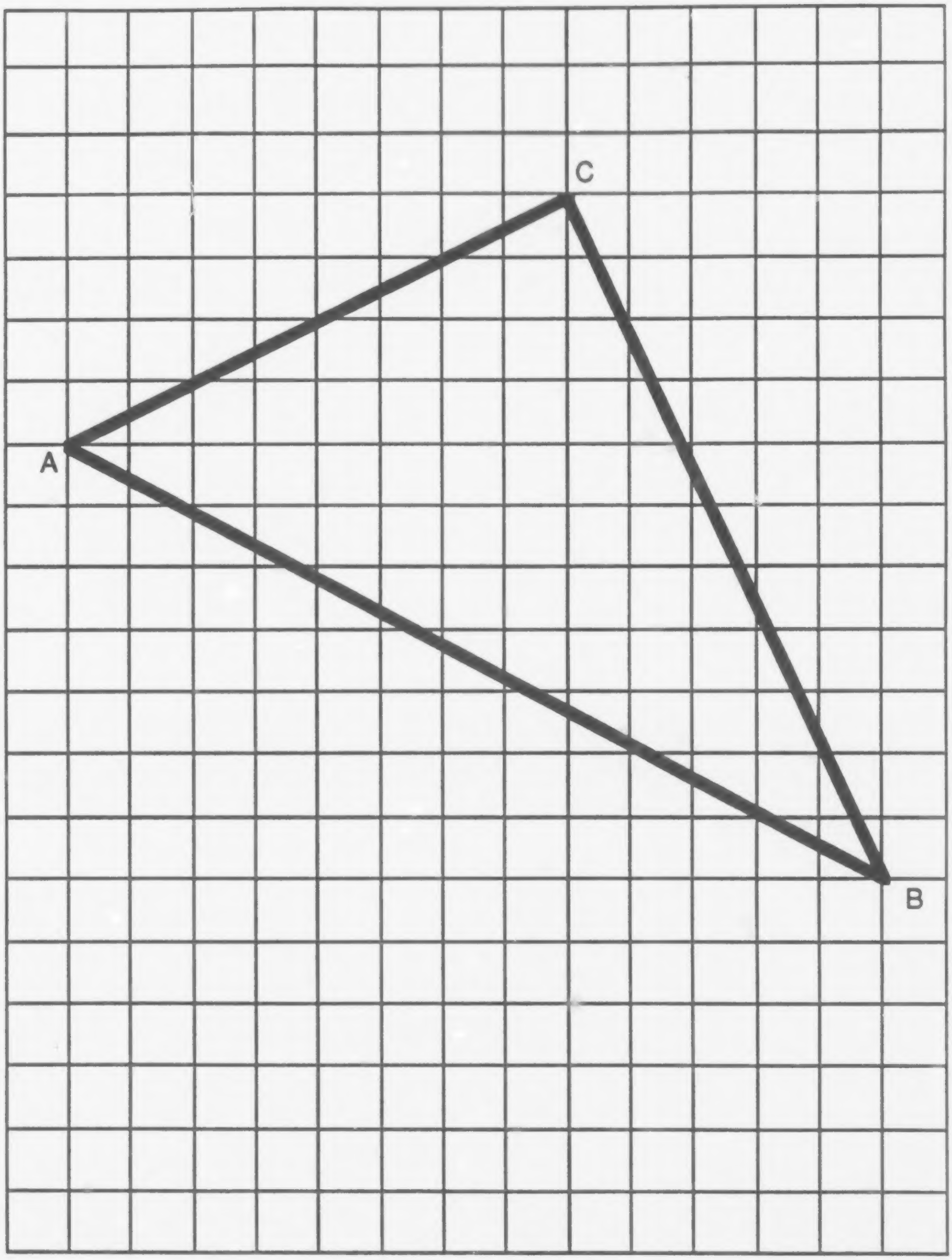
**Area of triangle = .5 x base x height**

| Triangle | Base | Height | Measure of Base round to the nearest tenth of a cm | Measure of Height round to the nearest tenth of a cm | .5 x B x H | ≈ Area round to the nearest tenth of a cm² |
|---|---|---|---|---|---|---|
| ACE | $\overline{AC}$ | | | | | |
| ACE | $\overline{EC}$ | | | | | |
| ACE | $\overline{AE}$ | | | | | |
| HKG | $\overline{HK}$ | | | | | |
| HKG | $\overline{GK}$ | | | | | |
| HKG | $\overline{GH}$ | | | | | |
| NPM | $\overline{PM}$ | | | | | |
| NPM | $\overline{MN}$ | | | | | |
| NPM | $\overline{PN}$ | | | | | |

Name ____________

## IT'S ALL IN HOW YOU LOOK AT IT

Student Activity Sheet 3, part 1



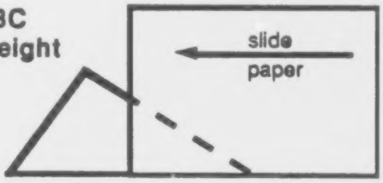
Name ____________________

## IT'S ALL IN HOW YOU LOOK AT IT

Student Activity Sheet 3, part 2

1. Estimate the area of triangle $\triangle ABC$. ________________________ sq. cm
2. Lightly shade in and then count the number of squares which are completely inside of $\triangle ABC$. There are __________________ squares. Is this greater than or smaller than the area of the triangle? ________________________
3. If the result from question 2 changes your estimate for the area of $\triangle ABC$, write your new estimate. ____________________ sq. cm
4. Lightly shade in all squares that have any part of them inside $\triangle ABC$. Count the total number of squares you have shaded so far in this lesson (All squares totally or partly in the triangle). There are ____________________________ squares. Is this greater than or smaller than the area of the triangle? __________________
5. If the result from question 4 changes your estimate for the area of $\triangle ABC$, write your new estimate. ____________________ sq. cm
6. Look at triangle $\triangle ABC$. Choose one side to be a base. Use a dash line for the height. (Hint: slide a piece of paper along the base you have chosen until it lines up with the opposite vertex. Use the edge of the paper as a guide for dashing in the height.)

| **Measure of Base** | **Measure of Height** | **Area of Triangle ABC .5 x base x height** |
|---|---|---|
| ____________ | ____________ | ______________ |



7. Which was closer to the area of $\triangle ABC$, your answer to question 1, 3, or 5? ___ ____________________

Why? ______________________________________________________________

______________________________________________________________

## LET'S GET IT "RIGHT"

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Geometry/Measurement |
| **SKILL:** | Make discoveries about triangles including the Pythagorean Theorem. |
| **MANAGEMENT**<br>**CLASS ORGANIZATION:** | Whole class, pairs |
| **TIME FRAME:** | One or two math periods |
| **MATERIALS:** | Calculators, Metric ruler |
| **VOCABULARY:** | Pythagorean Theorem, right triangle, right angle, obtuse triangle, acute triangle |
| **PREREQUISITE SKILL:** | Find the 2nd power of a number, measure to nearest millimeter, identify right, obtuse, and acute triangles |

**LESSON**

- **DIRECTED/ GUIDED INSTRUCTION:**
  - Hand out Student Activity Sheets 1 and 2 and metric rulers.
  - Students complete the Student Activity Sheets.
  - Discuss student observations.
  - Guide them to see:
    - If $a^2 + b^2 = c^2$, it is a right triangle. One angle measures 90°.
    - If $a^2 + b^2 < c^2$, it is an obtuse triangle. One angle measures greater than 90°.
    - If $a^2 + b^2 > c^2$, it is an acute triangle. All angles have a measure of less than 90°.

- **INDEPENDENT INSTRUCTION:**
  - Hand out Student Activity Sheets 3 and 4
  - Students complete the Student Activity Sheets.
  - Note: The measurements are given on the paper. Rulers are not needed.
  - Students round computation answers to the nearest tenth before entering them in the chart.
  - After students complete the sheets, discuss that the observations for Sheets 1 and 2 remain true for the triangles on Sheets 3 and 4.
  - Mention that $a^2 + b^2 = c^2$ if and only if the triangle is a right triangle. This discovery is called the Pythagorean Theorem.
  - Stress that measurements involve approximations so that numbers in the chart are not exact.

## LET'S GET IT "RIGHT"
Student Activity Sheet 1
Teacher Answer Sheet

1. Measure the lengths (to the nearest cm) of the triangles on Student Activity Sheet 2. Fill in the table below.

| Triangle | a | b | c | $a^2$ | $b^2$ | $c^2$ | $a^2 + b^2$ |
|---|---|---|---|---|---|---|---|
| ❶ | 3 | 15 | 16 | 9 | 225 | 256 | 234 |
| ❷ | 3 | 4 | 5 | 9 | 16 | 25 | 25 |
| ❸ | 6 | 9 | 10 | 36 | 81 | 100 | 117 |
| ❹ | 6 | 10 | 12 | 36 | 100 | 144 | 136 |
| ❺ | 8 | 15 | 17 | 64 | 225 | 289 | 289 |
| ❻ | 3 | 6 | 7 | 9 | 36 | 49 | 45 |
| ❼ | 5 | 12 | 13 | 25 | 144 | 169 | 169 |
| ❽ | 1 | 3 | 3 | 1 | 9 | 9 | 10 |
| ❾ | 3 | 4 | 6 | 9 | 16 | 36 | 25 |

2. For which triangles does $a^2 + b^2 = c^2$? *2, 5 and 7*

   What seems to be true about the triangles for which $a^2 + b^2 = c^2$?

   *They are right triangles. They have one 90° angle.*

3. For which triangle(s) is it true that $a^2 + b^2 > c^2$? *3 and 8*

   What seems to be true about the triangles for which $a^2 + b^2 > c^2$?

   *They are acute triangles. They are triangles with each angle measuring less than 90°.*

4. For which triangle(s) is it true that $a^2 + b^2 < c^2$? *1, 4, 6 and 9*

   What seems to be true about the triangles for which $a^2 + b^2 < c^2$?

   *They are obtuse triangles. They are triangles that have one angle measuring greater than 90°.*

# LET'S GET IT "RIGHT"

Student Activity Sheet 3

Teacher Answer Sheet

1. Record the lengths of the sides of the triangles from Student Activity Sheet 4. Fill in the table below. Round all numbers to the nearest tenth. (Answers may vary slightly.)

| Triangle | a | b | c | $a^2$ | $b^2$ | $c^2$ | $a^2 + b^2$ |
|---|---|---|---|---|---|---|---|
| ❶ | 4.0 | 5.1 | 8.4 | 16.0 | 26.0 | 70.6 | 42.0 |
| ❷ | 15.2 | 3.7 | 15.4 | 231.0 | 13.7 | 237.2 | 244.7 |
| ❸ | 3.3 | 18.0 | 18.3 | 10.9 | 324.0 | 334.9 | 334.9 |
| ❹ | 3.8 | 3.0 | 6.2 | 14.4 | 9.0 | 38.4 | 23.4 |
| ❺ | 1.5 | 2.0 | 2.5 | 2.3 | 4.0 | 6.3 | 6.3 |
| ❻ | 9.0 | 5.6 | 10.6 | 81.0 | 31.4 | 112.4 | 112.4 |
| ❼ | 2.8 | 3.9 | 4.2 | 7.8 | 15.2 | 17.6 | 23.0 |

2. For which triangles does $a^2 + b^2 = c^2$? *3, 5 and 6*

   What seems to be true about the triangles for which $a^2 + b^2 = c^2$?

   *They are right triangles.*

3. For which triangle(s) is it true that $a^2 + b^2 > c^2$? *2 and 7*

   What seems to be true about the triangles for which $a^2 + b^2 > c^2$?

   *They are acute triangles.*

4. For which triangle(s) is it true that $a^2 + b^2 < c^2$? *1 and 4*

   What seems to be true about the triangles for which $a^2 + b^2 < c^2$?

   *They are obtuse triangles.*

Name____________________________

## LET'S GET IT "RIGHT"

Student Activity Sheet 1

1. Measure the sides of the triangles on Student Activity Sheet 2 to the nearest millimeter (tenth of a cm). Fill in the table below.

| Triangle | a | b | c | $a^2$ | $b^2$ | $c^2$ | $a^2 + b^2$ |
|---|---|---|---|---|---|---|---|
| ❶ | | | | | | | |
| ❷ | | | | | | | |
| ❸ | | | | | | | |
| ❹ | | | | | | | |
| ❺ | | | | | | | |
| ❻ | | | | | | | |
| ❼ | | | | | | | |
| ❽ | | | | | | | |
| ❾ | | | | | | | |

2. For which triangles does $a^2 + b^2 = c^2$? ____________________
What seems to be true about the triangles for which $a^2 + b^2 = c^2$?

____________________________________________

____________________________________________

3. For which triangle(s) is it true that $a^2 + b^2 > c^2$ ?____________________

What seems to be true about the triangles for which $a^2 + b^2 > c^2$?

____________________________________________

____________________________________________

4. For which triangle(s) is it true that $a^2 + b^2 < c^2$?____________________

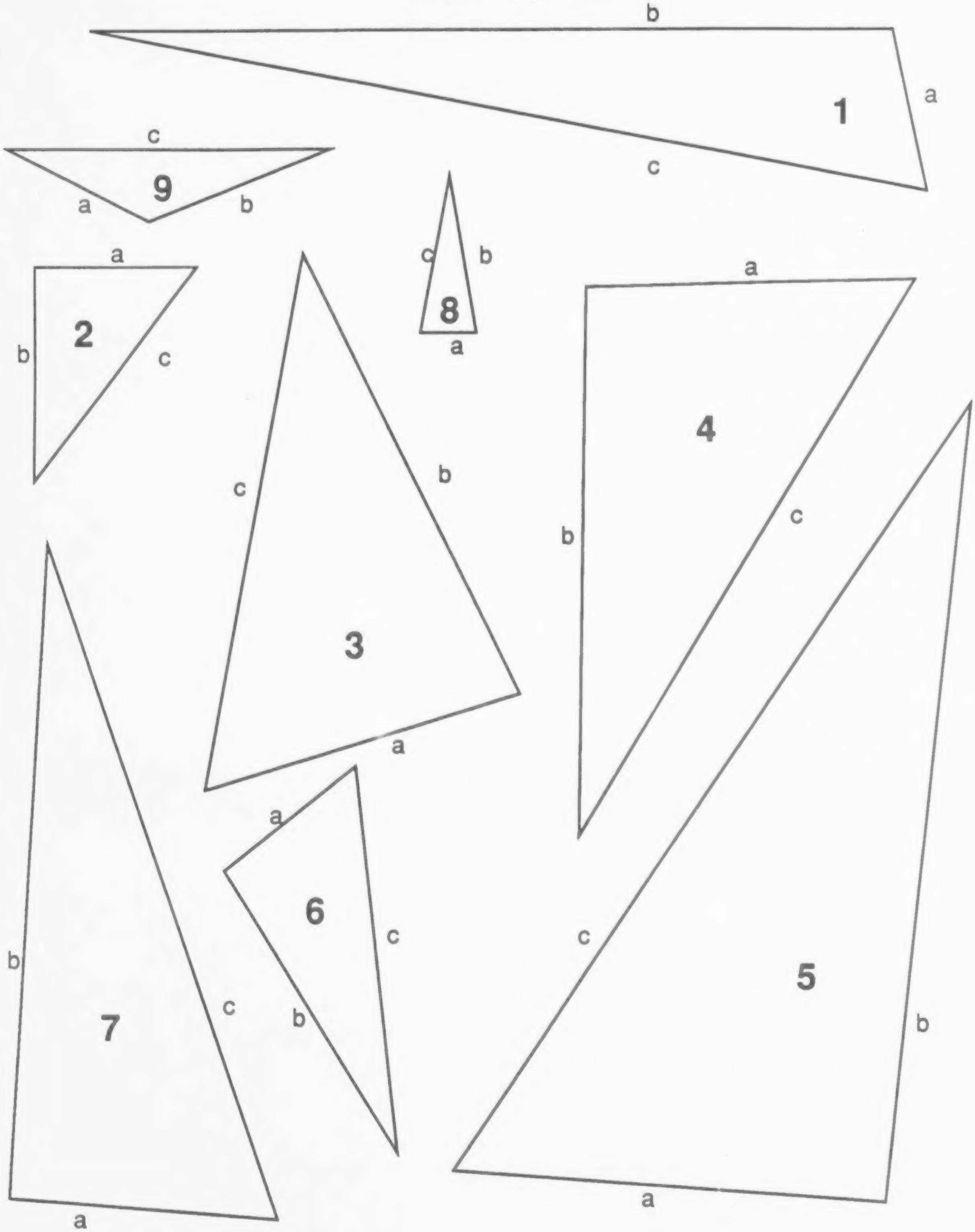
What seems to be true about the triangles for which $a^2 + b^2 < c^2$?________

____________________________________________

____________________________________________

**LET'S GET IT "RIGHT"** Name ____________________

Student Activity Sheet 2


b
a
1
c
c
9
a
b
c
b
8
a
a
2
b
c
c
b
3
a
a
4
b
c
a
6
c
b
b
7
c
a
c
5
b
a

Name ______________________

## LET'S GET IT "RIGHT"

Student Activity Sheet 3

1. Record the lengths of the sides of the triangles on Student Activity Sheet 4. Fill in the table below. Round all numbers to the nearest tenth.

| Triangle | a | b | c | $a^2$ | $b^2$ | $c^2$ | $a^2 + b^2$ |
|---|---|---|---|---|---|---|---|
| ❶ | | | | | | | |
| ❷ | | | | | | | |
| ❸ | | | | | | | |
| ❹ | | | | | | | |
| ❺ | | | | | | | |
| ❻ | | | | | | | |
| ❼ | | | | | | | |

2. For which triangles does $a^2 + b^2 = c^2$? ______________________

What seems to be true about the triangles for which $a^2 + b^2 = c^2$?

3. For which triangle(s) is it true that $a^2 + b^2 > c^2$? ______________________

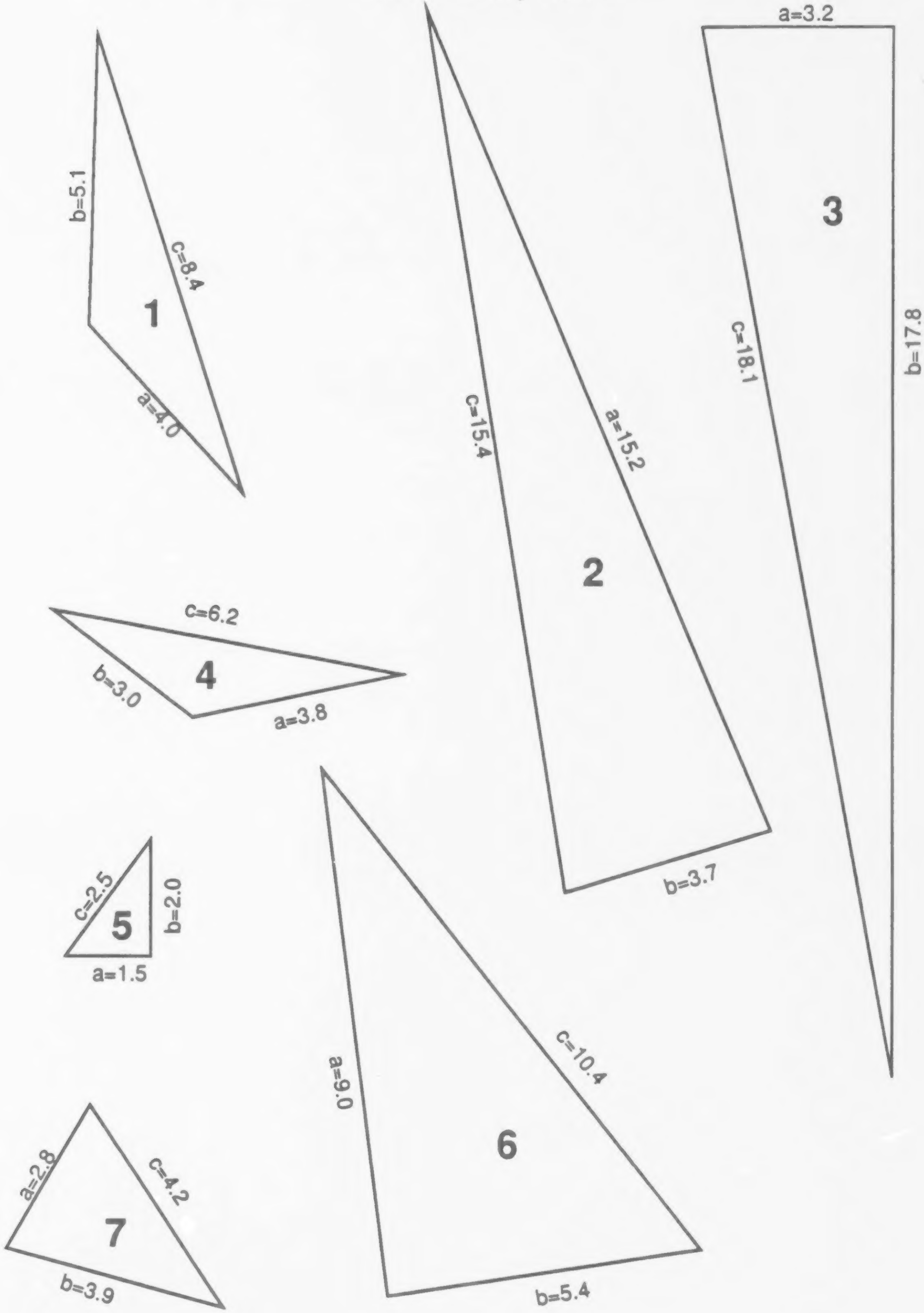
What seems to be true about the triangles for which $a^2 + b^2 > c^2$?

4. For which triangle(s) is it true that $a^2 + b^2 < c^2$? ______________________

What seems to be true about the triangles for which $a^2 + b^2 < c^2$? ____________

**LET'S GET IT "RIGHT"** Name________________________

Student Activity Sheet 4



1
b=5.1
c=8.4
a=4.0
2
c=15.4
a=15.2
b=3.7
3
a=3.2
c=18.1
b=17.8
4
c=6.2
b=3.0
a=3.8
5
c=2.5
b=2.0
a=1.5
6
a=9.0
c=10.4
b=5.4
7
a=2.8
c=4.2
b=3.9

# HERON'S FORMULA

**GRADE:** 7 - 8

**STRAND:** Geometry

**SKILL:** Measure to compute area using a formula.

**MANAGEMENT CLASS ORGANIZATION:** Whole class

**TIME FRAME:** Two or three math periods

**MATERIALS:** Calculator, meter sticks, centimeter rulers, square centimeter paper

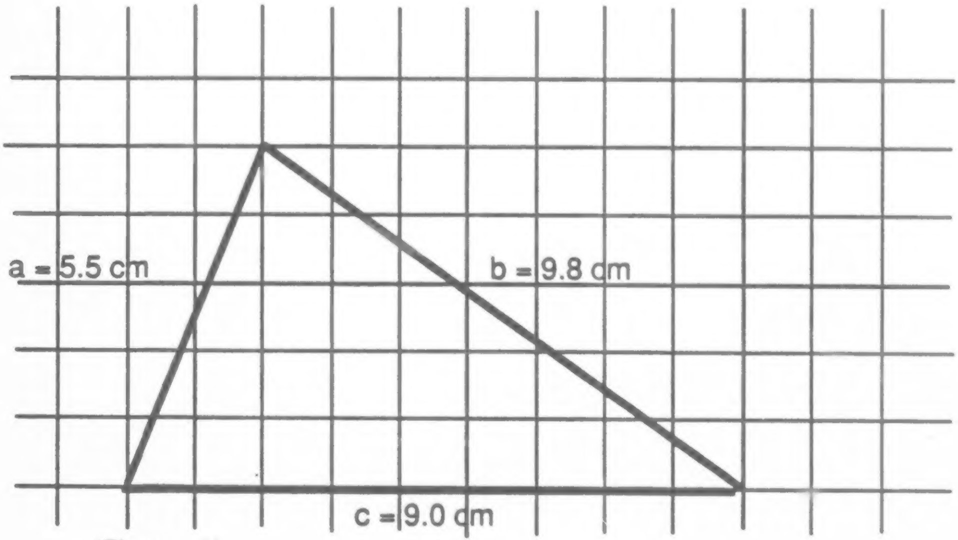
**VOCABULARY:** Triangular region, perimeter, semiperimeter, approximately equal to (≈)

**PREREQUISITE SKILL:** Ability to use a formula

## LESSON:

### DIRECTED LESSON:

1. Hand out Student Activity Sheet 1. Draw a triangle. Measure the lengths of the sides to the nearest tenth of a centimeter and write the measurements on the sides.
2. Students estimate the area of their triangle by counting.
3. Hand out Student Activity Sheet 2. Help students use Heron's Formula to compute the area of the triangle found on that page. (See page 2.)



(Figure 1)

- The perimeter of the triangle would be a + b + c. In order to find area using Heron's Formula, you must first find the semiperimeter. The **semiperimeter** "s", is half the perimeter. That is:

$$s = \frac{a + b + c}{2}$$

- Heron's Formula is

$$A = \sqrt{s\ (s-a)\ (s-b)\ (s-c)}$$

Use the measures for the triangle to compute the semiperimeter:

$$s = \frac{5.5 + 9.0 + 9.8}{2} = 12.15$$

Use Heron's Formula to compte the area of the triangle:

$$A = \sqrt{(12.15)(12.15-5.5)(12.15-9.0)(12.15-9.8)}$$

$$A = \sqrt{(12.15)(6.65)(3.15)(2.35)}$$

$$A = \sqrt{598.10348} = 24\ \text{cm}^2$$ rounded to the nearest square centimeter

Note: $\text{cm}^2$ is an abbreviation for square centimeter.

- **GUIDED PRACTICE:**
  4. Hand out Student Activity Sheet 3. Using Heron's Formula, students compute the area of the triangle they drew on Student Activity Sheet 1. Students compare their answer to their original estimate.

- **INDEPENDENT PRACTICE:**
  Hand out and have students complete Student Activity Sheets 4 and 5.

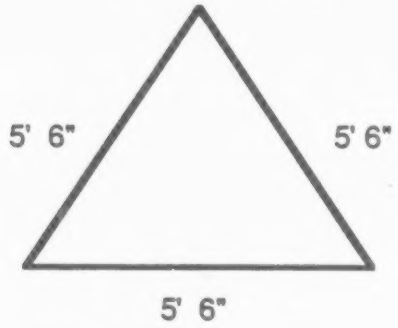
- **EVALUATION:**
  Teacher observation

- **EXTENSION:**
  Find a triangular shaped object. Use Heron's Formula to find its area.

## HERON'S FOMULA

### Student Activity Sheet 4
### Teacher Answer Sheet

Sarah's little brother has a sandbox that is in the shape of a triangle. She wants to use what she learned about Heron's Formula to find the area of the sandbox. Sarah measured the sandbox in feet and inches.



(Figure 3)

Use Heron's Formula to find the area of the triangular region in square feet. (Hint: 5' 6" = 5.5 feet.)

Answer: $s = \frac{5.5 + 5.5 + 5.5}{2} = \frac{3(5.5)}{2} = 8.25$

$A = \sqrt{(8.25)(8.25-5.5)(8.25-5.5)(8.25-5.5)}$

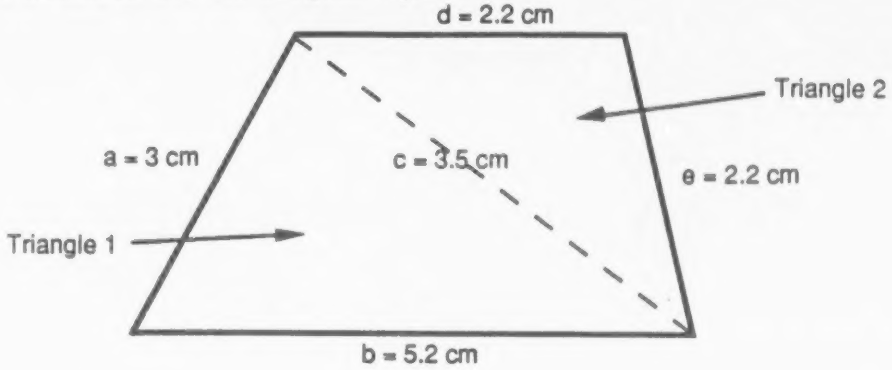
$A = \sqrt{(8.25)(2.75)(2.75)(2.75)}$

$A = \sqrt{171.57421} = 13\ \text{cm}^2$ rounded to the nearest sq. cm.

# HERON'S FORMULA

## Student Activity Sheet 5
## Teacher Answer Sheet

Use Heron's Formula to find the area of a quadrilateral region. A quadrilateral region can be subdivided into two triangular regions.



$s = \dfrac{a + b + c}{2}$

$A = \sqrt{s\ (s-a)\ (s-b)\ (s-c)}$

$s = \dfrac{3 + 5.2 + 3.5}{2} = 5.85$

$A = \sqrt{(5.85)(5.85-3)(5.85-5.2)(5.85-3.5)}$

$A = \sqrt{(5.85)(2.85)(.65)(2.35)}$

$A = \sqrt{2.5467243} \approx \underline{\ 5\ }\ \text{cm}^2$ rounded to the nearest sq. cm.

$s = \dfrac{c + d + e}{2}$

$A = \sqrt{s\ (s-c)\ (s-d)\ (s-e)}$

$s = \dfrac{3.5 + 2.2 + 2.2}{2} = 3.95$

$A = \sqrt{(3.95)(3.95-3.5)(3.95-2.2)(3.95-2.2)}$

$A = \sqrt{(3.95)(0.45)(1.75)(1.75)}$

$A = \sqrt{5.4435937} \approx \underline{\ 2\ }\ \text{cm}^2$ rounded to the nearest sq. cm.

$A \approx \underline{\ 5\ } + \underline{\ 2\ } = \underline{\ 7\ }\ \text{cm}$

Draw a quadrilateral on your centimeter graph paper. Divide it into 2 triangular regions. Use Heron's Formula to find its area.

Name ____________________

## HERON'S FORMULA

Student Activity Sheet 1

1. Draw a triangle on the centimeter grid using a centimeter ruler. Measure the lengths of the sides to the nearest tenth of a centimeter. Write the measurements on the sides of the triangle.

2. Estimate the area of the triangle by counting the number of centimeter squares.

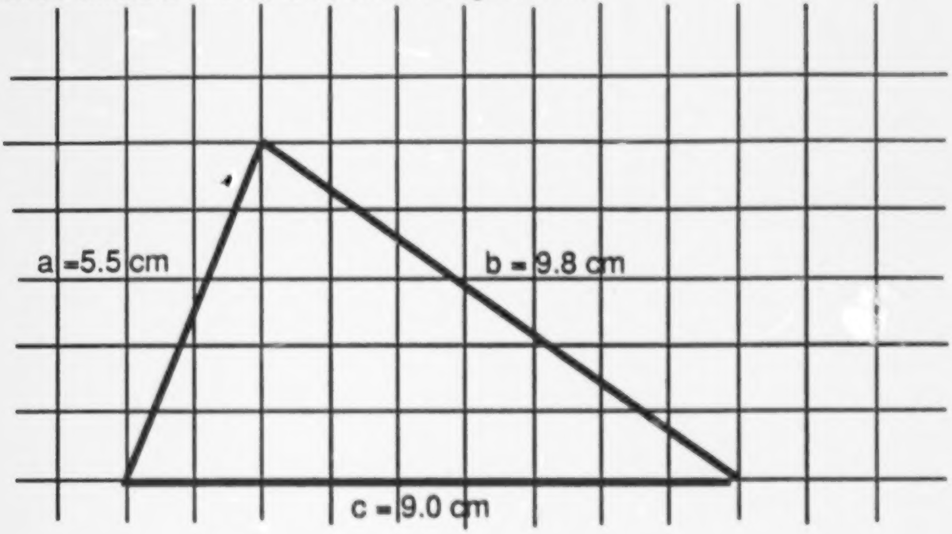
Estimate ____________________ $cm^2$

Name ____________________

## HERON'S FORMULA

### Student Activity Sheet 2

Use Heron's Formula to find the area of the triangle below.



The perimeter of the triangle would be a + b + c. In order to find area using Heron's Formula, you must first find the semiperimeter. The **semiperimeter "s"**, is half the perimeter. That is:

$$s = \frac{a + b + c}{2}$$

Heron's formula for the area of the triangle is

$$A = \sqrt{(s)\ (s-a)\ (s-b)\ (s-c)}$$

Use the measures to compute:

$$s = \frac{\_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_}{2}$$

s = ____________________

$$A = \sqrt{(\qquad)(\quad - \quad)(\quad - \quad)(\quad - \quad)}$$

$$A = \sqrt{(\qquad)(\qquad)(\qquad)(\qquad)}$$

$$A = \sqrt{\qquad\qquad}$$ = __________ $cm^2$ rounded to the nearest $cm^2$

Name ______________________________

## HERON'S FORMULA

### Student Activity Sheet 3

Use Heron's Formula to compute the area of the triangle you drew on Student Activity Sheet 1.

Reminder:

$$s = \frac{a + b + c}{2}$$

$$A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

a = ________ b = ________ c = ________

$$s = \frac{\qquad + \qquad + \qquad}{2}$$

s = ____________________

$$A = \sqrt{(\qquad)(\qquad)(\qquad)(\qquad)}$$

$$A = \sqrt{(\qquad)(\qquad)(\qquad)(\qquad)}$$

$A = \sqrt{\qquad\qquad\qquad}$ = __________ $cm^2$

rounded to the nearest $cm^2$

Was the answer using Heron's Formula greater than or less than your estimate?

____________________________________________________________

____________________________________________________________

What was the difference between your estimate and your Heron's Formula answer?

____________________________________________________________

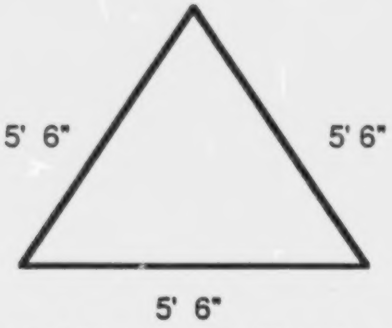
____________________________________________________________

Name ____________________

## HERON'S FORMULA

Student Activity Sheet 4

Sarah's little brother has a sand box that is in the shape of a triangle. She wants to use what she learned about Heron's Formula to find the area of the sandbox. Sarah measured the sand box in feet and inches.



- Use Heron's Formula to find the area of the triangular region in square feet. Hint: 5' 6" = 5.5 feet.

$$s = \frac{a + b + c}{2}$$

$$A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

$$s = \frac{\qquad + \qquad + \qquad}{2}$$

$$s = \underline{\qquad\qquad}$$

$$A = \sqrt{(\qquad)(\quad - \quad)(\quad - \quad)(\quad - \quad)}$$

$$A = \sqrt{(\qquad)(\qquad)(\qquad)(\qquad)}$$

$$A = \sqrt{\qquad\qquad} = \underline{\qquad}\ \text{cm}^2$$

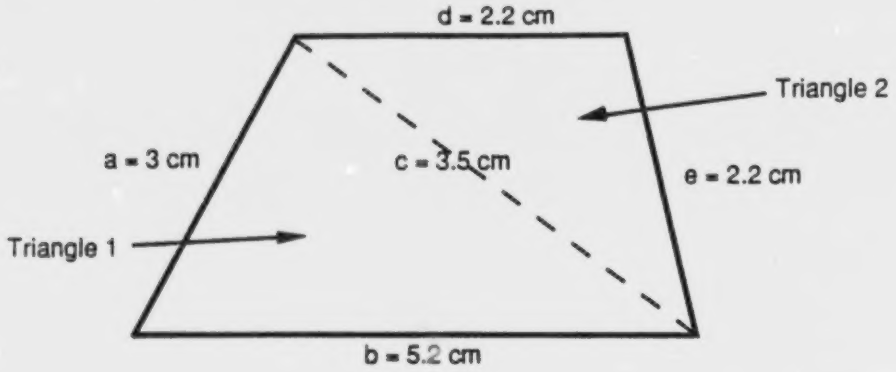
rounded to the nearest $\text{cm}^2$

Name ______________________________

# HERON'S FORMULA

## Student Activity Sheet 5

Use Heron's Formula to find the area of the quadrilateral region below by subdividing it into two triangular regions.



$s = \frac{a + b + c}{2}$ $s = \frac{c + d + e}{2}$

$A_1 = \sqrt{s\ (s-a)\ (s-b)\ (s-c)}$ $A_2 = \sqrt{s\ (s-c)\ (s-d)\ (s-e)}$

$s = \frac{\qquad + \qquad + \qquad}{2}$ $s = \frac{\qquad + \qquad + \qquad}{2}$

$A_1 = \sqrt{(\quad)(\quad)(\quad)(\quad)}$ $A_2 = \sqrt{(\quad)(\quad)(\quad)(\quad)}$

$A_1 = \sqrt{(\quad)(\quad)(\quad)(\quad)}$ $A_2 = \sqrt{(\quad)(\quad)(\quad)(\quad)}$

$A_1 = \sqrt{\qquad} \approx$ ________ $cm^2$ rounded to the nearest sq. cm. $A_2 = \sqrt{\qquad} \approx$ ________ $cm^2$ rounded to the nearest sq. cm.

$A_1 + A_2 =$ __________ + ________ = __________

Draw a quadrilateral on your centimeter graph paper. Divide it into 2 triangular regions. Use Heron's Formula to find its area.

# THE LONG & SHORT OF IT

| | |
|---|---|
| GRADE: | 7 - 8 |
| STRAND: | Geometry/Measurement |
| SKILL: | Determine perimeter and area of rectangles using formulas. Analyze data. Discover calculator limitations. |
| MANAGEMENT<br>CLASS ORGANIZATION: | Whole class, pairs or small groups |
| TIME FRAME: | One math period |
| MATERIALS: | Calculator |
| VOCABULARY: | Area, perimeter |
| PREREQUISITE SKILL: | Application of perimeter and area formulas to rectangles |

## LESSON

- **DIRECTED INSTRUCTION:**

  Review the concept of perimeter (distance around), and area (space inside) for a rectangle. Remind students that area can be found by multiplying length times width (l x w) and that perimeter can be found by adding all 4 sides or by adding one length to one width and multiplying that answer by two ((l + w) x 2).

- **GUIDED PRACTICE:**

  Hand out Student Activity Sheets 1 and 2. Together with the class, read the directions and work through rectangles 1 to 3.

- **INDEPENDENT PRACTICE:**

  Students complete Student Activity Sheets 1 and 2. After completion, discuss student results with your class. See Teacher Answer Sheet for many of the desired responses. Notice the unusual result, that the area remains constant at 4 $cm^2$, but the perimeter gets larger and larger.

- **EVALUATION:**

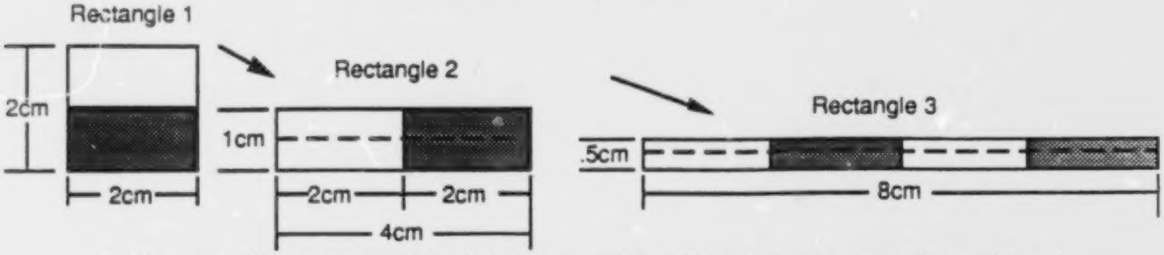
  Teacher observation, Student Activity Sheet responses.

- **EXTENSION:**

  Ask students to equate how long Rectangle 20, Student Activity Sheet 1, is in terms of a familiar quantity such as blocks long, miles long, etc.

## THE LONG AND SHORT OF IT

Teacher Answer Sheet
Student Activity Sheet 1



You are given a square that is 2 cm on a side. If it were cut across the middle of Rectangle 1 and the pieces were placed side by side, the new rectangle would be 4 cm long and 1 cm wide. If you repeat the process, the next new rectangle would be 8 cm long and 0.5 cm wide. Assume this process continues. Fill in the chart below with the requested information for each rectangle. Use your calculator for the computation.

| Rectangle | Length cm | Width cm | Area = l x w square cm | Perimeter = (l + w) x 2 cm |
|---|---|---|---|---|
| 1 | 2 | 2 | 4 | 8 |
| 2 | 4 | 1 | 4 | 10 |
| 3 | 8 | .5 | 4 | 17 |
| 4 | 16 | .25 | 4 | 32.5 |
| 5 | 32 | .125 | 4 | 64.25 |
| 6 | 64 | .0625 | 4 | 128.125 |
| 7 | 128 | .03125 | 4 | 256.0625 |
| 8 | 256 | .015625 | 4 | 512.03124 |
| 9 | 512 | .0078125 | 4 | 1024.0156 |
| 10 | 1024 | .0039062 | 3.9999488 | 2048.0078 |
| 11 | 2048 | .0019531 | 3.9999488 | 4096.0038 |
| 12 | 4096 | .0009765 | 3.99744 | 8192.0018 |
| 13 | 8192 | .0004882 | 3.993344 | 16384 |
| 14 | 16384 | .0002441 | 3.993344 | 32768 |
| 15 | 32768 | .000122 | 3.997696 | 65536 |
| 16 | 65536 | .000061 | 3.997696 | 131072 |
| 17 | 131072 | .0000305 | 3.997696 | 262144 |
| 18 | 262144 | .0000152 | 3.9845888 | 524288 |
| 19 | 524288 | .0000076 | 3.9845888 | 1048576 |
| 20 | 1048576 | .0000038 | 3.9845888 | 2097152 |

## THE LONG AND SHORT OF IT

Student Activity Sheet 2
Teacher Answer Sheet

1. What do you observe about the lengths of the rectangles? Explain? The lengths get larger. Each time the process occurs the length doubles.

2. What do you observe about the widths of the rectangles? Explain? Each time the process occurs the width is half as big. The calculator drops off extra decimal places that don't fit in the display so that the widths listed from Rectangle 10 on are really underestimates.

3. What do you observe about the areas of the rectangles? Explain? The real area remains 4. Since pieces are just rearranged no area is gained or lost in the process. The number for area beyond rectangle #10 is not shown as exactly 4 because many of the numbers in the width column are slight underestimates. This occurs due to limitations of the calculator.

4. What do you observe about the perimeters of the rectangles? Explain? The perimeters get larger. As the width progressively narrows, the calculator does not compute it accurately. In this example as the true number width gets smaller, the calculator cannot display beyond the 10 millionths column. In 8 digit display calculators, remaining digits to the right of the 10,000,000ths column are dropped off. Therefore, the number displayed is less than the true number. For example, Rectangle 11 the width = .0019531. In no. 12 width = .0009765, but no. 12 should read .0097655. Therefore, the calculator is off .00000005. The perimeter appears as twice the length because these widths are too tiny to calculate accurately using the calculator.

5. Look at your information for Rectangle 14. Is all the information accurate? Why do you think so? See answers 2, 3, and 4

6. What would happen in real life to the length, width, area, and perimeter if this process continued? The length would keep getting larger by doubling. The width would keep getting smaller by halving. The area would remain 4. The perimeter would become extremely close to twice the length.

7. Why isn't the calculator an appropriate tool to use if the chart is expanded to contain data up to Rectangle no. 40? The length would be too large to fit the display. The width would be so small only 0's would appear on the display.

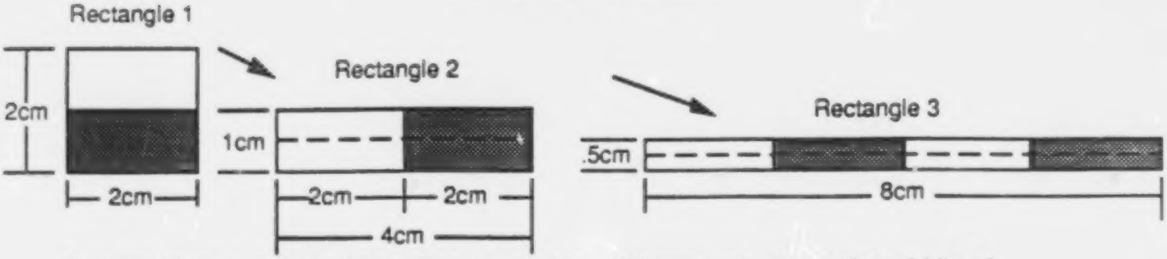
8. Write any other observations you have about the question. ______________

______________ answers will vary ______________

Name ____________________

## THE LONG AND SHORT OF IT

### Student Activity Sheet 1



You are given a square that is 2 cm on a side. If it were cut across the middle of Rectangle 1 and the pieces were placed side by side, the new rectangle would be 4 cm long and 1 cm wide. If you repeat the process, the next new rectangle would be 8 cm long and 0.5 cm wide. Assume this process continues. Fill in the chart below with the requested information for each rectangle. Use your calculator for the computation.

| Rectangle | Length (cm) | Width (cm) | Area = l x w (square cm) | Perimeter = (l + w) x 2 (cm) |
|---|---|---|---|---|
| 1 | 2 | 2 | 4 | 8 |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |
| 12 | | | | |
| 13 | | | | |
| 14 | | | | |
| 15 | | | | |
| 16 | | | | |
| 17 | | | | |
| 18 | | | | |
| 19 | | | | |
| 20 | | | | |

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Name ____________________

## THE LONG AND SHORT OF IT

### Student Activity Sheet 2

1. What do you observe about the lengths of the rectangles? Explain? ______
______________________________________________
______________________________________________

2. What do you observe about the widths of the rectangles? Explain? ______
______________________________________________
______________________________________________

3. What do you observe about the areas of the rectangles? Explain? ______
______________________________________________
______________________________________________

4. What do you observe about the perimeters of the rectangles? Explain? ___
______________________________________________
______________________________________________

5. Look at your information for Rectangle 14. Is all the information accurate? Why do you think so? ______________________
______________________________________________
______________________________________________

6. What would happen in real life to the length, width, area, and perimeter if this process continued? ______________________
______________________________________________
______________________________________________
______________________________________________
______________________________________________
______________________________________________

7. Why isn't the calculator an appropriate tool to use if the chart is expanded to contain data up to Rectangle no. 40? ______

______

______

______

______

8. Write any other observations you have. ______

______

______

______

# ANOTHER FENCE ON THE WALL

**GRADE:** 7 - 8

**STRAND:** Measurement/Geometry

**SKILL:** Investigate area of rectangles under a special condition.

## MANAGEMENT

**CLASS ORGANIZATION:** Whole class, pairs, or small groups

**TIME FRAME:** One math period

**MATERIALS:** Calculator

**VOCABULARY:** Dimensions, area, rectangle

**PREREQUISITE SKILL:** Area of a rectangle formula, graphing data

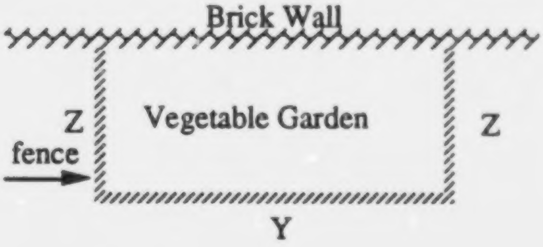
## LESSON

- **DIRECTED INSTRUCTION:**

Explain that you want to fence a rectangular area in your back yard to be used for a vegetable garden. One side will be an existing brick wall. The other three sides are to be enclosed by 30 feet of fencing. You want to find which dimensions will create a rectangle, so that the area of the garden is as large as possible.

Hand out Student Activity Sheet 1 and a graph sheet. Ask them to fill in the first line of the chart. Z is 1 foot. Discuss.

If Z is 1 ft., there are 2 sides this size, then 2 x 1 or 2 feet of the original 30 feet of fence are used in those 2 sides. This leaves 30 - 2 or 28 feet for side Y.



| Z | Y | Amount of fencing used 2 x Z + Y | Area of vegetable garden Z x Y sq.ft. |
|---|---|---|---|
| 1 | 28 | 30 | 28 |

2 x Z + Y is 2 x 1 + 28 = 30 feet, which shows that we didn't make a mistake in finding Y. Remember you should always end up using exactly 30 feet of fencing. Students record this answer on the graph.

- **GUIDED PRACTICE:**

Students fill in 2 on the chart for Z = 2 feet.

2 sides of 2 feet = 4 feet
1 side of 26 feet = 26 feet
Total = 30 feet

Students record this answer on the graph.

First use whole number choices for Z. When they have narrowed down their choices of whole numbers, they then start with decimals.

As an example of working with a decimal number, ask them to fill in a line on the chart for Z = 2.4 feet. They should realize 2 sides of 2.4 uses 2 x 2.4 or 4.8 feet of the fence, leaving 30 - 4.8 or 25.2 feet for the other side. So Y = 25.2 feet.

Checking for mistakes:

2 sides of 2.4 feet = 4.8 feet
1 side of 25.2 feet = 25.2 feet
Total fencing used = 30.0 feet

The area of the vegetable garden is 2.4 x 25.2 = 60.48 sq. ft.

| Z | Y | Amount of fencing used 2 x Z + Y | Area of vegetable garden Z x Y sq.ft. |
|---|---|---|---|
| 1 | 28 | 30 | 28 |
| 2.4 | 25.2 | 30 | 60.48 |

Ask what they observed. Guide them to notice that when Z increased by 1.4 ft., Y decreased by 2.8 ft., the area increased by 32.48 sq. ft., and the amount of fencing specified in the problem remains 30 feet. Students search for the largest possible area, using previous results as a guide.

Students record all trials on the graph.

- **INDEPENDENT PRACTICE:**

Students complete Student Activity Sheet 1, and graph each answer.

Upon completion of the Student Activity Sheet, discuss class observations from the chart and graph. They should find that the closer they chose Z to 7 or 8 inches the larger the area became, the further from 7 or 8 inches the smaller the area became. Those students who explore using decimals, between 7 and 8 find that the largest area occurs when Z = 7.5.

## ANOTHER FENCE ON THE WALL

Teacher Answer Sheet

Below is an organized list of possible answers. Student choices for Z may vary from this.

| Z | Y | Amount of fenced used 2 x Z + Y | Area of vegetable garden Z x Y sq.ft. |
|---|---|---|---|
| 1 | 28 | 30 | 28 |
| 2 | 26 | 30 | 52 |
| 3 | 24 | 30 | 72 |
| 4 | 22 | 30 | 88 |
| 5 | 20 | 30 | 100 |
| 6 | 18 | 30 | 108 |
| * 7 | 16 | 30 | 112 |
| * 8 | 14 | 30 | 112 |
| 9 | 12 | 30 | 108 |
| 10 | 10 | 30 | 100 |
| 11 | 8 | 30 | 88 |
| 12 | 6 | 30 | 72 |
| 13 | 4 | 30 | 52 |
| 14 | 2 | 30 | 28 |

| Z | Y | Amount of fenced used 2 x Z + Y | Area of vegetable garden Z x Y sq.ft. |
|---|---|---|---|
| 7.0 | 16 | 30 | 112.00 |
| 7.1 | 15.8 | 30 | 112.18 |
| 7.2 | 15.6 | 30 | 112.32 |
| 7.3 | 15.4 | 30 | 112.42 |
| 7.4 | 15.2 | 30 | 112.48 |
| * 7.5 | 15.0 | 30 | 112.50 |
| 7.6 | 14.8 | 30 | 112.48 |
| 7.7 | 14.6 | 30 | 112.42 |
| 7.8 | 14.4 | 30 | 112.32 |
| 7.9 | 14.2 | 30 | 112.18 |
| 8.0 | 14.0 | 30 | 112.00 |

*You may wish to have them look at the results of the area columns of these two charts.

## Another Fence On The Wall

Student Activity Sheet 1, page 2
Teacher Answer Sheet

Answers to student questions:

1. The dimensions for largest area are
   Z = 7.5 ft.
   Y = 15 ft.
   Area = 112.5 sq. ft.

   Since students are using guess and check they may not arrive at the exact answer.

2. A. Y is twice as big as Z. (15 = 2 x 7.5)

   B. Y is $\frac{1}{2}$ of the amount of fencing used. (15 = $\frac{1}{2}$ x 30) Z is $\frac{1}{4}$ of the amount of fencing used. (7.5 = $\frac{1}{4}$ x 30)

   C. You may wish to have students do the extension below or discuss the fact that 1, 2, and 3 above would be true for any size fence. If instead of 30 ft. you had 100 ft. of fencing, then
      a) Y would be $\frac{1}{2}$ x 100 or 50 ft.
      b) Z would be $\frac{1}{4}$ x 100 or 25 ft.
      c) Y would be as big as Z

- **EVALUATION:**
  Teacher observation and Student Activity Sheet.

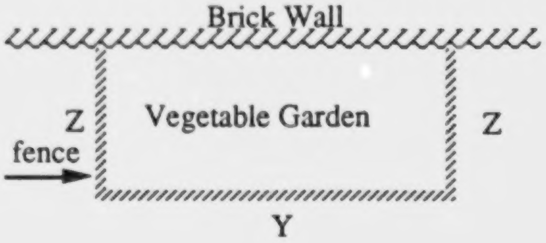
- **EXTENSION:**
  Student solve the same problem with 40 ft. or 100 ft of fencing.

Name ____________________

## ANOTHER FENCE ON THE WALL

Student Activity Sheet 1, page 1

You want to fence in a rectangular area in your backyard to be used for a vegetable garden. One side will be an existing brick wall. The other three sides are to be enclosed by exactly 30 ft. of fencing. You need to find which dimensions will create a rectangle, so that the area of the garden is as large as possible. Reminder: Area of a Rectangle = length x width.



| $Z$ (feet) | $Y$ (feet) | Amount of fencing used $2 \times Z + Y$ (feet) | Area of vegetable garden $Z \times Y$ (square feet) |
|---|---|---|---|
| 1 | | | |
| | | | |
| | | | |
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Name ____________________

**Another Fence On The Wall**
Student Activity Sheet 1, page 2

Additional workspace.

| Z (feet) | Y (feet) | Amount of fencing used 2 x Z + Y (feet) | Area of vegetable garden Z x Y (square feet) |
|---|---|---|---|
| | | | |
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Name ____________________

## ANOTHER FENCE ON THE WALL

Student Activity Sheet 1, page 3

1. What dimensions gave the biggest area?

   Z = _______________ Y = _______________ Area = ____________

2. Use your Z and Y in question 1 to answer the following questions.

   A. What do you observe about the relationship of the width to the length in your solution? ________________________________________________

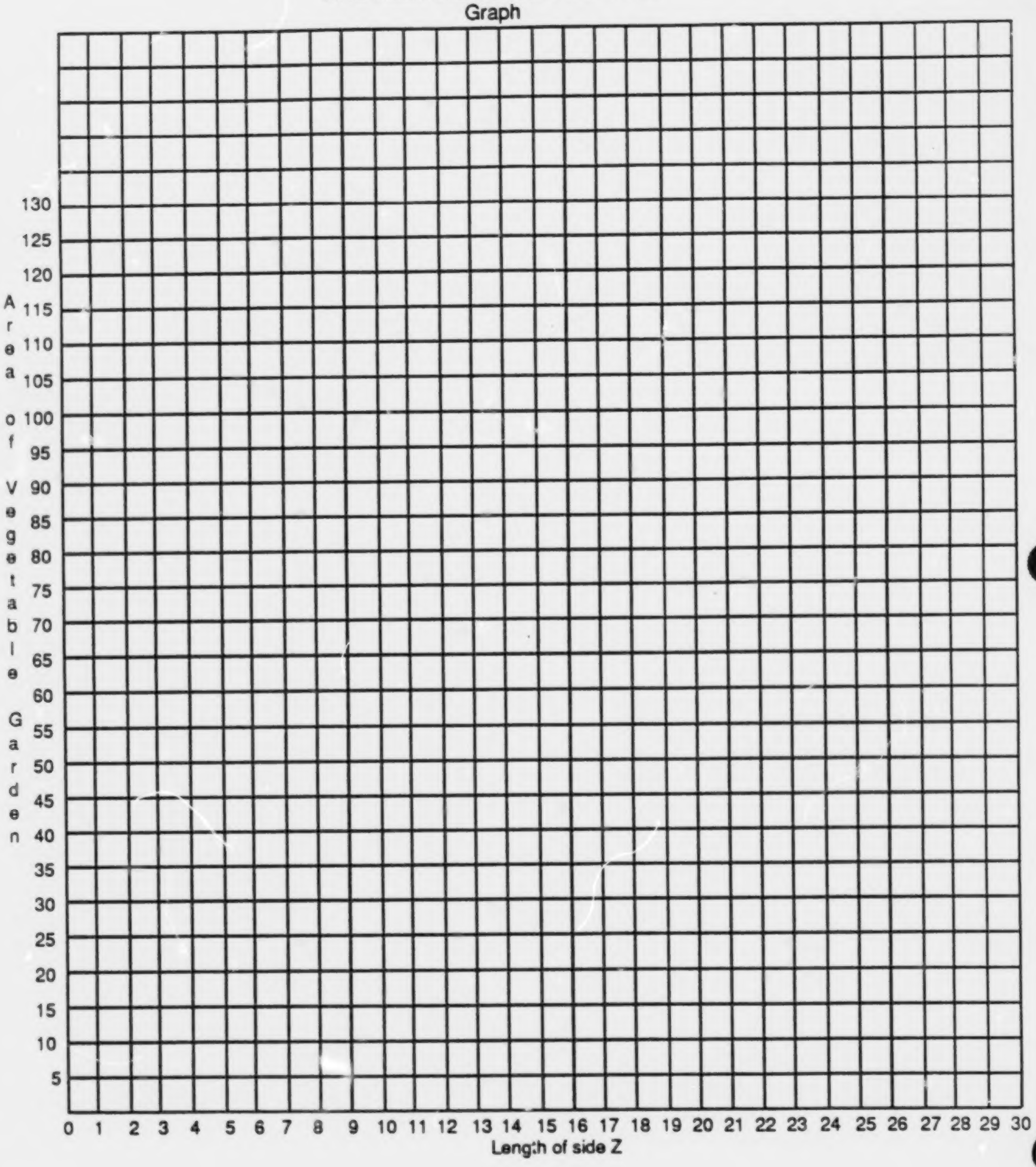
   B. What do you observe about the relationships between the width and the amount of fencing, and the length and the amount of fencing? _________

   C. Other observations. ________________________________________

Name ____________________

## ANOTHER FENCE ON THE WALL

Graph



# THE WHEELS ON THE BIKE GO ROUND AND ROUND

**GRADE:** 7 - 8

**STRAND:** Geometry/Measurement

**SKILL:** Apply circumference formula to real life situations.

**MANAGEMENT**

**CLASS ORGANIZATION:** Whole class, pairs

**TIME FRAME:** One math period

**MATERIALS:** Calculator, trundle wheel (or wheel) or round basket

**VOCABULARY:** Diameter, circumference, revolution, pi ($\pi$), inches, feet, and mile

**PREREQUISITE SKILL:** Calculator Lesson 23 in Book 3.

**LESSON**

- **DIRECTED INSTRUCTION/ GUIDED PRACTICE**

  In Book 3, lesson 23, students discovered circumference divided by diameter equals pi, $C \div d = \pi$. In this lesson they will use the equation, $C = \pi \times d$. This lesson will relate these formulas to the wheels of their bicycles and the distances they travel.

  Teacher rolls a wheel once along a line to demonstrate that CIRCUMFERENCE is the DISTANCE ONE REVOLUTION of the wheel will cover. For example, a wagon wheel with a 10" diameter will roll about 3.14 x 10" = 31.4 inches in one revolution. Pass out Student Activity Sheet.

  Guide students through questions 1-5. Discuss results. See Teacher Answer Sheet.

- **INDEPENDENT PRACTICE:**

  Students complete Student Activity Sheet.

- **HOME EXTENSION ACTIVITY:**

  Hand out Home Activity Sheet and have students compare their results the following day. Answers will vary.

## THE WHEELS ON THE BIKE GO ROUND AND ROUND

Student Activity Sheet
Teacher Answer Sheet

CIRCUMFERENCE = π X d = pi x diameter
use 3.14 for π

1. How far will a bicycle with a 20" diameter wheel roll in one revolution ? 62.8 or about 63 inches
2. There are 5280 feet in one mile. How many inches are in one mile? 63360
3. Estimate how many revolutions the 20" diameter bicycle wheel will make in one mile. 63360 ÷ 63 is about 1,000 revolutions
4. What operation can you use to solve question 3? (÷) divide
5. Use your calculator to solve question 3. Round to the nearest whole number. 63360 ÷ 63 = 1006 or 63360 ÷ 62.8 = 1009
6. If you have a bicycle with a 24" diameter wheel, will it take more or less revolutions than the 20" wheel to go one mile? less
7. Find the number of revolutions the 24" bicycle wheel will make in one mile. 24" x 3.14 = 75 inches 63360 ÷ 75 = 844.8 = 845
8. On a 5 mile bicycle trip, how many revolutions will a 24" bicycle wheel make? 845 x 5 = 4225
9. Find how many revolutions the 24" bicycle wheel will make on a 10 mile trip. Do this mentally. Record your answer 4225 x 2 = 8450 or 845 x 10 = 8450 Verify with your calculator. (Hint: you can use the answer to problem 7 or 8.)

NAME______________

## THE WHEELS ON THE BIKE GO ROUND AND ROUND
Student Activity Sheet

CIRCUMFERENCE = $\pi$ X d = pi x diameter
use 3.14 for $\pi$

1. How far will a bicycle with a 20" diameter wheel roll in one revolution ?

_______________________________________________

2. There are 5280 feet in one mile. How many inches are in one mile?

_______________________________________________

3. Estimate how many revolutions the 20" diameter bicycle wheel will make in one mile. _______________________________________________

4. What operation can you use to solve question 3? ____________________

5. Use your calculator to solve question 3. Round to the nearest whole number.

_______________________________________________

6. If you have a bicycle with a 24" diameter wheel, will it take more or less revolutions than the 20" wheel to go one mile? ____________________

7. Find the number of revolutions the 24" bicycle wheel will make in one mile.

_______________________________________________

8. On a 5 mile bicycle trip, how many revolutions will a 24" bicycle wheel make?

_______________________________________________

9. Find how many revolutions the 24" bicycle wheel will make on a 10 mile trip. Do this mentally. Record your answer ______________________________

Verify with your calculator. (Hint: you can use the answer to problem 7 or 8.)

Name ____________

## THE WHEELS ON THE BIKE GO ROUND AND ROUND

### Home Activity Sheet

$$\boxed{C = \pi d}$$

1. Measure the diameter of a tire on a car in inches. ____________________
2. Determine the number of revolutions the tire will make in one mile. Record ____________________________________________
3. Tires last about 30,000 miles. How many revolutions would that be? ____________________________________________
4. What other vehicles have larger wheel sizes? ____________________
5. How would these larger sizes affect the number of revolutions in one mile? ____________________________________________ ____________________________________________ ____________________________________________ ____________________________________________

## WHICH HOLDS MORE?

**GRADE:** 7 - 8

**STRAND:** Geometry

**SKILL:** Estimate and find volume of solids.

### MANAGEMENT

**CLASS ORGANIZATION:** Whole class for directed lesson, then in small groups.

**TIME FRAME:** Two math periods

**MATERIALS:** Calculator, 8.5" by 11" paper, metric and customary rulers, masking or cellophane tape, different cylindrical containers

**VOCABULARY:** Volume, circumference, diameter, radius, cylinder

**PREREQUISITE SKILL:** Understand $\pi$ and use of formulas.

### LESSON

**DIRECTED INSTRUCTION AND GUIDED PRACTICE:**

- Give each student 2 pieces of 8.5 by 11 inch paper and a ruler. With the longer side in a vertical position, curl the paper so you get a hollow cylinder. Tape the edges so they meet but do not overlap. See figure 1.

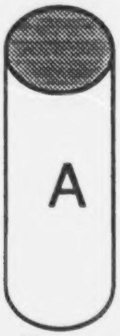


Figure 1

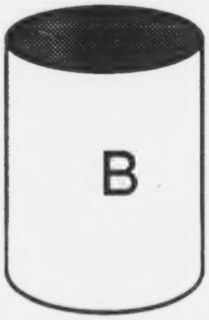


Figure 2

- Students use the other piece of paper and with the shorter side in a vertical position, curl the paper so you get a hollow cylinder. Tape the edges. See Figure 2.
- Hand out Student Activity Sheet 1.
- Students have two different cylinders made from 8.5" by 11" paper.
- Students estimate which holds more.

  Be sure students notice the circumference of cylinder A equals the width of the original paper. The circumference of cylinder B equals the length of the original paper.

- Students measure the diameter of each cylinder and record.
- To find the diameter, students will also use the formula: Circumference $\div \pi = d$. The circumference for figure 1 is 8.5". The circumference for figure 2 is 11".
- Discuss the differences between the answers to questions 2 and 3.
- The radius is $\frac{1}{2}$ of the diameter (.5 x d).
- Volume $= \pi r^2 h$. Volume will be in cubic inches ($in^3$). The heights of the cylinders are shown in the picture at the top of Student Activity Sheet 1.
- Be sure students have time to compare calculated volumes with their original estimates. Ask them if they repeat the procedure with 2 other matching sheets of paper, "Will the volume of the shorter cylinder always be more?" Explain.

- **INDEPENDENT PRACTICE:**
  - Hand out Student Activity Sheet 2 and metric rulers.
  - Select three different cylindrical containers.
  - Label them A, B, C.
  - Using estimation, order the cylinders from the least to the greatest volume.

Example: volume of B < volume of A < volume of C

Students:

- Measure the diameter of the circular base and the height of each container to the nearest tenth of a cm.
- Compute the radius of each container.
- Record their measurement on Student Activity Sheet 2.
- Find the volume of each container. [$V = \pi r^2 h$]
- Record answers on the Student Activity Sheet.
- Discuss how their estimates compare to the actual volume of each container.

- **HOME ACTIVITY**

  Students measure the diameter and height of 3 cans and use their calculator to find the volume. Record diameter, height, and volume for each can.

## WHICH HOLDS MORE?

Student Activity Sheet 1
Teacher Answer Sheet

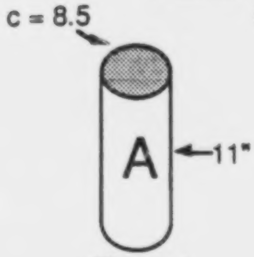


Figure 1

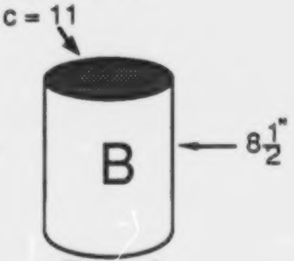


Figure 2

1. Estimate which cylinder holds more. ______________________

Note: The circumference of cylinder A equals the width of your original paper. The circumference of cylinder B equals the length of your original paper.

2. Measure the diameter of each cylinder.

Diameter of cylinder A = about 2 3/4 or 2.75 in.

Diameter of cylinder B = about 3 1/2 or 3.50 in.

circumference ÷ π = diameter

3. Use the fomula c ÷ π = d (π = 3.14) to compute the diameter of each cylinder.

Diameter of cylinder A = 2.707 in.

Diameter of cylinder B = 3.5031847 in.

4. Are the answers to questions 2 and 3 approximately the same ? ____

Answer will vary

Explain: Using an inch ruler, measurements will be in fractional parts, either fourths, eighths or sixteenths.

5. Radius = Diameter ÷ 2 Use the diameter from questions 3 to compute the radii of the cylinders.

Radius of cylinder A = 1.35 in.

Radius of cylinder B = 1.75 in.

VOLUME OF A CYLINDER
Volume = π x radius of base x radius of base x height = $\pi r^2 h$
Volume = area of circular base x height of cylinder.

6. Find the volume of the cylinders.

Volume of cylinder A = 63.94915 cubic in.

Volume of cylinder B = 81.738125 cubic in.

7. Compare results to your estimate in question 1. Which cylinder holds more? ______________________

Name ____________________

## WHICH HOLDS MORE?

Student Activity Sheet 1

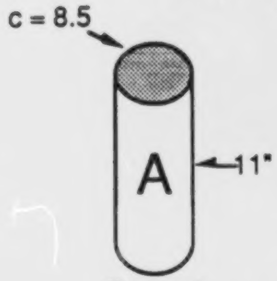


Figure 1

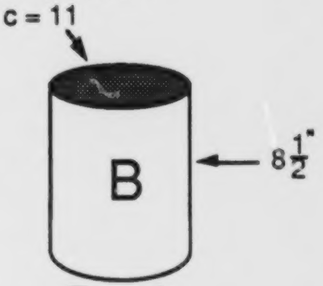


Figure 2

1. Estimate which cylinder holds more. ____________________

Note: The circumference of cylinder A equals the width of your original paper. The circumference of cylinder B equals the length of your original paper.

2. Measure the diameter of each cylinder.

Diameter of cylinder A = ____________________ in.

Diameter of cylinder B = ____________________ in.

circumference + $\pi$ = diameter

3. Use the fomula $c + \pi = d$ ($\pi = 3.14$) to compute the diameter of each cylinder.

Diameter of cylinder A = ____________________ in.

Diameter of cylinder B = ____________________ in.

4. Are the answers to questions 2 and 3 approximately the same ?

Explain: ____________________

____________________

5. Radius = Diameter + 2 Use the diameter from questions 3 to compute the radii of the cylinders.

Radius of cylinder A = ____________________ in.

Radius of cylinder B = ____________________ in.

VOLUME OF A CYLINDER

Volume = $\pi$ x radius of base x radius of base x height = $\pi r^2 h$

Volume = area of circular base x height of cylinder.

6. Find the volume of the cylinders.

Volume of cylinder A = ____________________cubic in. (in$^3$).

Volume of cylinder B = ____________________cubic in. (in$^3$).

7. Compare results to your estimate in question 1. Which cylinder holds more? ____________________

Name ____________________

## WHICH HOLDS MORE?

Student Activity Sheet 2

1. Select three different cylindrical containers.
2. Label them A, B, C.
3. Using estimation, order the cylinders from the least to the greatest volume.

cylinder with least volume

cylinder with greatest volume

______ < ______ < ______

4. Measure the diameter of the circular base and the height of each container. What is the radius of each container?
5. Record your measurement.

| | diameter nearest tenth of cm (d) | radius nearest tenth of cm (r) | height nearest tenth of cm (h) |
|---|---|---|---|
| A | | | |
| B | | | |
| C | | | |

6. Find the volume of each container.

$$\pi = 3.14 \quad V = \pi r^2 h$$

| | $V = \pi r^2 h$ |
|---|---|
| A | |
| B | |
| C | |

7. How did your estimate compare to the actual volume of each container?

______________________________________________

______________________________________________

______________________________________________

# CHRIS' UP AND DOWN DAY

**GRADE:** 7 - 8

**STRAND:** Measurement

**SKILL:** Apply temperature conversion formulas

**MANAGEMENT**

**CLASS ORGANIZATION:** Whole class

**TIME FRAME:** One math period

**MATERIALS:** Calculator, thermometers (optional)

**VOCABULARY:** Conversion, Fahrenheit, Celsius

**PREREQUISITE SKILL:** Understanding decimals

## LESSON

### Activity 1

- **DIRECTED INSTRUCTION:**

  "Today we are going to read a story about Chris and use the story to help us learn about Fahrenheit and Celsius temperatures." Hand out Student Activity Sheet. "There are adjective blanks for you to fill in to describe the temperature, and temperature blanks for you to fill in the conversion of the given Fahrenheit or Celsius temperatures."

  "Use the sheet with the thermometers to estimate your temperatures. Use the words at the top of the worksheet for the adjectives." Students might find it easier to do all the estimating of temperatures before writing the adjectives. Each adjective is used once.

- **GUIDED PRACTICE:**

  Do the first paragraph with the students before they work on their own. Discuss how they can look at the thermometers and estimate the converted temperature.

- **INDEPENDENT PRACTICE:**

  Students estimate the rest of Student Activity Sheet, filling in their answers in the blanks. They will not need their calculators for this part of the period. Give them about 10-15 minutes.

- **EVALUATION:**

  When Student Activity Sheet is completed, students compare their recorded responses. Discuss any major differences.

## LESSON

### Activity 2

- **DIRECTED INSTRUCTION:**

  Hand out a second copy of Student Activity Sheet. Students will use formulas for converting Fahrenheit to Celsius and Celsius to Fahrenheit.

| Celsius to Fahrenheit | Fahrenheit to Celsius |
|---|---|
| C x 9 ÷ 5 + 32 = F | (F - 32) x 5 ÷ 9 = C |

  Use calculators to do these conversions.

  Students do the first paragraph. This time using the formulas to find the correct answers for the temperatures. Ask students to compare their estimates with their computed answers.

- **INDEPENDENT PRACTICE:**

  Students complete Student Activity Sheet on their own using the formulas. They check to see how close their estimates were to the computed answer.

- **EVALUATION:**

  Discuss results.

- **HOME ACTIVITY:**

  Students write a creative story problem involving varied temperatures using °F and °C. They must have at least ten conversions.

### CHRIS' UP AND DOWN DAY

Student Activity Sheet 2
Teacher Answer Sheet

| Vocabulary Needed for Blanks (Use each adjective once) | | | | | |
|---|---|---|---|---|---|
| **frigid** | **hot** | **warm** | **steaming** | **sunny** | **surfing** |

C x 9 ÷ 5 + 32 = F

(F - 32) x 5 ÷ 9 = C

Chris and his sister Jan woke up Saturday morning and looked out the window and saw it was not a ____sunny____ day. The temperature gauge read 15° C which is____59 °F___.
#1 adj. #2

Since Chris and Jan had planned to go to the beach later, this wasn't the kind of weather they expected. They had hoped it would be ____surfing____ weather with
#3 adj.

a temperature of 80 °F which is____26.7 °C.
#4

Both decided since they couldn't go to the beach, they didn't want to do their Saturday chores either. Chris quickly jumped into bed and groaned loudly, "Mom, I'm sick today. I bet I have a temperature of 39 °C which is__102.2 °F__."
#5

Jan exclaimed, " And I have a stomach ache!" Mom came in the room with a thermometer and put it in Chris' mouth. When she took it out later she said, "Why Chris, your temperature is 96° F which is__35.6 °C__. That's
#6

too low, so you must be sick. I'll go fix you some ____hot____oatmeal, with a
#7 adj.

temperature of 88° C, which is____190.4 °F__. That will warm your body so that you
# 8

can have a normal reading of 98.6° F which is____37° C__. As Chris lay in bed, he
#9

daydreamed about being in sunny, ___warm___ Hawaii, surfing in the 27° C which is
#10 adj.

___80.6___ °F ocean. Chris certainly didn't want to be in the ___frigid___
#11 #12 adj.

water near Alaska, with temperatures near 0° C which is ___32 °F___. As
#13

Chris was daydreaming, Mom came in again with a ___steaming___ cup of cocoa
#14 adj.

that looked like it must be 110° C which is ___230 °F___. While Chris was sipping the cocoa,
#15

the sun came streaming in his window. Jan jumped up and looked at the temperature

gauge. It read 75° F which is ___23.9 °C___. "Oh, boy!", Jan and Chris thought, "I bet by afternoon
#16

the water will be around 20° C which is ___68 °F___ and the beach will be 80° F which
#17

is ___26.7 °C___. I'm going to do my chores now so I can go surfing afterward." Mom laughed as
# 1 8

Chris hurried out the door for she knew Chris' temperature had been 37° C which is ___98.6 °F___
#19

all along.

Name ____________________

## CHRIS' UP AND DOWN DAY

### Student Activity Sheet

| Vocabulary Needed for Blanks (Use each adjective once) | | | | | |
|---|---|---|---|---|---|
| frigid | hot | warm | steaming | sunny | surfing |

°C x 9 ÷ 5 + 32 = °F

(°F - 32) x 5 ÷ 9 = °C

Chris and his sister Jan woke up Saturday morning and looked out the window and saw it was not a ____________ day. The temperature gauge read 15° C which is ____________ °F.
#1 adj. #2

Since Chris and Jan had planned to go to the beach later, this wasn't the kind of weather they expected. They had hoped it would be ________________ weather with
#3 adj.

a temperature of 80 °F which is ____________ °C.
#4

Both decided since they couldn't go to the beach, they didn't want to do their Saturday chores either. Chris quickly jumped into bed and groaned loudly,

"Mom, I'm sick today. I bet I have a temperature of 39 °C which is ____________ °F."
#5

Jan exclaimed, " And I have a stomach ache!" Mom came in the room with a thermometer and put it in Chris' mouth. When she took it out later she said, "Why Chris, your temperature is 96° F which is ________ °C. That's
#6

too low, so you must be sick. I'll go fix you some ______________ oatmeal, with a
#7 adj.

temperature of 88° C, which is ____________ °F. That will warm your body so that you
# 8

can have a normal reading of 98.6° F which is ____________ °C. As Chris lay in bed, he
#9

daydreamed about being in sunny, ______________ Hawaii, surfing in the 27° C which is
#10 adj.

__________________°F ocean. Chris certainly didn't want to be in the ________________
#11 #12 adj.

water near Alaska, with temperatures near 0° C which is__________°F. As
#13

Chris was daydreaming, Mom came in again with a ______________________cup of cocoa
#14 adj.

that looked like it must be 110° C which is__________°F. While Chris was sipping the cocoa,
#15

the sun came streaming in his window. Jan jumped up and looked at the temperature

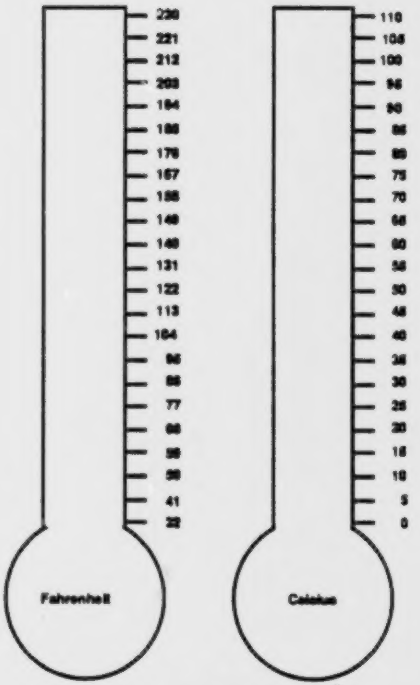
gauge. It read 75° F which is_____°C. "Oh, boy!", Jan and Chris thought, "I bet by afternoon
# 1 6

the water will be around 20° C which is________°F and the beach will be 80° F which
#17

is ________°C. I'm going to do my chores now so I can go surfing afterward." Mom laughed as
# 1 8

Chris hurried out the door for she knew Chris' temperature had been 37° C which is________°F
#19

all along.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Celsius

225

## SCALING FOR SPECIAL EFFECTS

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Measurement/ Geometry |
| **SKILL:** | Measure lengths to the nearest tenth of a centimeter; use scale factors |

### MANAGEMENT

| | |
|---|---|
| **CLASS ORGANIZATION:** | Whole class, pairs, or individuals |
| **TIME FRAME:** | One math period |
| **MATERIALS:** | Calculator, centimeter rulers, 3-cm paper clip (optional) |
| **VOCABULARY:** | Scale factor |
| **PREREQUISITE SKILL:** | Measurement to the nearest tenth of a centimeter |

### LESSON

- **DIRECTED INSTRUCTION/GUIDED PRACTICE:**

Tell your class that a company is going to produce a television advertisement for office supplies in which a person is supposed to look much smaller than usual. The plan is to create this effect by showing the person next to some common objects (paperclips, pencils, etc.) that are greatly increased in size. In comparison, the person will look tiny. Their problem is how big to make the models of the real objects so that the effect will be realistic. Each real object must be enlarged by the same amount so that everything will appear to be in the same scale. To do this the length (and other dimensions) of each object will be multiplied by a number called a scale factor.

Hand out Student Activity Sheet 1. Tell the class that the paper clip in the bottom left corner is a life-size picture of a real paper clip. The paper clip on the right side of the page is the size that the advertising company wants to make the model.

Working as a full class, the students should fill in the Student Activity Sheet through question 5. In question 3, some students will divide the length of the large paper clip, 19, by the length of the real paper clip, 3, to get 6.3, while others might find 6.3 by trial and error. In any case, it is important to confirm (in question 4) that 6.3 works as a scale factor, transforming 3 cm into about 19 cm.

- **INDEPENDENT PRACTICE:**

The students should complete the page independently as individuals or in pairs. When most of the class has completed the Student Activity Sheet discuss their results. Probably some of the students will have slightly different measurements for some of the objects.

If there is time you can ask questions like these: "Would any of the model objects be longer than a meter?" Yes. [The pencil would be a little more than 103 cm long, which is longer than a meter.]

"If you were going to build the model paper clip, how would you find out how long a piece of wire you would need?"

This might be approached in three ways:

a. Try to estimate the length of the model paper clip, somehow turning your ruler to follow the curves.
b. Lay a piece of string along the model, following the wire, and then measure its length.
c. Straighten out a real paper clip, measure its length, and then multiply by the scale factor 6.3.

You would need a piece of wire about 68.5 cm long.

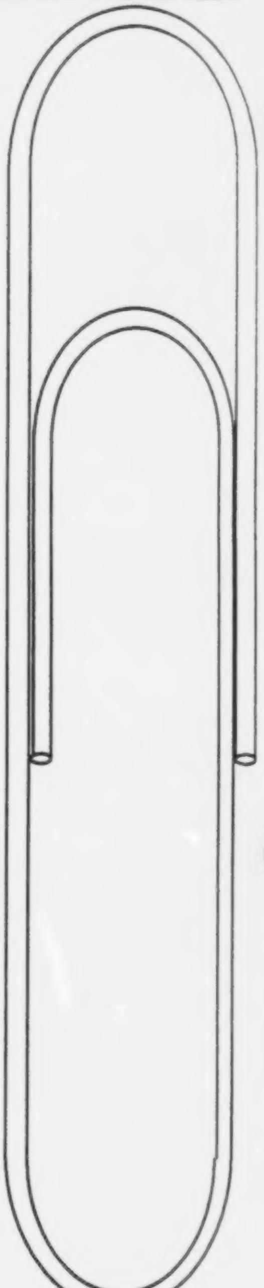
"Remember that we're making the models large so that an actor would look small in comparison. How large would an actor look if he's really 6 feet tall?" The viewers do not know that the objects are large models. They assume they are the normal size. So in comparison, the actor appears to be shrunken by the scale factor. The actor appears to be 6 ÷ 6.3 feet tall, or 0.9524 feet, or 0.9524 x 12 = 11.43 inches.

- **EVALUATION:**

  Teacher observation

Name ______________________

## SCALING FOR SPECIAL EFFECTS
Student Activity Sheet
Teacher Answer Sheet



**Make all measurements to the nearest tenth of a centimeter.**

Answers may vary slightly due to accuracy of measurement.

1. How long is the real paper clip shown below? 3 cm
2. How long is the model paper clip shown at right ? 19 cm
3. What number can you multiply the measurement of the real paper by to get the measurement of the model? 6.3

The answer in question number 3 is the SCALE FACTOR.

4. Try it!

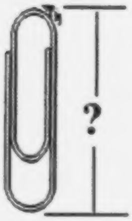
Scale Factor

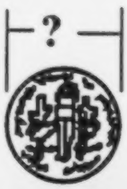
3 cm $\xrightarrow{6.3\ \times}$ 18.9 ≈ 19 cm

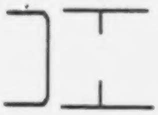
length of real paperclip: → length of model paperclip

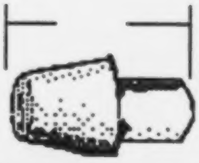
Use the same scale factor to find how large all of the models should be.

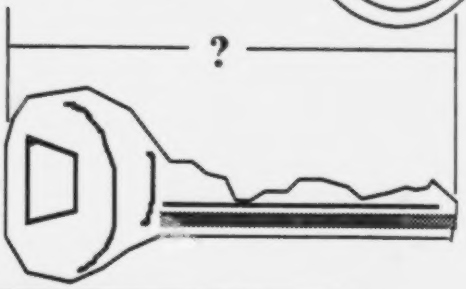
5. key: 6.6 cm $\xrightarrow{6.3\ \times}$ 41.58 ≈ 41.6 cm
6. dime: 1.7 cm $\xrightarrow{6.3\ \times}$ 10.71 ≈ 10.7 cm
7. pencil: 16.3 cm $\xrightarrow{6.3\ \times}$ 102.69 ≈ 102.7 cm
8. eraser: 2.7 cm $\xrightarrow{6.3\ \times}$ 17.01 ≈ 17.0 cm
9. staple: 1.4 cm $\xrightarrow{6.3\ \times}$ 8.82 ≈ 8.8 cm

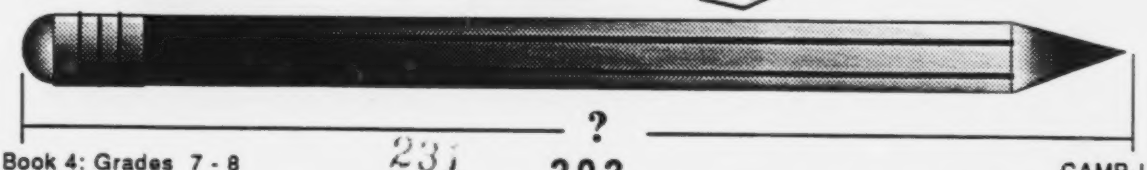












Name ____________________

## SCALING FOR SPECIAL EFFECTS

Student Activity Sheet

**Make all measurements to the nearest tenth of a centimeter.**

1. How long is the real paper clip shown below? cm
2. How long is the model paper clip shown at right ? ____________ cm
3. What number can you multiply the measurement of the real paper by to get the measurement of the model?

The answer in question number 3 is the **SCALE FACTOR**.

4. Try it!

Scale Factor

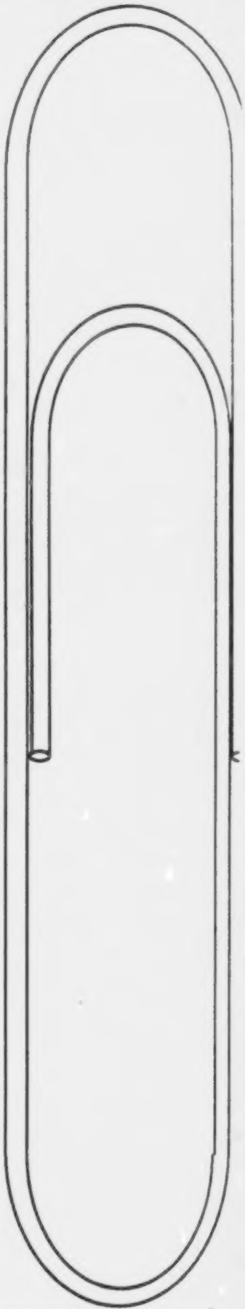
______ x

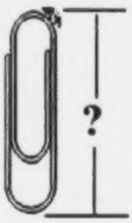
__________ cm → __________ cm

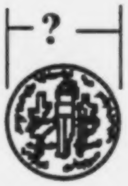
length of real paperclip: | length of model paperclip

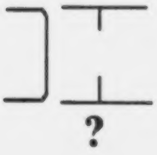
Use the same scale factor to find how large all of the models should be.

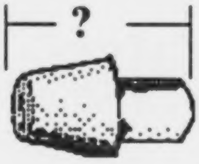
5. key: ______ x __________ cm → __________ cm
6. dime: ______ x __________ cm → __________ cm
7. pencil: ______ x __________ cm → __________ cm
8. eraser: ______ x __________ cm → __________ cm
9. staple: ______ x __________ cm → __________ cm

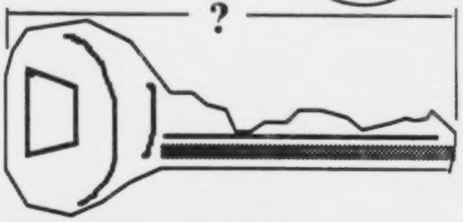


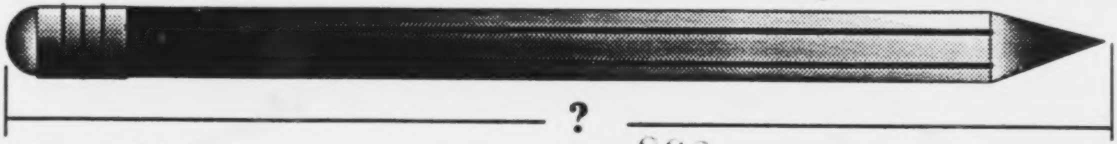












# MORE SCALING FOR SPECIAL EFFECTS

**GRADE:** 7 - 8

**STRAND:** Number

**SKILL:** Determine how area and volume are affected by a change of scale.

**MANAGEMENT**

**CLASS ORGANIZATION:** Whole group and individuals, or pairs

**TIME FRAME:** One math period

**MATERIALS:** Calculator, centimeter rulers, an object shaped like a rectangular prism for each individual student or pair

**VOCABULARY:** Scale factor, rectangular prism, square centimeters ($cm^2$), cubic centimeters ($cm^3$)

**PREREQUISITE SKILL:** Lesson 26

## LESSON

- **DIRECTED INSTRUCTION/GUIDED PRACTICE:**

Remind your class of the problem of building model objects to use in a TV advertisement. Recall that each measurement of a real object was multiplied by a SCALE FACTOR of 6.3 to get the corresponding length of the model.

Distribute the Student Activity Sheet. To build the model of the stick of gum we need to know the length and the width. Students should measure the length and width of the picture, and then multiply each by the scale factor 6.3 to get the length and width of the model.

Someone may suggest that we should consider the thickness of a stick of gum as well. That can't be shown easily in the picture. Either measure a real piece of gum or tell them that a stick of gum is about 0.2 cm thick, so the model would have to be about 1.3 cm thick ($6.3 \times 0.2 = 1.26 \approx 1.3$).

- **INDEPENDENT PRACTICE:**

Students find a rectangular prism. This could be something like a paperclip box, a chalk box, a shoe box, a book, or a gift box. They will use the prism to complete question 2.

Upon completion of the Student Activity Sheet, discuss their results. Question 5 asks for a comparison of results from questions 3 and 4. It is important for students to understand that the area of the top of the model is NOT 6.3 times the area of the top of the real object. Since length and width are both increased by a factor of 6.3, the area is increased by a factor of 6.3 x 6.3, or almost 40.

So, whatever a student's answer to question 3 is, the answer to question 4 should be about 40 times larger.
Similarly, in question 6, the volume increases by a factor of 6.3 x 6.3 x 6.3, or about 250. Notice that this means that a model of a 1 pound box of sugar, if filled with sugar, would weigh more than 250 pounds!

- **EVALUATION:**

  Teacher observation. Answer to questions 2 - 8 will vary.

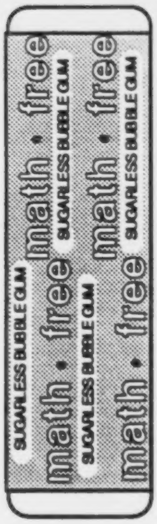
Name______________

## MORE SCALING FOR SPECIAL EFFECTS
### Student Activity Sheet

Use a scale factor of 6.3

Make all measurements to the nearest tenth of a centimeter.

1. This is a life-size picture of a real stick of gum. How large should the model stick of gum be?


SUGARLESS BUBBLE GUM
math • free
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math • free

| | Real object | Scale Factor | Model object |
|---|---|---|---|
| **length:** | ________ cm | 6.3 X → | ________ cm |
| **width:** | ________ cm | 6.3 X → | ________ cm |
| **height:** | ________ cm | 6.3 X → | ________ cm |

2. Find a real object that looks like a rectangular prism. Record its measurements, and then figure out how large the model should be.

| | Real object | Scale Factor | Model object |
|---|---|---|---|
| **length:** | ________ cm | 6.3 X → | ________ cm |
| **width:** | ________ cm | 6.3 X → | ________ cm |
| **height:** | ________ cm | 6.3 X → | ________ cm |

3. What is the surface area of the top of your object?

Real: ____________cm x ____________cm = ____________$\text{cm}^2$

**length** **width** **area**

4. What would be the surface area of the top of your model?

Model: ____________cm x ____________cm = ____________$\text{cm}^2$

**length** **width** **area**

5. How does the area of the top of the model compare with the area of the top of the real object?

________________________________________________

________________________________________________

6. What is the volume of your object?

____________cm x ____________cm x ____________cm = ____________$\text{cm}^3$

**length** **width** **height** **volume**

7. What is the volume of your model?

____________cm x ____________cm x ____________cm = ____________$\text{cm}^3$

**length** **width** **height** **volume**

8. How does the volume of the model compare with the volume of the real object?

________________________________________________

________________________________________________

# MONUMENTAL SCALING

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Number |
| **SKILL:** | Determine scale factors that will result in models of certain sizes. |

## MANAGEMENT

| | |
|---|---|
| **CLASS ORGANIZATION:** | Whole class, small groups or pairs |
| **TIME FRAME:** | One or two math periods |
| **MATERIALS:** | Calculators, one yardstick |
| **VOCABULARY:** | Scale factor |
| **PREREQUISITE SKILL:** | The lessons on scale factor entitled Lesson 26 and 27 |

## LESSON

- **DIRECTED INSTRUCTION/GUIDED PRACTICE:**

Tell your class about a person who decided she wanted to make a set of models of famous monuments. Write this list of monuments and their heights on the board:

| | Real height |
|---|---|
| Totem Pole (British Columbia) | 175 ft. |
| Statue of Liberty (New York) | 305 ft. |
| Washington Monument (DC) | 555 ft. |
| Gateway Arch (Missouri) | 570 ft. |

She wanted to have the models small enough to fit in a regular-sized room. Ask what scale factor she should use.

After some discussion, say that the person decided to use a scale factor of $\frac{1}{100}$.

Be sure that everyone understands that $\frac{1}{100}$ is shown as 0.01 on the calculator. To confirm this, press $\boxed{1}$ $\boxed{\div}$ $\boxed{100}$ $\boxed{=}$.

On the board draw an arrow after each height and label it with .01 X, like this:

| | Real height | Scale factor | Model height |
|---|---|---|---|
| **Totem pole:** | 175 ft. | $\xrightarrow{0.01\text{ X}}$ | ________ ft. |
| **Statue of Liberty:** | 305 ft. | $\xrightarrow{0.01\text{ X}}$ | ________ ft. |
| **Washington Monument:** | 555 ft. | $\xrightarrow{0.01\text{ X}}$ | ________ ft. |
| **Gateway Arch:** | 570 ft. | $\xrightarrow{0.01\text{ X}}$ | ________ ft. |

The background of your class will affect how the model heights are computed. Some students may be able to do the computation mentally. If there is any doubt, use the calculator, for example, .01 $\boxed{X}$ 175 $\boxed{=}$ .

Complete the column under "Model height" on the board:

| | Real height | Scale factor | Model height |
|---|---|---|---|
| **Totem pole:** | 175 ft. | $\xrightarrow{0.01\text{ X}}$ | 1.75 ft. |
| **Statue of Liberty:** | 305 ft. | $\xrightarrow{0.01\text{ X}}$ | 3.05 ft. |
| **Washington Monument:** | 555 ft. | $\xrightarrow{0.01\text{ X}}$ | 5.55 ft. |
| **Gateway Arch:** | 570 ft. | $\xrightarrow{0.01\text{ X}}$ | 5.70 ft. |

Interpret these model heights in terms that are real to the students. For example, 1.75 is the same as $1\frac{3}{4}$ feet, or 1 foot 9 inches, or 21 inches. Be sure that students connect these numbers with a measurement on the yardstick. Similarly, 5.55 feet is a little more than $5\frac{1}{2}$ feet.

Continue by saying that the person has decided to include the Eiffel Tower as well. Add it to your list.

| | Real height | Scale factor | Model height |
|---|---|---|---|
| **Eiffel Tower:** | 986 ft. | $\xrightarrow{0.01 \text{ X}}$ | 9.86 ft. |

Calculate what the height of the model Eiffel Tower would be (9.86 ft.) Unfortunately the height of her ceiling was only 8 feet, so if she made a model of the Eiffel Tower using the same scale it would not fit.

Discuss what she might do. Possibilities include:

a) Abandon the plan to build a model of the Eiffel Tower.
b) Cut a hole in her ceiling so that the model Eiffel Tower will fit.
c) Change the scale factor for the Eiffel Tower. (leaving it at 0.01 for the other structures)
d) Change the scale factor for all the structures.

The students should understand that choice (c) is unsatisfactory if she is trying to keep all the models in the same relative scale.

Choice (d), changing the scale factor for all the structures, is the most reasonable approach. She wants to find a scale factor for which the Eiffel Tower model will be shorter than 8 feet. Clearly the scale factor must be less than .01. On the other hand she would like all the models to be at least one foot tall.

Encourage discussion on the value of using scaling.

- **INDEPENDENT PRACTICE:**

Distribute the Student Activity Sheet to pairs or small groups. Be sure that the students understand that:

- the same scale factor is to be used with all the structures
- the models of the Eiffel Tower should be less than 8 feet tall
- all the models should be at least 1 foot tall

Answers will vary. Possible answers for the scale factor will range between .006 and .008. Some students may discover that the range can be found by finding the decimal equivalent of $\frac{1}{175}$ for the low end of the scale factor and the decimal equivalent of $\frac{8}{986}$ for the high end of the scale factor (see Extension). Answer to question 2: use a scale factor of .00092 or smaller ($\frac{11}{12}$ ft. ÷ 986 ft.).

- **EXTENSION:**

Find the smallest and largest possible scale factors that will produce models that satisfy the requirements of the problem. [To two significant figures, the scale factors are all numbers from 0.0058 to 0.0081.]

Name ____________________

## MONUMENTAL SCALING

### Student Activity Sheet

Find a scale factor that will make the model of the Eiffel Tower fit under an 8 foot ceiling.

All the models should be at least 1 foot tall. Experiment using various scale factors until you find one that meets both requirements. Fill in the chart using this scale factor.

| | Real height | Scale factor | Model height |
|---|---|---|---|
| **Totem pole:**<br>(British Columbia) | 175 ft. | ________ X | ____________ ft. |
| **Statue of Liberty:**<br>(New York City) | 305 ft. | ________ X | ____________ ft. |
| **Washington Monument:**<br>(District of Columbia) | 555 ft. | ________ X | ____________ ft. |
| **Gateway Arch:**<br>(St. Louis, MO) | 570 ft. | ________ X | ____________ ft. |
| **Eiffel Tower:**<br>(Paris, France) | 986 ft. | ________ X | ____________ ft. |

1. Can you find another scale factor that you could have used? ________________

   If so, what is it? ________________________

2. How could you draw the Eiffel Tower on $8\frac{1}{2}$" x 11" paper. What scale could you possibly use?

   ______________________________________________

   ______________________________________________

   ______________________________________________

3. Why is scaling important? ______________________________

   ______________________________________________

   ______________________________________________

# SCALING UNKNOWN HEIGHTS

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Measurement / Geometry |
| **SKILL:** | Use a scale factor to determine the height of an inaccessible object. |

**MANAGEMENT**

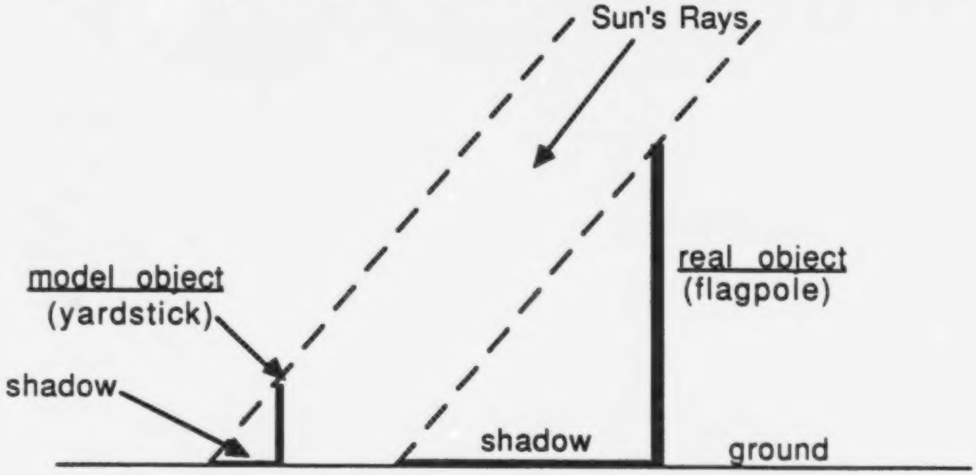
| | |
|---|---|
| **CLASS ORGANIZATION:** | Whole class/ small groups |
| **TIME FRAME:** | One or two math period |
| **MATERIALS:** | Calculators, long customary measuring tape, yardstick |
| **VOCABULARY:** | Scale factor |
| **PREREQUISITE SKILL:** | Lessons 26-28 |

## LESSON

- **DIRECTED INSTRUCTION:**

The student activity part of this lesson must be done outside on a sunny day. The directed instruction part can be done anytime prior to having students make their own measurements.

Introduce the lesson by saying that someone wanted to measure the height of a particular flagpole. It was too tall for him to climb, and he did not want to cut it down. After careful thought, he found a way to measure it indirectly by using a scale factor. As you continue your story, draw and label the following picture. Be sure to draw each part as it is described in your story. Start with the flagpole only; then add the sun's rays creating the flagpole's shadow; then add the yardstick etc.



One sunny day he placed a yardstick vertically, near the flagpole. Since the sun's rays are parallel, he said that the yardstick and its shadow is a scale model of the flagpole, and its shadow. Emphasize that the yardstick, is a model of the flagpole. He drew a chart like this. [draw on the board]

| | Real object | | Scale Factor | Model object | |
|---|---|---|---|---|---|
| **Flagpole:** | ______ | f t | ______ X<br>→ | ______ | f t |
| **Shadow:** | ______ | f t | ______ X<br>→ | ______ | f t |

Be sure that the class understands that this situation is similar to previous lessons on scale factors. The measurements of the "real objects," the flagpole, and its shadow, will be multiplied by a scale factor to get the corresponding measurements of the model, the yardstick, and its shadow.

The height of the model is easy: 3 feet. He found the lengths of the shadows by measuring them along the ground. Filling in these numbers, your chart might look like this:

| | Real object | | Scale Factor | Model object | |
|---|---|---|---|---|---|
| **Flagpole:** | ______ | ft | ______ X<br>→ | 3 | ft |
| **Shadow:** | 38 | ft | ______ X<br>→ | 2.3 | ft |

Note: all of these measures are in feet. [If your class needs practice in converting inches to feet you can say that he measured the yardstick's shadow and got $27\frac{1}{2}$ inches. Then convert $27\frac{1}{2}$ inches to feet by computing 27.5 $\boxed{\div}$ 12 $\boxed{=}$ and round to 2.3.]

His next step was to find the scale factor. He knew it must be a number that you can multiply by 38 to get 2.3. By trial and error, or by dividing 2.3 $\boxed{\div}$ 38 $\boxed{=}$, your class can determine that a scale factor of 0.06 is very close. That can be filled in over the arrows.

| | Real object | Scale Factor | Model object |
|---|---|---|---|
| **Flagpole:** | ______ ft | $\xrightarrow{\underline{0.06}\ \times}$ | 3 ft |
| **Shadow:** | 38 ft | $\xrightarrow{\underline{0.06}\ \times}$ | 2.3 ft |

Finally we can compute the height of the flagpole, 50 feet, either by trial and error or dividing 3 $\boxed{\div}$ 0.06 $\boxed{=}$.

However the numbers are found, the class should confirm that the scale factor gives the results claimed. That is, when all the numbers have been filled into the chart, check to see that they are right. Put the measurements on your diagram on the board.

- **GUIDED PRACTICE:**

Erase the measurements on the board for the flagpole, the model object, their shadows, and the scale factor. Students copy the blank chart from the board. Tell the students that there's a flagpole at another school that is taller. Ask what they need to know to find its height. As they tell you what they need to know in order to find the height, supply the following information for them to fill in on their chart.

| | Real object | Scale Factor | Model object |
|---|---|---|---|
| **Flagpole:** | ______ ft | $\xrightarrow{\underline{\quad\quad}\ \times}$ | 3 ft |
| **Shadow:** | 100 ft | $\xrightarrow{\underline{\quad\quad}\ \times}$ | 4'7" ft<br>(4.58 ft) |

Give them time to figure out the scale factor and then the height of the flagpole. Assist only if necessary. [Answer: Scale Factor = .046

Height = 65 $\frac{1}{2}$ feet]

- **INDEPENDENT PRACTICE:**

Hand out Student Activity Sheet and review with the class

The Student Activity Sheet provides a form for individuals or small groups to record measurements they can make on a sunny day. The object they use should be something like a lightpole, flagpole, or corner of a building, so the top of it will cast a distinctive shadow and its shadow can be measured starting from a point on the ground directly below the top.

Each group should make its own measurements, not necessarily of the same object. The groups do not all have to make their measurements on the same day and the activity can be done before or after school.

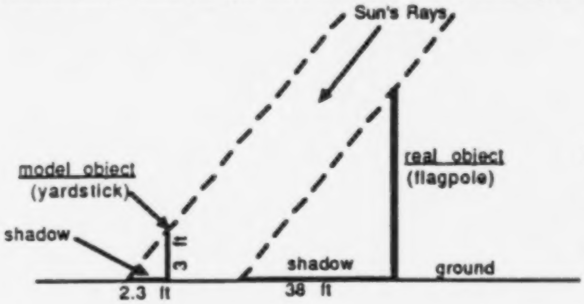
If the object is one that everyone is familiar with, it is often instructive to have the class estimate its height before the measurements are made. Then compare the estimates with the calculated height.

Name ____________

## SCALING UNKNOWN HEIGHTS

Student Activity Sheet

Reminder: Illustration below is from the example you did in class.


Sun's Rays
real object
(flagpole)
model object
(yardstick)
shadow
3 ft
shadow
38 ft
ground
2.3 ft

| | Real object | Scale Factor | Model object |
|---|---|---|---|
| Flagpole: | ________ ft | 0.06 x | 3 ft |
| Shadow: | 38 ft | 0.06 x | 2.3 ft |

What and where is the object whose height you are going to find? ____________

________________________________________

What unit are you using? Feet? Meters? ____________

Find the scale factor for your model so you can determine the height of the object you have chosen.

| | Real object | Scale Factor | Model object |
|---|---|---|---|
| ________<br>Name of real object | ________ ft | ____ x | ________ ft |
| Shadow: | ________ ft | ____ x | ________ ft |

Sketch the diagram of the object and its shadow below. Label each part with your measurements.

# CHAPTER 3 ASSESSMENT:

## MEASUREMENT AND GEOMETRY

1. Jorge spends 48 minutes in each of 6 classes in junior high school. If he arrives at school at 8:15 am and leaves at 2:40 pm, what percent of his school day is spent in class?

   Student response: 6 x 48 = 288 minutes in class
   6(60) + 25 = 360 + 25 = 385 minutes in school
   288/385 = 75% of school day spent in class

2. Choose three of the following geometric shapes and write a complete definition to describe them:

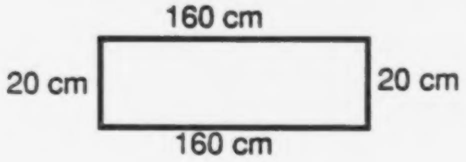
   Scalene triangle, equilateral triangle, isosceles triangle, rhombus, parallelogram, rectangle, trapezoid.

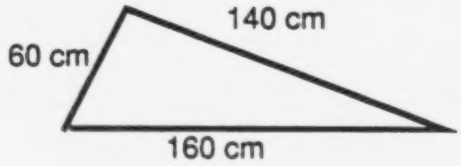
   Student responses will vary as students will use their own language. Possible definitions are listed below.

   Scalene triangle - A triangle with 3 unequal sides and 3 unequal angles.
   Equilateral triangle - A triangle with 3 equal sides and 3 equal angles.
   Isosceles triangle - A triangle with 2 equal sides and 2 equal angles.
   Rhombus - A parallelogram with equal sides.
   Parallelogram - A quadrilateral whose opposite sides are parallel.
   Rectangle - A quadrilateral with 4 right angles.
   Trapezoid - A quadrilateral with 2 sides parallel and 2 side not parallel.

3. Sketch 3 different polygons with perimeters of 360mm. Name the polygons and label their dimensions. The polygons do not have to be drawn to scale.

   Student responses will vary. Some examples are:





4. How many different rectangles with only whole number measurements for length and width can you make using a 24 in piece of string? Make a chart showing the dimensions of the different rectangles.

Student response should be 6 different rectangles.

| Width | Length | Perimeter |
|---|---|---|
| 1 | 11 | 24 |
| 2 | 10 | 24 |
| 3 | 9 | 24 |
| 4 | 8 | 24 |
| 5 | 7 | 24 |
| 6 | 6 | 24 |

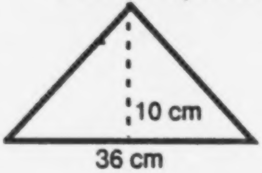
A 3 by 9 rectangle is not considered different from a 9 by 3 rectangle.

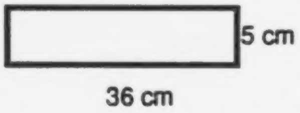
5. Design a swimming pool. Show its dimensions.
   a. Compute the length of fencing you would need to enclose it
   b. Compute the area of a solar pool cover.

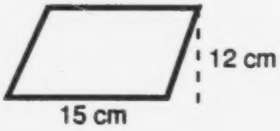
Student responses will vary.

6. Sketch three different polygons with areas of 180 $cm^2$.
Name the polygons and label their dimensions. The polygons do not have to be drawn to scale.

Student responses will vary. Some examples are:







7. List real-life situations where knowing how to find area or perimeter would be useful.

Student responses may include purchasing a rug, paint, fence, seeds, or wallpaper.

8. If you were an author writing a mathematics textbook, how would you explain area and perimeter?

Student responses should include some mention of distance around in the perimeter explanation. A discussion of area should refer to the amount of space inside the borders of a closed flat figure.

9. Write a word problem using area and another using perimeter. Solve the problems.

Student responses will vary.

10. a. The rectangular floor of your kitchen has an area of 400 square feet. Write several possible dimensions for the room. Make a chart to display results.

| Length | Width | Area |
|---|---|---|
| | | |
| | | |

b. How many possibilities are there?

Student responses may include:

| Length | Width | Area |
|---|---|---|
| 1 | 400 | 400 |
| 2 | 200 | 400 |
| 4 | 100 | 400 |
| 5 | 80 | 400 |
| 8 | 50 | 400 |
| 10 | 40 | 400 |
| 16 | 25 | 400 |
| 20 | 20 | 400 |

If only whole numbers are considered there are 8 possibilities. This assumes a 2 by 200 room is the same as a 200 by 2 room. If fractional dimensions are allowed. There is an infinite number of possibilities.

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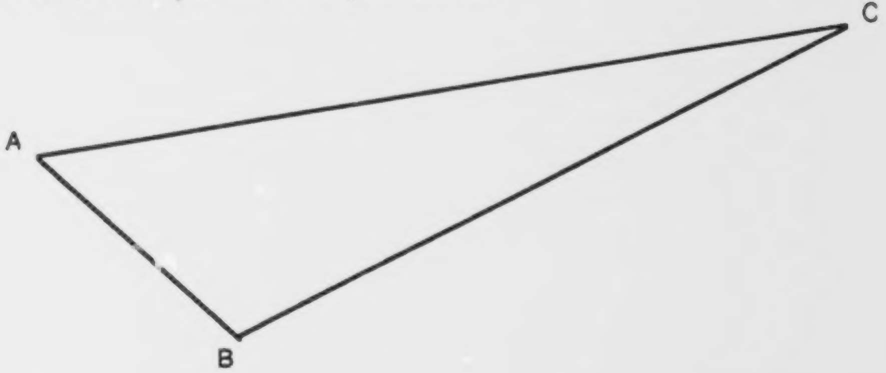
11. A carpenter brought in the 60 ft. of wood molding needed to exactly fit around three of the four walls of a rectangular bedroom. Record on a chart, possible dimensions (length and width) of the bedroom. Determine the largest possible area a room like this could have.

Student results should reflect the information shown in the chart below.

| Room Width | Room Length | Total Length of 3 Walls | Area |
|---|---|---|---|
| 1 | 58 | 60 | 58 |
| 2 | 56 | 60 | 112 |
| 3 | 54 | 60 | 162 |
| 4 | 52 | 60 | 208 |
| 5 | 50 | 60 | 250 |
| 6 | 48 | 60 | 288 |
| 7 | 46 | 60 | 322 |
| 8 | 44 | 60 | 352 |
| 9 | 42 | 60 | 378 |
| 10 | 40 | 60 | 400 |
| 11 | 38 | 60 | 418 |
| 12 | 36 | 60 | 432 |
| 13 | 34 | 60 | 442 |
| 14 | 32 | 60 | 448 |
| 15 | 30 | 60 | 450 |
| 16 | 28 | 60 | 448 |
| 17 | 26 | 60 | 442 |
| 18 | 24 | 60 | 432 |
| 19 | 22 | 60 | 418 |
| 20 | 20 | 60 | 400 |
| 21 | 18 | 60 | 378 |
| 22 | 16 | 60 | 352 |
| 23 | 14 | 60 | 322 |
| 24 | 12 | 60 | 288 |
| 25 | 10 | 60 | 250 |
| 26 | 8 | 60 | 208 |
| 27 | 6 | 60 | 162 |
| 28 | 4 | 60 | 112 |
| 29 | 2 | 60 | 58 |

Under these conditions a room width of 15 and length of 30 provides the largest possible area.

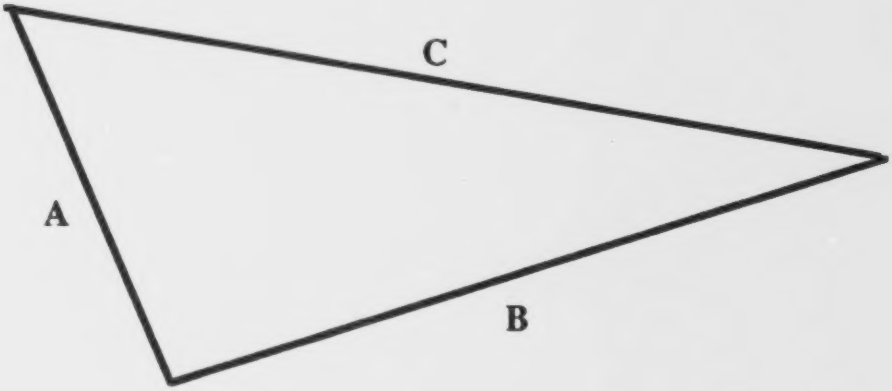
12. a. Find the area of the following triangle in square centimeter.
b. Describe how you arrived at your answer.



Student responses will vary because of the inaccuracies of measurement. They may use A = .5 x base x height and measure to find the height, or they can use Heron's formula. Answers should range between 18 to 20.

13. Measure the sides to the nearest cm. Use Heron's Formula to compute its area.

$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ (round the area to the nearest hundredth)



Student response:

$s = \frac{6 + 11 + 13}{2} = 15$

$A = \sqrt{15(15 - 6)(15 - 11)(15 - 13)}$

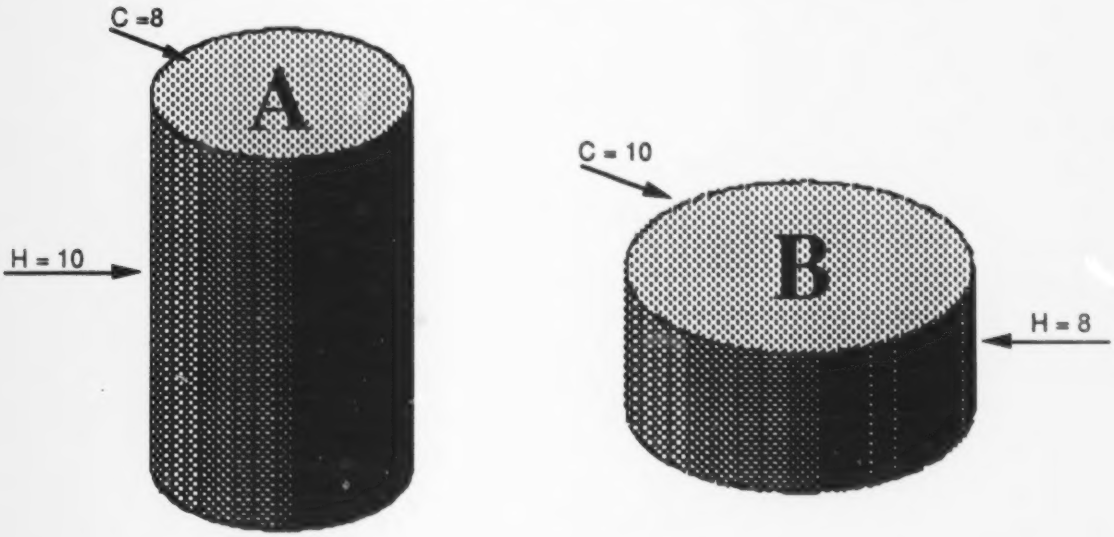
$A = \sqrt{15 \times 9 \times 4 \times 2}$

$A = \sqrt{1080}$

$A = 32.86\ \text{cm}^2$

14. An 8 inch by 10 inch paper has been folded two different ways to form a cylinder. Which one will have the greater volume? Show your work. (Round quotients to the nearest hundredth.)

| $C \div \pi = d$ | $d \div 2 = r$ | $V = \pi r^2 h$ |
|---|---|---|



Student responses:
Figure A

$C = 8$
$d = C \div \pi = 8 \div 3.14 = 2.55$
$r = 2.55 \div 2 = 1.275 = 1.28$
$V = \pi r^2 h = 3.14\ (1.25)^2\ (10) = 49.06\ \text{in}^3$

Figure B

$C = 10$
$d = C \div \pi = 10 \div 3.14 = 3.18$
$r = 3.18 \div 2 = 1.59$
$V = \pi r^2 h = 3.14\ (1.59)^2\ (8) = 63.51\ \text{in}^3$

Figure B has the greater volume.

15. Write a set of "I Have, Who Has" cards for concepts you learned from Measurement or Geometry lessons. Be sure no two cards have the same answer.

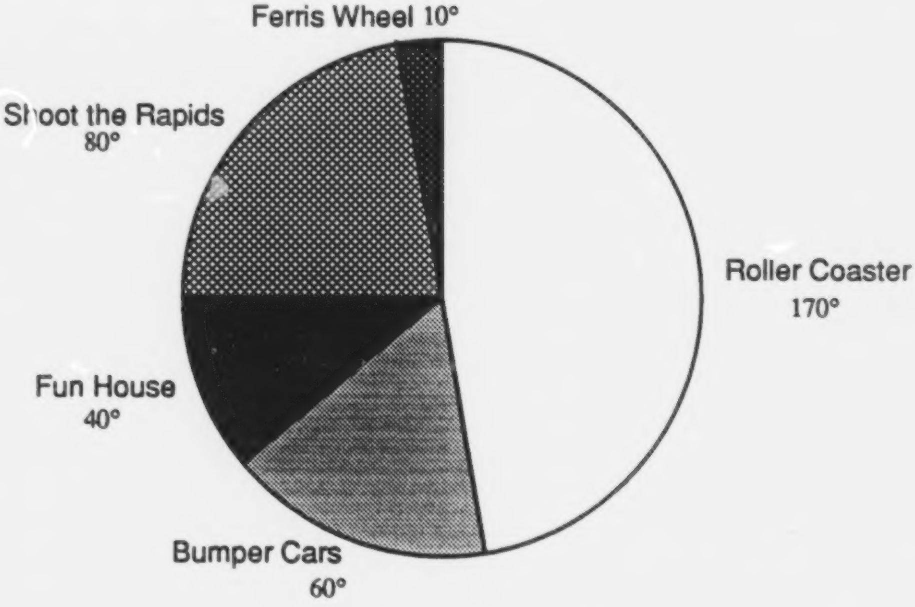
Student responses will vary.

16. Complete a circle graph to show the following data:
There are 36 students in Mrs. Javor's sixth grade class, and they chose their favorite rides in a local amusement park. (Round off answers where necessary.)

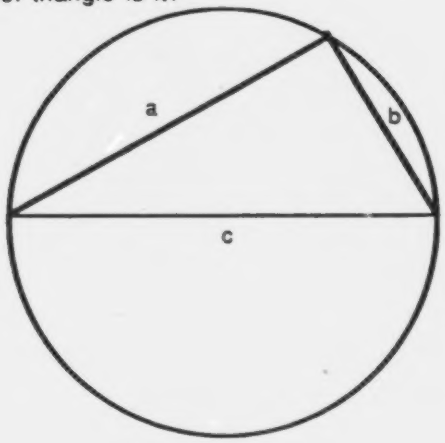
| | |
|---|---|
| Bumper Cars | 6 Students |
| Roller Coaster | 17 Students |
| Ferris Wheel | 1 Students |
| Shoot the Rapids | 8 Students |
| Fun House | 4 Students |

Student responses:

| | |
|---|---|
| Bumper Cars | $\frac{6}{36} \times 360 = 60^\circ$ |
| Roller Coaster | $\frac{17}{36} \times 360 = 170^\circ$ |
| Ferris Wheel | $\frac{1}{36} \times 360 = 10^\circ$ |
| Shoot the Rapids | $\frac{8}{36} \times 360 = 80^\circ$ |
| Fun House | $\frac{4}{36} \times 360 = 40^\circ$ |

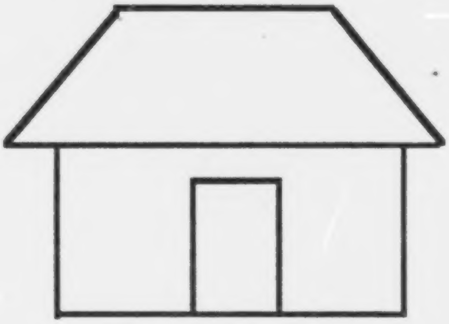


17. Measure the lengths of sides a, b, and c. What is the relationship between $a^2$, $b^2$, and $c^2$? What kind of triangle is it?



Student response:
It is a right triangle because $a^2 + b^2 = c^2$. Note: Because of the inaccuracies involved in measurement, $a^2 + b^2$ may not appear to be exactly equal to c.

18. Use a scale factor to compute the dimensions for the front of the doghouse you will build. It should be similar to the one shown below and designed for a dog that is 2.5 feet tall.



Student response:

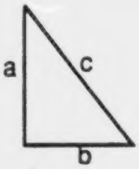
| measurement in drawing (inches) | Scale Factor | real measurement (feet) |
|---|---|---|
| | 3.33... X | |
| .75 in | $\longrightarrow$ | 2.5 ft |
| height of door | | height of dog |

A scale factor larger than 2.5 + .75 = 3.33 ... will make the door high enough for the dog. Students may use any reasonable number such as 3.4, 3.5, or 4 as a scale factor to do their computation.

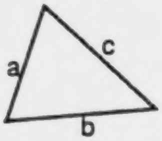
Answers will vary.

19. Match each inequality with the correct triangle. Explain how you arrived at your answer.

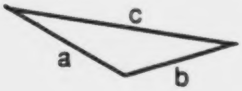
$a^2 + b^2 > c^2$



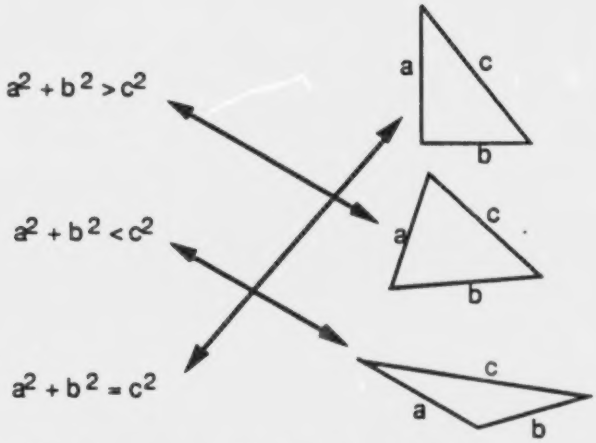
$a^2 + b^2 < c^2$



$a^2 + b^2 = c^2$



Student responses:



Note $a^2 + b^2 = c^2$ matches with the right triangle.
$a^2 + b^2 < c^2$ matches with the obtuse triangle.
$a^2 + b^2 > c^2$ matches with the acute triangle.

20. Use the following formulas to convert Fahrenheit into Celsius (Centigrade) and Celsius into Fahrenheit. (F - 32) x 5 ÷ 9 = C and C x 9 ÷ 5 + 32 = F (Round to the nearest degree.)

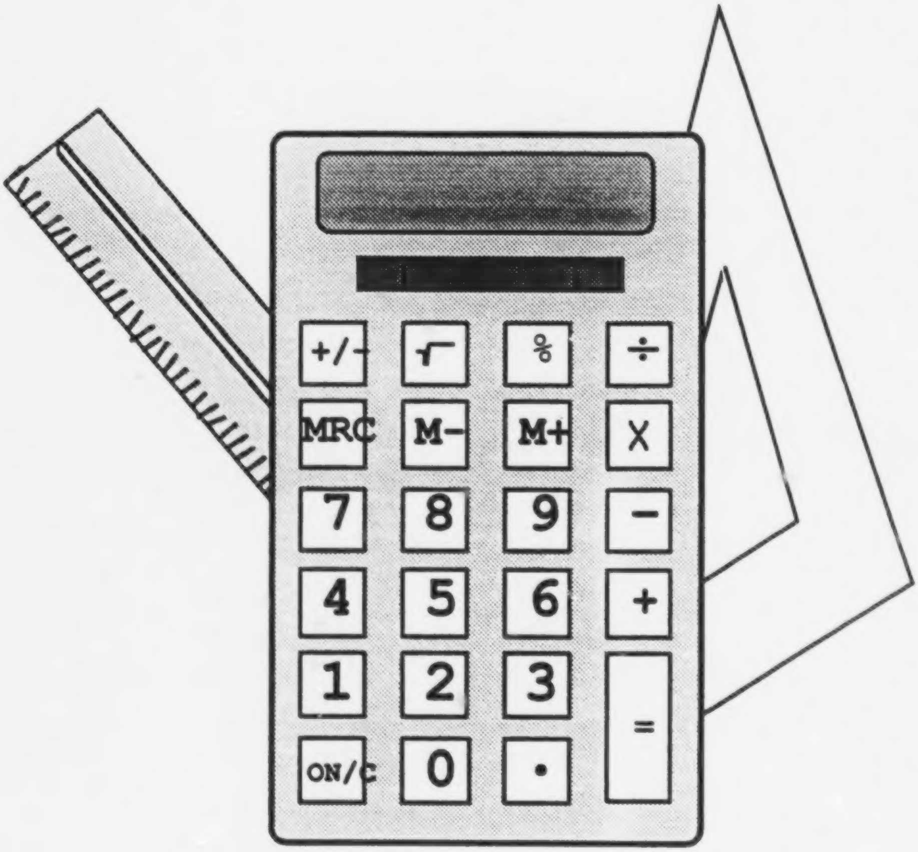
72 degrees Fahrenheit = ____________________ Celsius
72 degrees Celsius = ________________________ Fahrenheit

Student response:

| | |
|---|---|
| C = (F - 32) x 5 ÷ 9 | F = C x 9 ÷ 5 + 32 |
| C = (72 - 32) x 5 ÷ 9 | F = 72 x 9 ÷ 5 + 32 |
| C = 40 x 5 ÷ 9 | F = 648 ÷ 5 + 32 |
| C = 200 ÷ 9 | F = 129.6 + 32 |
| C = 22° | F = 161.6 = 162° |

# CAMP-LA

## Calculators and Mathematics Project, Los Angeles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## Grades 7 - 8

# CHAPTER 4

## NUMBER/ ALGEBRA

# EZ MILLION TRIVIA PURSUIT

**GRADE:** 7 - 8

**STRAND:** Number

**SKILL:** Use whole numbers in problem situations.

**MANAGEMENT**

**CLASS ORGANIZATION:** Pairs or small groups

**TIME FRAME:** Two or three math periods

**MATERIALS:** Calculator

**VOCABULARY:** Trivia, million, ream, equivalent

**PREREQUISITE SKILL:** Experience working with large numbers.

**LESSON**

- **DIRECTED INSTRUCTION:**

Think of the number 1,000,000. Will it fit on the calculator?
Hand out Student Activity Sheet 1.
The first problem solving situation should be done by the teacher with the class.

1. If your family were to win a $1,000,000 lottery, tax free, and collect $200 a week, how long would it take to collect the full amount?
    - Estimate the number of weeks.
    - Estimate the number of years.
    - Compute the actual amounts.
    - Answer: 5000 weeks = 96.153846 = 96 years

- **GUIDED PRACTICE:**

2. Pretend you are a banker and can count $1 bills at the rate of one hundred per minute. If you work six hours a day, how long would it take you to count $1,000,000? How many people would be needed to complete this task in one working day (counting at the same rate)?

- Estimate how long it would take.
- Estimate the number of people needed.
- Compute the actual amounts.
- Answer: 100 per minute x 60 minutes = $6000 per hour.
  $6000 x 6 hours = $36,000 per day.
  How long: 1,000,000 ÷ 36,000 = 27.77... = 28 days
  People: 28 people working 1 day.

- **INDEPENDENT PRACTICE:** (Answers are rounded.)

Hand out and have students complete Student Activity Sheets 2-4.

3. A. If a car travels 65 miles per hour for 24 hours a day. How long would it take to complete a journey of 1,000,000 miles?

Answer: 65 miles x 24 hours = 1560 miles per day
1,000,000 ÷ 1560 = 641 days, or 1 year and 276 days.

B. If the car in problem A traveled only 55 miles per hour, how long would it take?

Answer: 55 miles x 24 hours = 1320 miles per day
1,000,000 ÷ 1320 = 758 days or 2 years and 28 days.

C. How long would it take a car traveling 8 hours each day at 65 miles per hour to complete a journey of 1,000,000 miles?

Answer: 65 miles x 8 hours = 520 miles per day
1,000,000 ÷ 520 = 1923 days or 5 years and 98 days.

D. How long would it take a car traveling 8 hours each day at 55 miles per hour to complete a journey of 1,000,000 miles?

Answer: 55 x 8 hours = 440 miles per day
1,000,000 ÷ 440 = 2273 days or 6 years and 83 days.

4. A ream of typing paper contains 500 sheets of paper. How many reams of paper are needed to assemble 1,000,000 sheets? If the reams were placed in a single stack, and each ream is 2 inches thick, then how high would a million pieces of paper be?

Answer: 1,000,000 sheets ÷ 500 sheets = 2,000 reams
2,000 reams x 2 inches = 4,000 inches

If each ream is about 5 cm thick, then how many meters high would the stack be?

Answer: 2,000 reams x 5 cm = 10,000 cm
10,000 cm ÷ 100 cm = 100 meters

If a flagpole is 45 feet high, then how many flagpoles high would 1,000,000 pieces of paper be? Estimate first.

Flagpole: 45 feet or 540 inches
1,000,000 pieces (4000 inches) would be a little more than 7 flag poles.

5. The Ancient Romans defined a mile as the length of 1000 paces. Each pace is the distance covered walking forward two steps. If a man's pace is about $2\frac{1}{2}$ feet long, then how long would a million paces be?

Answer: $2\frac{1}{2}$ x 1,000,000 = 2,500,000 feet
2,500,000 feet ÷ 5280 feet in a mile = 473 miles

6. A dollar bill is about 6 inches long and $2\frac{1}{2}$ inches wide. If you place dollar bills end to end in a straight line, how many dollar bills would it take to cover a distance of 1,000,000 inches? 1,000,000 inches is equivalent to how many miles?

Answer: 1,000,000 inches ÷ 6 inches = 166,667 bills
1,000,000 inches ÷ 12 inches in a foot = 83,333 feet
83,333 feet ÷ 5280 ft in a mile = 16 miles

- **EXTENSION:**

Hand out Student Activity Sheet 5. Discuss. Students make a rough estimate first then gather data for a better estimate. How many sheets of paper are used by your class each year? By your school?
Note: How many sheets are used by a class each day? A week?
Remember there are 500 sheets in a ream of paper.

Name ____________________

## EZ MILLION TRIVIA PURSUIT
Student Activity Sheet 1

Directions: Discuss each problem with your group and plan the best way to solve it. Estimate first. Compute the actual amounts.

1. If your family were to win a $1,000,000 lottery, tax free, and would collect $200 a week, how long would it take to collect the full amount? Estimate the number of weeks. Estimate the number of years.

| | Weeks | Years |
|---|---|---|
| Estimate | | |

| | Weeks | Years |
|---|---|---|
| Actual | | |

2. Pretend you are a banker and can count one dollar bills at the rate of one hundred per minute. If you work six hours a day, how long would it take you to count $1,000,000? How many people would be needed to complete this task in one working day (counting at the same rate)? Estimate how long it would take. Estimate the number of people needed. Compute the actual amounts.

| | How long | How many people |
|---|---|---|
| Estimate | | |

| | How long | How many people |
|---|---|---|
| Actual | | |

Name ____________________

## EZ MILLION TRIVIA PURSUIT

Student Activity Sheet 2

Directions:

Read the following problem. Record your estimates on the chart below then solve. (Round answers to the nearest day)

3. A. If a car travels 65 miles per hour for 24 hours a day. How long would it take to complete a journey of 1,000,000 miles?

B. If the car in problem A traveled only 55 miles per hour, how long would it take?

C. How long would it take a car traveling 8 hours each day at 65 miles per hour to complete a journey of 1,000,000 miles?

D. How long would it take a car traveling 8 hours each day at 55 miles per hour to complete a journey of 1,000,000 miles?

| TIME TO DRIVE 1,000,000 MILES | | | | |
|---|---|---|---|---|
| | | | ESTIMATE | ACTUAL |
| | Hours Driving Per Day | Miles Per Hour | Years and Days | Years and Days |
| A | 24 hour per day | 65 | | |
| B | 24 hour per day | 55 | | |
| C | 8 hours per day | 65 | | |
| D | 8 hours per day | 55 | | |

How did you find the actual number of years and days from the given information?

______________________________________________

______________________________________________

Name ____________________

## EZ MILLION TRIVIA PURSUIT

Student Activity Sheet 3

4. A ream of typing paper contains 500 sheets of paper. How many reams of paper are needed to assemble 1,000,000 sheets? If the reams were placed in a single stack, and each ream is 2 inches thick, then how high would a million pieces of paper be?

__________________reams ________________________inches

If each ream is about 5 cm thick, then how many meters high would the stack be? ________________________meters

If a flagpole is 45 feet high, then how many flagpoles high would 1,000,000 pieces of paper be? Estimate first.

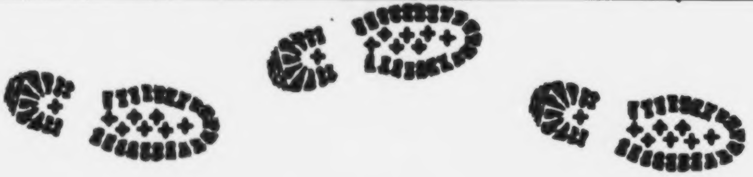
|  |  |
|---|---|
| Estimate | How many flag poles? |
|  |  |

|  |  |
|---|---|
| Actual | How many flag poles? |
|  |  |

5. The Ancient Romans defined a mile as the length of 1000 paces. Each pace is the distance covered walking forward two steps. If a man's pace is about $2\frac{1}{2}$ feet long, then how long would a million paces be?

|  |  |  |  |  |
|---|---|---|---|---|
| Estimate | One pace | Million paces | ÷ 5280 ft | ≈ miles |
|  | $2\frac{1}{2}$ feet |  |  |  |

|  |  |  |  |  |
|---|---|---|---|---|
| Actual | One pace | Million paces | ÷ 5280 ft | ≈ miles |
|  | $2\frac{1}{2}$ feet |  |  |  |



Name ______________________

## EZ MILLION TRIVIA PURSUIT

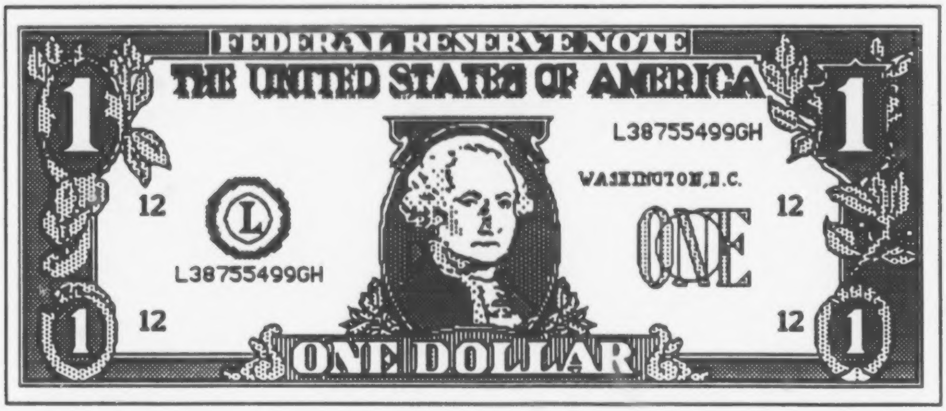
### Student Activity Sheet 4

6. A dollar bill is about 6 inches long and $2\frac{1}{2}$ inches wide. If you place dollar bills end to end in a straight line, how many dollar bills would it take to cover a distance of 1,000,000 inches?

| | |
|---|---|
| Estimate | How many dollar bills? |
| | |
| Actual | How many dollar bills? |
| | |

1,000,000 inches is equivalent to how many miles?

| | |
|---|---|
| Estimate | How many miles? |
| | |
| Actual | How many miles? |
| | |


FEDERAL RESERVE NOTE
THE UNITED STATES OF AMERICA
1
1
L38755499GH
WASHINGTON, D.C.
12
L
12
ONE
L38755499GH
1
12
12
1
ONE DOLLAR

NAME ____________________

## EZ MILLION TRIVIA PURSUIT

Student Activity Sheet 5

Directions: Discuss each problem with your group and plan the best way to solve it. Estimate first. Then gather data and compute an improved estimate.

How many sheets of paper are used by your class each year? By your school? Estimate first.

Note: How many sheets are used by a class each day?
A week?
Remember there are 500 sheets in a ream of paper.

Original estimate.

| | Estimate | Sheets per day | Sheets per week | Sheets per year | Reams |
|---|---|---|---|---|---|
| Class | | | | | |
| School | | | | | |

Improved estimate based on gathered data.

| | Estimate based on gathered data | Sheets per day | Sheets per week | Sheets per year | Reams |
|---|---|---|---|---|---|
| Class | | | | | |
| School | | | | | |

## BUTCHER MATH

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Number |
| **SKILL:** | Multiply and divide decimals. |
| **MANAGEMENT**<br>**CLASS ORGANIZATION:** | Whole class or pairs |
| **TIME FRAME:** | One math period |
| **MATERIALS:** | Calculator |
| **VOCABULARY:** | Net weight (wt.), price per pound (lb.), total price, factor, almost equal symbol (≈) |
| **PREREQUISITE SKILL:** | Round to the nearest hundredth |

**LESSON**

- **DIRECTED INSTRUCTION:**

In this lesson you use information from grocery store labels. You will estimate answers to help determine the arithmetic operation to use. You will use calculators for the actual computation. You will develop formulas to apply to similar situations.

Draw a label that lists net weight, price per pound, and total price using the numbers in the chart below.

Note: Markets round net weight and money to the nearest hundredth.

The teacher will cover up the total price and ask students to estimate the covered amount.

**BEEF LOIN TOP SIRLOIN**

| Net Weight | Price per lb. | Total Price |
|---|---|---|
| 1.51 lb | $2.49 | $3.76 |

1lb. cost: $2.49 ≈ $2.50
2 lb. cost: about $5.00

1.5 lb. would cost: between $3 and $4.

Think about the mathematical operation and process you would use to calculate the total price.
Would you add, subtract, multiply or divide 1.51(net weight) and $2.49 (price per pound) to get the total price?

Try it with 1.51(lbs.) and 2.49 (price/lb.) on your calculator.
Find the total price.
What answer did you get?
What formula shows how to find total price when we know net weight and price per lb.? Remember to round amounts to the nearest hundredth.

Net weight x price per lb. = total price
1.51 x $2.49 = 3.76

**3.7599 rounds to $3.76**

- **GUIDED PRACTICE:**
  Hand out Student Activity Sheet 1. Students will answer the questions with their partners as teacher observes and assists as needed. Students will verify answers with their calculator. Discuss and correct student answers.

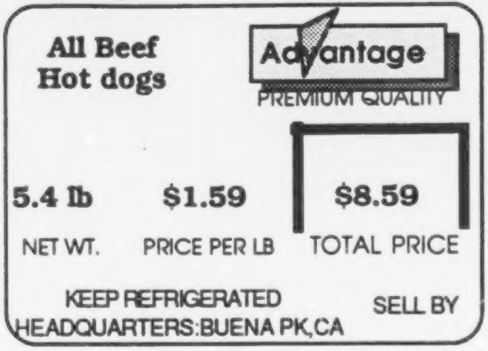
- **INDEPENDENT PRACTICE:**
  Hand out and have students complete Student Activity Sheet 2.

- **HOME ACTIVITY:**
  Find 4 food labels (fish, meat, cheese) that include net weight, price per pound, and total price. Bring in the labels or copy the information (including name of the item) on the blanks provided. Fill in the amounts and verify their accuracy with your calculator.

## BUTCHER MATH
Student Activity Sheet 1
Teacher Sheet

**Example**

1.



**All Beef Hot dogs**

Advantage
PREMIUM QUALITY

**5.4 lb** NET WT.

**$1.59** PRICE PER LB

**$8.59** TOTAL PRICE

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA

SELL BY

Look at the label above.

1. The net wt. is 5.4 lb

2. The price per lb. is $1.59

3. The total price is $8.59

4. Write the number sentence and formula you would use with net wt. and price per lb. to get the total price 5.4 lbs. x $1.54/lb. = $8.59

net wt. x price per pound = total price

5. Write the number sentence and formula you would use if you knew the total price and net wt. and wanted to find the price per lb.

$8.59 ÷ 5.4 lbs. = $1.59/lb.

Total price ÷ net weight = price per pound

6. Write the number sentence and formula you would use if you knew the total price and the price per lb. and wanted to find the net wt. $8.59 ÷ $1.59 lb. = 5.4 lbs.

Total price ÷ price per pound= net weight

## BUTCHER MATH

Student Activity Sheet 2
Teacher Answer Sheet

Use the appropriate formulas from #4, 5, 6 on Sheet 1 to complete the labels.

1. LEAN GROUND BEEF

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| .75lb | $1.87 | $1.40 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

2. BEEF CHUCK BONELESS ROUND ROAST

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| 2.25lb | $1.82 | $4.10 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

3. TOP SIRLOIN

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| 0.89lb | $1.87 | $1.66 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

4. PORK FEET

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| 2.5 lb | $.94 | $2.35 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

5. 3 FRESH HALF BREAST/RIBS

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| 1.94 lb | $2.19 | $4.25 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

6. YELLOWTAIL

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| .93 lb | $4.67 | $4.34 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

7. LONGHORN CHEDDAR CHEESE

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| 1.34 lb | $2.79 | $3.74 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

8.

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| .8 lb | $11.59 | $9.27 |

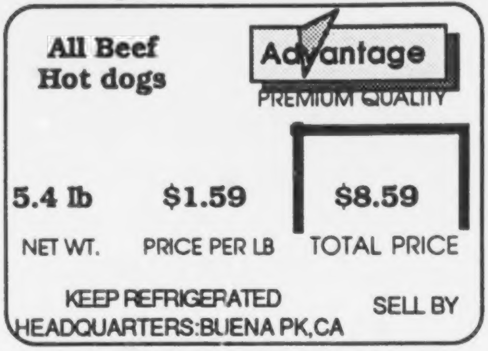
KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

Name ____________________

## BUTCHER MATH
Student Activity Sheet 1

**Example**

1.



**All Beef Hot dogs**

Advantage
PREMIUM QUALITY

| **5.4 lb** | **$1.59** | **$8.59** |
|---|---|---|
| NET WT. | PRICE PER LB | TOTAL PRICE |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA
SELL BY

Look at the label above.

1. The net wt. is ____________________
2. The price per lb. is ________________
3. The total price is _________________
4. Write the number sentence and formula you would use with net wt. and price per lb. to get the total price. ____________________________________________
5. Write the number sentence and formula you would use if you knew the total price and net wt. and wanted to find the price per lb.
6. Write the number sentence and formula you would use if you knew the total price and the price per lb. and wanted to find the net wt. ____________________

Name ____________________

## BUTCHER MATH

Student Activity Sheet 2

Use the appropriate formulas from numbers 4, 5, 6 on sheet 1 to complete the labels.

1. LEAN GROUND BEEF

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| .75lb | $1.87 | |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

2. BEEF CHUCK BONELESS ROUND ROAST

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| 2.25lb | $1.82 | |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

3. TOP SIRLOIN

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| 0.89lb | | $1.66 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

4. PORK FEET

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| | $.94 | $2.35 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

5. 3 FRESH HALF BREAST/RIBS

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| | $2.19 | $4.25 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

6. YELLOWTAIL

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| .93 lb | | $4.34 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

7. LONGHORN CHEDDAR CHEESE

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| | $2.79 | $3.74 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

8.

Advantage
PREMIUM QUALITY

| NET WT. | PRICE PER LB | TOTAL PRICE |
|---|---|---|
| | $11.59 | $9.27 |

KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

Name ____________________

## BUTCHER MATH
### Home Activity Sheet

Find 4 labels on foods that include net weight, price per pound, and total price. Copy the information on the blanks provided. Verify their accuracy with your calculator.

1\.

Advantage
PREMIUM QUALITY
NET WT. PRICE PER LB TOTAL PRICE
KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

Verify

____________________

2\.

Advantage
PREMIUM QUALITY
NET WT. PRICE PER LB TOTAL PRICE
KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

Verify

____________________

3\.

Advantage
PREMIUM QUALITY
NET WT. PRICE PER LB TOTAL PRICE
KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

Verify

____________________

4\.

Advantage
PREMIUM QUALITY
NET WT. PRICE PER LB TOTAL PRICE
KEEP REFRIGERATED
HEADQUARTERS:BUENA PK,CA SELL BY

Verify

____________________

# GET THE BEST BUY

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Number |
| **SKILL:** | Multiply or divide numbers involving money.<br>Solve story problems of real life situations using a calculator. |

**MANAGEMENT**

| | |
|---|---|
| **CLASS ORGANIZATION:** | Individual or small group |
| **TIME FRAME:** | One math period |
| **MATERIALS:** | Calculator |
| **VOCABULARY:** | Best buy, price per unit |
| **PREREQUISITE SKILL:** | Round decimal numbers |

## LESSON

- **DIRECTED INSTRUCTION:**

Discuss: price per unit means cost of 1 unit. It is found by computing: total cost + number of units. Round division answers to the nearest penny.

Examples:
A 12 oz. jar of jam selling for $1.69 costs 1.69 +12 = 0.1408333 which rounds to $0.14 per ounce.

A 16 oz. jar of jam selling for $1.89 costs 1.89 + 16 = 0.118125 which rounds to $0.12 per ounce. The 16 oz. jar is a better buy by $0.14 -$0.12 = $0.02 per ounce.

- **GUIDED PRACTICE:**

Store A has a 28 oz. jar of peanut butter on sale for $2.99. Store B has an 18 oz. jar of peanut butter on sale for $1.79.

Students use calculators to find the unit price at store A and store B. Tell which is a better buy, and find the approximate amount of savings per unit. Check to see that they do the following:
Store A: $2.99 + 28 = 0.0167857 rounds to $0.11 per ounce.
Store B: $179 + 18 = 0.0994444 rounds to $0.10 per ounce.
Store B is a better buy.
Store B saves you about $0.11 - $0.10 = $0.01 per ounce.

- **INDEPENDENT PRACTICE:**

Do Student Activity Sheet

- **EVALUATION:** Student Activity Sheet

- **HOME ACTIVITY:**

Select 10 items at the market. Record their price and their weight, size, or quantity. Use your calculator to find their unit price. (Examples; price per ounce, price per candy bar, price per piece of gum) Record your results.

## GET THE BEST BUY

Teacher Answer Sheet

Unit price means: Price for 1 unit, it is found by

Price ÷ number of units

- Use your calculator to find unit prices.
- Round your answers to the nearest cent.
- Fill in the chart

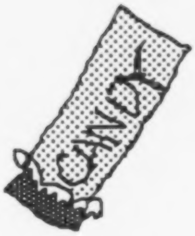
| | Store A | | | Store B | | |
|---|---|---|---|---|---|---|
| Item | Size | Price | Unit Price (round to the nearest cent) | Size | Price | Unit Price (round to the nearest cent) |
| Apple Juice | 64 oz. | 1.29 | .02 | 48 oz. | .99 | .02 |
| Barbecue Sauce | 18 oz. | .99 | .06 | 19 oz. | 1.49 | .08 |
| Pineapple Juice | 40 oz. | 1.69 | .04 | 20 oz. | .75 | .04 |
| Salad Dressing | 12 oz. | 1.49 | .12 | 16 oz | 1.99 | .12 |
| Potato Chips | 7 oz. | .99 | .14 | 16 oz. | 1.79 | .11 |
| Batteries | 4 (pkg) | 1.59 | .40 | 4 /(pkg) | 1.89 | .47 |
| Soda | 32 oz. | .99 | .03 | 32 oz. | 1.49 | .05 |
| Chicken | 5 pieces | 4.12 | .82 | 8 pieces | 6.80 | .85 |
| Steak | 2.5 lbs | 7.55 | 3.02 | 1.2 lbs | 4.20 | 3.50 |
| Chocolate bars | 6 bars | 4.22 | .70 | 4 bars | 2.60 | .65 |

| **Item** | **Less expensive store (rounded unit price)** | **Approximate amount of savings per unit** |
|---|---|---|
| Apple Juice | same | .00 |
| Barbecue Sauce | Store A | .02 |
| Pineapple Juice | same | .00 |
| Salad Dressing | same | .00 |
| Potato Chips | Store B | .03 |
| Batteries | Store A | .07 |
| Soda | Store B | .07 |
| Chicken | Store A | .03 |
| Steak | Store A | .48 |
| Chocolate bars | Store B | .05 |

Is the largest size always the best buy? no

Is it worth it to drive to a store that has more best buys? What other factors are important in choosing where to shop? Transportation expense to store. Value to you of time expended. Selection of items available in the store: Convenience, etc.

Name ____________________



# GET THE BEST BUY

Student Activity Sheet

Unit price means: price for 1 unit. It is found by:

**Unit Price = Price ÷ number of units**

- Use your calculator to find unit prices.
- Round your answers to the nearest cent.
- Fill in the chart.

| | Store A | | | Store B | | |
|---|---|---|---|---|---|---|
| Item | Size | Price | Unit Price (round to the nearest cent) | Size | Price | Unit Price (round to the nearest cent) |
| Apple Juice | 64 oz. | 1.29 | | 48 oz. | .99 | |
| Barbecue Sauce | 18 oz. | .99 | | 19 oz. | 1.49 | |
| Pineapple Juice | 40 oz. | 1.69 | | 20 oz. | .75 | |
| Salad Dressing | 12 oz. | 1.49 | | 16 oz | 1.99 | |
| Potato Chips | 7 oz. | .99 | | 16 oz. | 1.79 | |
| Batteries | 4 (pkg) | 1.59 | | 4 (pkg) | 1.89 | |
| Soda | 32 oz. | .99 | | 32 oz. | 1.49 | |
| Chicken | 5 pieces | 4.12 | | 8 pieces | 6.80 | |
| Steak | 2.5 lbs | 7.55 | | 1.2 lbs | 4.20 | |
| Chocolate bars | 6 bars | 4.22 | | 4 bars | 2.60 | |

| Item | Less expensive store (using rounded unit price) | Approximate amount of savings per unit |
|---|---|---|
| Apple Juice | | |
| Barbecue Sauce | | |
| Pineapple Juice | | |
| Salad Dressing | | |
| Potato Chips | | |
| Batteries | | |
| Soda | | |
| Chicken | | |
| Steak | | |
| Chocolate bars | | |

Is the largest size always the best buy? ____________________

Is it worth it to drive to a store that has more best buys? What other factors are important in choosing where to shop? ____________________

____________________

____________________

# WATCH YOUR MONEY GROW

**GRADE:** 7 - 8

**STRAND:** Number

**SKILL:** Use powers and multiples of powers to explore large numbers.

## MANAGEMENT

**CLASS ORGANIZATION:** Pairs

**TIME FRAME:** One or two math periods

**MATERIALS:** Calculator

**VOCABULARY:** Millions, thousands

**PREREQUISITE SKILL:** Place value

## LESSON

- **DIRECTED INSTRUCTION:**

Tell students the purpose of this lesson is to discover how rapidly numbers grow through multiplication. Give students Student Activity Sheet 1. Read the situation with your class. Everyone records an estimate and completes the worksheet.

> Note: The constant function of the calculator may be used in this activity. For example, on Student Activity Sheet 1 you may press 2 x 1===...

Discuss students' results and comments.

- **INDEPENDENT PRACTICE:**

Hand out Student Activity Sheet 2. Working in pairs, students complete the worksheet. Discuss results with the class.

Hand out Student Activity Sheet 3. Working in pairs, students complete the worksheet. Discuss results with the class.

Hand out Home Activity. Encourage Students to do the activity with their parents.

- **EVALUATION:**

Teacher observation and Student Activity Sheets.

### WATCH YOUR MONEY GROW
Student Activity Sheet 1
Teacher Answer Sheet

Somebody gives you a magic dollar. It is magic because every night it doubles so that the next day instead of one dollar you have two magic dollars.

Estimate how many days it will take for your dollar to become over a million dollars.

Record your estimate: ____________________

Complete the chart below using your calculator.
(Note: on the calculator you are continually multiplying the number shown on the display by 2 without clearing the calculator.)

| Day number | Number of magic dollars |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 4 |
| 4 | 8 |
| 5 | 16 |
| 6 | 32 |
| 7 | 64 |
| 8 | 128 |
| 9 | 256 |
| 10 | 512 |
| 11 | 1024 |
| 12 | 2048 |
| 13 | 4096 |
| 14 | 8192 |
| 15 | 16384 |
| 16 | 32768 |
| 17 | 65536 |
| 18 | 131072 |
| 19 | 262144 |
| 20 | 524288 |
| 21 | 1048576 |
| 22 | |
| 23 | |
| 24 | |

At what point were you surprised by the number of magic dollars?

________________________________________

________________________________________

How did the result differ from your expectations?

________________________________________

# WATCH YOUR MONEY GROW
Student Activity Sheet 2
Teacher Answer Sheet

Somebody gives you a magic nickel. Each magic nickel grows overnight to three magic nickels. Estimate how many days it would take until your magic nickel becomes at least one million dollars.

Record your estimate: ______________________

Complete the chart below using your calculator. (Note: You continually multiply the number shown on the display by 3 without clearing the calculator or use your calculators constant function.)

| Day number | Number of magic dollars |
|---|---|
| 1 | .05 |
| 2 | .15 |
| 3 | .45 |
| 4 | 1.35 |
| 5 | 4.05 |
| 6 | 12.15 |
| 7 | 36.45 |
| 8 | 109.35 |
| 9 | 328.05 |
| 10 | 984.15 |
| 11 | 2952.45 |
| 12 | 8857.35 |
| 13 | 26572.05 |
| 14 | 79716.15 |
| 15 | 239148.45 |
| 16 | 717445.35 |
| 17 | 2152336.05 |
| 18 | |
| 19 | |
| 20 | |

How did the result differ from your expectations?

Answer will vary.

Compare these results to those you found in Student Activity Sheet 1.

______________________________________________

______________________________________________

## WATCH YOUR MONEY GROW

Student Activity Sheet 3
Teacher Answer Sheet

Magic quarters double every night to 2 magic quarters. Magic pennies change every night to 4 magic pennies.

If you could borrow one of these coins for only 5 days, which coin would you choose?

Magic Quarter

If you could borrow one of these coins for 13 days, which would you choose?

Magic Penny

Now complete the chart. Use it to decide if you made the correct choices.
Complete one column before starting the other.

| Day number | Magic Quarter (multiply by 2) | Magic Penny (multiply by 4) |
|---|---|---|
| 1 | .25 | .01 |
| 2 | .50 | .04 |
| 3 | 1 | .16 |
| 4 | 2 | .64 |
| 5 | 4 | 2.56 |
| 6 | 8 | 10.24 |
| 7 | 16 | 40.96 |
| 8 | 32 | 163.84 |
| 9 | 64 | 655.36 |
| 10 | 128 | 2621.44 |
| 11 | 256 | 10485.76 |
| 12 | 512 | 41943.04 |
| 13 | 1024 | 167772.16 |

What did you discover? Why do you think this happened?

## WATCH YOUR MONEY GROW
Teacher Answer Sheet

**HOME ACTIVITY:**

Discuss the following problem at home. If you were to sign a contract with your family that in return for keeping your room clean for an entire year you would to be given 1 penny the first day of February, two pennies the second day of February, 4 pennies the next, doubling each day until the month was over, would this be a fair deal? Would your family be willing to pay you that much?________________________________________

Why or why not?____________________________________

Using the calendar for February, fill in the amount of money that you would be paid each day.

| SUNDAY | MONDAY | TUESDAY | WEDNESDAY | THURSDAY | FRIDAY | SATURDAY |
|---|---|---|---|---|---|---|
| | 1<br>$.01 | 2<br>$.02 | 3<br>$.04 | 4<br>$.08 | 5<br>$.16 | 6<br>$.32 |
| 7<br>$.64 | 8<br>$1.28 | 9<br>$2.56 | 10<br>$5.12 | 11<br>$10.24 | 12<br>$20.48 | 13<br>$40.96 |
| 14<br>$81.92 | 15<br>$163.84 | 16<br>$327.68 | 17<br>$655.36 | 18<br>$1310.72 | 19<br>$2621.44 | 20<br>$5242.88 |
| 21<br>$10485.76 | 22<br>$20971.52 | 23<br>$41943.04 | 24<br>$83886.08 | 25<br>$167772.16 | 26<br>$335544.32 | 27<br>$671088.64 |
| 28<br>$1342177.28 | (calculator shows 1342177.2) | | | | | |

What did you discover?________________________________________________

________________________________________________________________

________________________________________________________________

How did the results differ from your family's expectation?_________________

________________________________________________________________

________________________________________________________________

________________________________________________________________

Share your discoveries from Student Activity Sheets 1, 2, and 3 with your family.

Name ______________

## WATCH YOUR MONEY GROW

### Student Activity Sheet 1

Somebody gives you a magic dollar. It is magic because every night it doubles so that the next day instead of one dollar you have two magic dollars.

Estimate how many days it will take for your dollar to become over a million dollars.

Record your estimate: ____________________

Complete the chart below using your calculator.
(Note: on the calculator you are continually multiplying the number shown on the display by 2 without clearing the calculator.)

| Day number | Number of magic dollars |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 4 |
| 4 | 8 |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 16 | |
| 17 | |
| 18 | |
| 19 | |
| 20 | |
| 21 | |
| 22 | |
| 23 | |
| 24 | |

At what point were you surprised by the number of magic dollars?

______________________________________________

______________________________________________

How did the result differ from your expectations?

______________________________________________

______________________________________________

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Name ____________________

## WATCH YOUR MONEY GROW

### Student Activity Sheet 2

Somebody gives you a magic nickel. Each magic nickel grows overnight to three magic nickels. Estimate how many days it would take until your magic nickel becomes at least one million dollars.

Record your estimate: ________________________

Complete the chart below using your calculator.(Note: You continually multiply the number shown on the display by 3 without clearing the calculator or use your calculators constant function.)

| Day number | Number of magic dollars |
|---|---|
| 1 | .05 |
| 2 | .15 |
| 3 | .45 |
| 4 | 1.35 |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 16 | |
| 17 | |
| 18 | |
| 19 | |
| 20 | |

How did the result differ from your expectations?

______________________________________________________________________

______________________________________________________________________

Compare these results to those you found in Student Activity Sheet 1.

______________________________________________________________________

______________________________________________________________________

Name ____________________

## WATCH YOUR MONEY GROW

Student Activity Sheet 3

Magic quarters double every night to 2 magic quarters. Magic pennies change every night to 4 magic pennies.

If you could borrow one of these coins for only 5 days, which coin would you choose?

______________

If you could borrow one of these coins for 13 days, which would you choose?

__________________

Now complete the chart. Use it to decide if you made the correct choices.
Complete one column before starting the other.

| Day number | Magic Quarter (multiply by 2) | Magic Penny (multiply by 4) |
|---|---|---|
| 1 | .25 | .01 |
| 2 | .50 | .04 |
| 3 |  | .16 |
| 4 |  | .64 |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |

What did you discover? Why do you think this happened?

________________________________________________

________________________________________________

________________________________________________

Name____________________

## WATCH YOUR MONEY GROW

Home Activity Sheet

Discuss the following problem at home. If you were to sign a contract with your family that in return for keeping your room clean for an entire year you would to be given 1 penny the first day of February, two pennies the second day of February, 4 pennies the next, doubling each day until the month was over, would this be a fair deal? Would your family be willing to pay you that much?______________________________

Why or why not?______________________________

______________________________

Using the calendar for February, fill in the amount of money that you would be paid each day.

| SUNDAY | MONDAY | TUESDAY | WEDNESDAY | THURSDAY | FRIDAY | SATURDAY |
|---|---|---|---|---|---|---|
| | 1<br>$.01 | 2<br>$.02 | 3<br>$.04 | 4<br>$.08 | 5 | 6 |
| 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| 28 | (This is a non-Leap year February) | | | | | |

What did you discover?

______________________________

______________________________

How did the results differ from your family's expectations?

______________________________

______________________________

Share your discoveries from Student Activity Sheets 1, 2, and 3 with your family.

# MULTIPLICATION THE EASY EXPONENTIAL WAY

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Number |
| **SKILL:** | Use exponents to multiply and to find square roots. |

## MANAGEMENT

| | |
|---|---|
| **CLASS ORGANIZATION:** | Individual or pairs |
| **TIME FRAME:** | Two or more math periods |
| **MATERIALS:** | Calculator |
| **VOCABULARY:** | Power, exponent, exponential form, base |
| **PREREQUISITE SKILL:** | Whole number square root, find powers (exponents) of a number |

## LESSON

- **DIRECTED INSTRUCTION:**
    - Hand out Student Activity Sheet 1.
    - Step 1
        - Students complete the Powers of 2 column. Use the calculator: Press $\boxed{2}\,\boxed{\times}\,\boxed{1}\,\boxed{=}\,\boxed{=}\,\boxed{=}$... Students need to understand exponents. For example: $2^5 = 2 \times 2 \times 2 \times 2 \times 2$. Check that students complete the Powers of 2 column correctly.
    - Step 2
        - Two numbers were chosen from the list of Powers of ? Example: 128 and 256. Discuss headings for columns A to F of their chart. In column C is the exponential form of 128, and in column D is the exponential form of 256.
        - Multiply $128 \times 256 = 32{,}768$. This product in exponent form is in Column F.
        - Students choose five other pairs of numbers that are powers of 2, and whose exponents add to less than 26, and fill in the chart. Encourage students to look at their results to try to find a way to complete the chart without doing the actual computations for column E. Students complete the chart and answer questions on the bottom of the sheet. Discuss class observations.
        - Students should conclude there is an alternate way to find products of numbers which can be written as powers of two.
        - When the bases are the same, you can multiply the factors or numbers by adding the exponents: $2^7 \times 2^8 = 2^{15}$
        - Look up the equivalent number for the new exponential, for example: $128 \times 256 = 2^7 \times 2^8 = 2^{7+8} = 2^{15} = 32768$
          Verify that $128 \times 256 = 32{,}768$ with your calculator.

- **GUIDED PRACTICE:**
  - Hand out Student Activity Sheet 2. Use 4096 from the list of powers of 2 from Student Activity Sheet 1. Students use their calculators to find $\sqrt{4096.}$ Answer is 64. (This means 64 x 64 = 4096). Since 64 is a whole number, 4096 and 64 are placed in the chart.
  - Enter the exponential form of 4096 in column C.
    Enter the exponential form of 64 in column D.

| A | B | C | D |
|---|---|---|---|
| Square Root of Number in Column A | Whole Number Square root of Number | Number in Column A in Exponential Form | Square Root of Number in Column A Exponential Form |
| 4096 | 64 | $2^{12}$ | $2^{6}$ |

  - Students complete the chart using numbers which can be written as powers of 2 and which have whole number square roots. Use your calculator to verify solutions.
  - Students answer questions 1, 2, and 3. After they finish, discuss their observations. Guide them to see that from the list of powers of 2, only those with even numbered powers had whole number square roots.
  - The power of 2 in the square root of a number is half the power of 2 of the original number.

eg. The square root of $2^{10} = 2^{5}$
because $2^{5} \times 2^{5} = 2^{5+5} = 2^{10}$
Verify with your calculator using the square root key.

$1024 = 2^{10}$ $\quad\quad \sqrt{2^{10}} = 2^{5} \quad 2^{5} = 32$

Therefore $\quad \sqrt{1024} = 32$

  - Hand out Student Activity Sheet 3 parts A and B.
    Students fill in the powers of 3 (using 3 x 1 = = = ...)
    Students fill in the powers of 4 (using 4 x 1 = = = ...)
    Students fill in the powers of 5 and the powers of 6, using the same key strokes as above.
  - Students answer the questions. Discuss class results and observations. Guide them to observe the following:

$$\begin{aligned} 2^{3} \times 3^{3} &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \\ &= (2 \cdot 3)(2 \cdot 3)(2 \cdot 3) \\ &= 6 \cdot 6 \cdot 6 \\ \text{or } 2^{3} \times 3^{3} &= (2 \cdot 3)^{3} = 6^{3} \end{aligned}$$

- **EVALUATION:**
  Teacher observation and Student Activity Sheets.

- **EXTENSION:**

  If $2^4 \times 2^0 = 2^{4+0} = 2^4$ and

  if $3^4 \times 3^0 = 3^{4+0} = 3^4$,

  then what number must $2^0$ and $3^0$ be?

**Answer Sheet for Extension**

Since $2^0$ and $3^0$ leave numbers unchanged by multiplication they must equal 1.

Another explanation of why $2^0 = 1$ and $3^0 = 1$ is to look at the patterns below.

$$\begin{array}{ll} 2^4 = 16 & \\ & \big\} \div 2 \\ 2^3 = 8 & \\ & \big\} \div 2 \\ 2^2 = 4 & \\ & \big\} \div 2 \\ 2^1 = 2 & \\ & \big\} \div 2 \\ 2^0 = 1 & \end{array} \qquad\qquad \begin{array}{ll} 3^4 = 81 & \\ & \big\} \div 3 \\ 3^3 = 27 & \\ & \big\} \div 3 \\ 3^2 = 9 & \\ & \big\} \div 3 \\ 3^1 = 3 & \\ & \big\} \div 3 \\ 3^0 = 1 & \end{array}$$

## MULTIPLICATION THE EASY EXPONENTIAL WAY

### Student Activity Sheet 1
### Teacher Answer Sheet

1. Answer key for Powers of 2 is on next page.
2. What do you observe about columns C, D, and F?

   All numbers in column C and D are even numbers. The exponent of the number in column C added to the exponent of the number in column D gives you the exponent of the number in column F.
3. How could you use the "powers of 2" list to find the product of 2048 x 4096?

$$2048 = 2^{11} \qquad 4096 = 2^{12} \text{ so}$$

$$2048 \times 4096 = 2^{11} \times 2^{12} = 2^{11+12} = 2^{23} = 8{,}388{,}608$$

---

### Student Activity Sheet 2
### Teacher Answer Sheet

1. What do you notice about the exponents in column C? All the exponents are even numbers.
2. What do you observe about the relationship between the exponential form in column C and the exponential in Column D?

   The exponents in column D are $\frac{1}{2}$ as large as the exponents in the same row in column C.
3. How could you use the powers of 2 list to find the answer to $\sqrt{262144}$ without using your calculator? $262.144 = 2^{18}$ so the $\sqrt{262144}$ is $2^9 = 512$ (Note $\frac{1}{2}$ of 18 is 9)

   This can be checked on the calculator by pressing 262.144 $\boxed{\sqrt{\ }}$ to get 512 or by multiplying 512 x 512 to get 262.144.

## MULTIPLICATION THE EASY EXPONENTIAL WAY

Student Activity Sheet 3, part A

Teacher Answer Sheet

| | | | | |
|---|---|---|---|---|
| $2^{1}$ =2 | $3^{1}$ =3 | $4^{1}$ =4 | $5^{1}$ =5 | $6^{1}$ =6 |
| $2^{2}$ =4 | $3^{2}$ =9 | $4^{2}$ =16 | $5^{2}$ =25 | $6^{2}$ =36 |
| $2^{3}$ =8 | $3^{3}$ =27 | $4^{3}$ =64 | $5^{3}$ =125 | $6^{3}$ =216 |
| $2^{4}$ =16 | $3^{4}$ =81 | $4^{4}$ =256 | $5^{4}$ =625 | $6^{4}$ =1296 |
| $2^{5}$ =32 | $3^{5}$ =243 | $4^{5}$ =1024 | $5^{5}$ =3125 | $6^{5}$ =7776 |
| $2^{6}$ =64 | $3^{6}$ =729 | $4^{6}$ =4096 | $5^{6}$ =15625 | $6^{6}$ =46656 |
| $2^{7}$ =128 | $3^{7}$ =2187 | $4^{7}$ =16384 | $5^{7}$ =78125 | $6^{7}$ =279936 |
| $2^{8}$ =256 | $3^{8}$ =6561 | $4^{8}$ =65536 | $5^{8}$ =390625 | $6^{8}$ =1679616 |
| $2^{9}$ =512 | $3^{9}$ =19683 | $4^{9}$ =262144 | $5^{9}$ =1953125 | $6^{9}$ =10077696 |
| $2^{10}$ =1024 | $3^{10}$ =59049 | $4^{10}$ =1048576 | $5^{10}$ =9765625 | $6^{10}$ =60466176 |
| $2^{11}$ =2048 | $3^{11}$ =177147 | $4^{11}$ =4194304 | $5^{11}$ =48828125 | |
| $2^{12}$ =4096 | $3^{12}$ =531441 | $4^{12}$ =16777216 | | |
| $2^{13}$ =8192 | $3^{13}$ =1594323 | $4^{13}$ =67108864 | | |
| $2^{14}$ =16384 | $3^{14}$ =4782969 | | | |
| $2^{15}$ =32768 | $3^{15}$ =14348907 | | | |
| $2^{16}$ =65536 | $3^{16}$ =43046721 | | | |
| $2^{17}$ =131,072 | | | | |
| $2^{18}$ =262,144 | | | | |
| $2^{19}$ =524,288 | | | | |
| $2^{20}$ =1,048,576 | | | | |
| $2^{21}$ =2,097,152 | | | | |
| $2^{22}$ =4,194,304 | | | | |
| $2^{23}$ =8,388,608 | | | | |
| $2^{24}$ =16,777,216 | | | | |
| $2^{25}$ =33,554,432 | | | | |
| $2^{26}$ =67,108,864 | | | | |

- Why do you think each column stopped where it did?
  Continuing the pattern would create numbers too large for the calculator display.

1. Use the Powers of 3 column to multiply 729 x 81?
   To multiply numbers with exponents, add the powers of the two exponential numbers where the bases are the same. Answer 729 x 81 = $3^{6}$ x $3^{4}$ = $3^{10}$ = 59049

2. Find the product of 64 and 16384 using the powers of 4 chart. Use the exponential number for 64 and the exponential number for 16384. Verify your answer using the calculator.
   64 = $4^{3}$ 16384 = $4^{7}$ 64 x 16384 = $4^{3}$ x $4^{7}$ = $4^{10}$ = 1048576

3. Find the product of 3125 and 15625 using the powers of 5 chart. Use the exponential number for 3125 and the exponential number for 15625. Verify your answer using the calculator.
   3125 = $5^{5}$ 15625 = $5^{6}$ 3125 x 15625 = $5^{5}$x$5^{6}$ = $5^{11}$=48.828.125

# MULTIPLICATION THE EASY EXPONENTIAL WAY
## Student Activity Sheet 3, part B
## Teacher Answer Sheet

Use Student Activity Sheet 3, part A to complete this part.

To find the product of 36 and 46656 you can use the exponential number for 36 and the exponential number for 46656.

$36 = 6^2$ $\quad$ $46656 = 6^6$ so $36 \times 46656 = 6^2 \times 6^6 = 6^8 = 1679616$

Note: The base (6) was the same, but the powers were different (2 and 6). What happens if the bases are different and the powers are the same? Do the following problems . What do you observe?

$2^3 \times 3^3 = \underline{\quad 8 \quad} \times \underline{\quad 27 \quad} = \underline{\quad 216 \quad} = \boxed{6}^3$

$3^2 \times 5^2 = \underline{\quad 9 \quad} \times \underline{\quad 25 \quad} = \underline{\quad 225 \quad} = \boxed{15}^2$

$3^4 \times 2^4 = \underline{\quad 81 \quad} \times \underline{\quad 16 \quad} = \underline{\quad 1296 \quad} = \boxed{6}^4$

$4^2 \times 3^2 = \underline{\quad 16 \quad} \times \underline{\quad 9 \quad} = \underline{\quad 144 \quad} = \boxed{12}^2$

$6^2 \times 3^2 = \underline{\quad 36 \quad} \times \underline{\quad 9 \quad} = \underline{\quad 324 \quad} = \boxed{18}^2$

1. What do you observe in each of the problems above?

   If you do not have matching bases, but you have matching exponents, you can multiply the bases and leave the exponent unchanged.

2. Write a similar problem and verify that your solution works?

   Answer will vary. To be a similar problem the powers of both of the numbers must be the same.

Name ______________________

## MULTIPLICATION THE EASY EXPONENTIAL WAY

Student Activity Sheet 1

**Powers of 2**

$2^{1}$ =
$2^{2}$ =
$2^{3}$ =
$2^{4}$ =
$2^{5}$ =
$2^{6}$ =
$2^{7}$ =
$2^{8}$ =
$2^{9}$ =
$2^{10}$ =
$2^{11}$ =
$2^{12}$ =
$2^{13}$ =
$2^{14}$ =
$2^{15}$ =
$2^{16}$ =
$2^{17}$ =
$2^{18}$ =
$2^{19}$ =
$2^{20}$ =
$2^{21}$ =
$2^{22}$ =
$2^{23}$ =
$2^{24}$ =
$2^{25}$ =
$2^{26}$ =

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| Factor A | Factor B | Factor A in exponential form | Factor B in exponential form | Product of factors A & B | Products in exponential form |
| 128 | 256 | $2^{7}$ | $2^{8}$ | 32,768 | $2^{15}$ |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
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| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

1. Complete the Powers of 2 chart first. Use the information to complete the second chart.

2. What do you observe about columns C, D, and F? ______________________

______________________________________________

______________________________________________

3. How could you use the "powers of 2" list to find the product of 2048 x 4096?

______________________________________________

______________________________________________

Name ____________________

# MULTIPLICATION THE EASY EXPONENTIAL WAY

## Student Activity Sheet 2

- Select a number from the Powers of 2 chart. Find its square root. If the square root is a whole number, record the number and its square root on the chart.

Example: $\sqrt{64}$ = 8, a whole number. Record 64 and 8 on the chart.
$\sqrt{32}$ = 5.5668542 The square root is not a whole number. Do not record on the chart.

- Experiment with numbers from the powers of 2 chart to see which have **whole number** square roots. Record them.
- Complete the chart.

| A | B | C | D |
|---|---|---|---|
| Number | Whole Number Square root of Number | Number in Column A in Exponential Form | Square Root of Number in Column A in Exponential Form |
| 4096 | 64 | $2^{12}$ | $2^{6}$ |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

1. What do you notice about the exponents in column C? ____________________

____________________________________________

2. What do you observe about the relationship between the numbers in column C and the numbers in Column D? ____________________________________

____________________________________________

3. How could you use the powers of 2 list to find the answer to $\sqrt{262144}$ without using your calculator? ____________________________________

____________________________________________

Name ______________

## MULTIPLICATION THE EASY EXPONENTIAL WAY

### Student Activity Sheet 3, part A

| Powers of 2 | Powers of 3 | Powers of 4 | Powers of 5 | Powers of 6 |
|---|---|---|---|---|
| $2^{1}$ =2 | $3^{1}$ = | $4^{1}$ = | $5^{1}$ = | $6^{1}$ = |
| $2^{2}$ =4 | $3^{2}$ = | $4^{2}$ = | $5^{2}$ = | $6^{2}$ = |
| $2^{3}$ =8 | $3^{3}$ = | $4^{3}$ = | $5^{3}$ = | $6^{3}$ = |
| $2^{4}$ =16 | $3^{4}$ = | $4^{4}$ = | $5^{4}$ = | $6^{4}$ = |
| $2^{5}$ =32 | $3^{5}$ = | $4^{5}$ = | $5^{5}$ = | $6^{5}$ = |
| $2^{6}$ =64 | $3^{6}$ = | $4^{6}$ = | $5^{6}$ = | $6^{6}$ = |
| $2^{7}$ =128 | $3^{7}$ = | $4^{7}$ = | $5^{7}$ = | $6^{7}$ = |
| $2^{8}$ =256 | $3^{8}$ = | $4^{8}$ = | $5^{8}$ = | $6^{8}$ = |
| $2^{9}$ =512 | $3^{9}$ = | $4^{9}$ = | $5^{9}$ = | $6^{9}$ = |
| $2^{10}$ =1024 | $3^{10}$ = | $4^{10}$ = | $5^{10}$ = | $6^{10}$ = |
| $2^{11}$ =2048 | $3^{11}$ = | $4^{11}$ = | $5^{11}$ = | |
| $2^{12}$ =4096 | $3^{12}$ = | $4^{12}$ = | | |
| $2^{13}$ =8192 | $3^{13}$ = | $4^{13}$ = | | |
| $2^{14}$ =16384 | $3^{14}$ = | | | |
| $2^{15}$ =32768 | $3^{15}$ = | | | |
| $2^{16}$ =65536 | $3^{16}$ = | | | |
| $2^{17}$ =131,072 | | | | |
| $2^{18}$ =262,144 | | | | |
| $2^{19}$ =524,288 | | | | |
| $2^{20}$ =1,048,576 | | | | |
| $2^{21}$ =2,097,152 | | | | |
| $2^{22}$ =4,194,304 | | | | |
| $2^{23}$ =8,388,608 | | | | |
| $2^{24}$ =16,777,216 | | | | |
| $2^{25}$ =33,554,432 | | | | |
| $2^{26}$ =67,108,864 | | | | |

- Why do you think each column stopped where it did?

______________________________________________

______________________________________________

1. Use the Powers of 3 column to multiply 729 x 81?

______________________________________________

______________________________________________

2. Find the product of 64 and 16384 using the powers of 4 chart. Use the exponential number for 64 and the exponential number for 16384. Verify your answer using the calculator.

3. Find the product of 3125 and 15625 using the powers of 5 chart. Use the exponential number for 3125 and the exponential number for 15625. Verify your answer using the calculator.

Name ____________________

## MULTIPLICATION THE EASY EXPONENTIAL WAY

Student Activity Sheet 3, part B

Use Student Activity Sheet 3, part A to complete this part.

To find the product of 36 and 46656 you can use the exponential number for 36 and the exponential number for 46656

$36 = 6^2, \quad 46656 = 6^6$ so $36 \times 46656 = 6^2 \times 6^6 = 6^8 = 1679616$

Note: The base (6) was the same, but the powers were different (2 and 6).

What happens if the bases are different and the powers are the same? Do the following problems. What do you observe?

$2^3 \times 3^3 =$ ____8____ x ____27____ = ____216____ = $\boxed{\phantom{00}}^3$

$3^2 \times 5^2 =$ __________ x __________ = __________ = $\boxed{\phantom{00}}^2$

$3^4 \times 2^4 =$ __________ x __________ = __________ = $\boxed{\phantom{00}}^4$

$4^2 \times 3^2 =$ __________ x __________ = __________ = $\boxed{\phantom{00}}^2$

$6^2 \times 3^2 =$ __________ x __________ = __________ = $\boxed{\phantom{00}}^2$

1. What do you observe in each of the problems above?

______________________________________________________________

______________________________________________________________

2. Write a similar problem and verify that your solution works?

x = __________ x __________ = __________ = $\boxed{\phantom{00}}$

x = ______ x __________ = __________ = $\boxed{\phantom{00}}$

x = ______ x __________ = __________ = $\boxed{\phantom{00}}$

x = ______ x __________ = __________ = $\boxed{\phantom{00}}$

x = ______ x __________ = __________ = $\boxed{\phantom{00}}$

# YOUR GAIN - YOUR LOSS

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Number |
| **SKILL:** | Multiply or divide a number by a multi-digit number. |

**MANAGEMENT**

| | |
|---|---|
| **CLASS ORGANIZATION:** | Individual or pairs |
| **TIME FRAME:** | One or two math periods |
| **MATERIALS:** | Calculator |
| **VOCABULARY:** | Calories |
| **PREREQUISITE SKILL:** | Multiply or divide decimals, round decimals |

**LESSON**

- **DIRECTED INSTRUCTION/ GUIDED PRACTICE:**
  Note: Students relate mathematics to weight loss and gain.
  - Hand out Calorie Chart. Ask questions about doubling or tripling portions and its effect on the number of calories.
  - Hand out Student Activity Sheet 1. Do days 1-4 with the students and discuss the results. Students complete the chart.

- **INDEPENDENT PRACTICE:**
  - Hand out Student Activity Sheet 2. Students calculate the hours of each activity needed for Mad-Man to lose one pound.

    A class discussion should follow to determine which activities produce the quickest or slowest loss. Discuss that doctors are consulted for the most healthful weight loss program.

- **EVALUATION:**
  Teacher observation and Student Activity Sheets.

- **EXTENSION:**
  - Hand out Student Activity Sheet 3. Students complete the sheet independently. Answers will vary.

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## YOUR GAIN - YOUR LOSS

Calorie Chart

| Food | Measure | Calories |
|---|---|---|
| Apple | 1 medium | 80 |
| Bacon (cooked) | 1 slice | 46 |
| Banana | 1 medium | 81 |
| Beef Stew | 1 cup | 218 |
| Biscuits | 4 oz | 104 |
| Blintz, Cheese | 1 | 70 |
| Bologna Sandwich | 1 | 369 |
| Cake, with icing | 1 slice | 274 |
| Cantaloupe | 1 | 70 |
| Carrot | 1 | 21 |
| Cheerios | 1 oz | 112 |
| Cheese, American | 1 slice | 113 |
| Cheese Cake | 1 slice | 214 |
| Chicken, Fried | 1/2 chicken | 437 |
| Chocolate Milk Shake | 11 oz serving | 355 |
| Chocolate Pudding | 1/2 cup | 174 |
| Cookies, Chocolate Chip | 1 | 58 |
| Cookies, Oreo | 1 | 50 |
| Corned Beef Hash | 4 oz serving | 405 |
| Doughnut | 1 | 165 |
| Egg, Hard Boiled | 1 | 81 |
| Enchilada | 1 | 259 |
| Frog Legs | 4 oz serving | 81 |
| Fruit Cocktail | 1/2 cup | 89 |
| Fudgsicle | 1 | 102 |
| Grapes | 1 cup | 104 |
| Grapefruit | 1 | 98 |
| Hamburger (Fast Food) | 1 | 251 |
| Hot Dog with bun | 1 | 255 |
| Ice Cream | 1 cup | 272 |
| Ice Cream Sandwich | 1 | 173 |
| Jellybeans | 1 oz serving | 104 |
| Lasagna | 8 oz serving | 255 |
| Lemon Pie | 1 slice | 227 |
| Lettuce | 1 head | 59 |
| Liver | 4 oz serving | 260 |
| Lobster | 4 oz serving | 109 |
| Macaroni and Cheese | 1 cup | 430 |
| Muffin, Blueberry | 1 | 115 |
| Oatmeal | 1 cup | 150 |
| Orange | 1 medium | 77 |
| Pancake | 1 | 84 |
| Pizza (Cheese) | 1 slice | 184 |
| Popcorn | 1 cup | 25 |
| Potato Chips | 1 oz | 158 |
| Spaghetti & Meat Balls | 1 cup | 322 |
| Steak | 1 lb | 1596 |
| Turkey | 4 oz | 253 |
| Twinkies | 1 | 144 |

From The Dictionary of Calories & Carbohydrates by Barbara Kraus

## YOUR GAIN - YOUR LOSS

Student Activity Sheet 1
Teacher Answer Sheet

Mad-Man David LeRoque the wrestler needs to gain weight.
His trainer has computed that Mad-Man must eat 4000 calories per day more than he usually eats. To get his additional 4000 calories, he decides to pick a different food for each of the 15 days in his weight gaining program. Help him plan his menu. Record your answers in the chart below.

Day 1: Mad-Man decides to eat his regular menu plus 4000 calories' worth of chocolate milkshakes. 1 milk shake has 355 calories.
To find how many milkshakes are needed, divide 4000 by 355 on your calculator. Record your answer. The calculator answer, 11.267605 is between 11 and 12. Eleven milkshakes will not provide the minimum 4000 calories, so he will drink twelve.

Use the calories chart provided to select additional foods to complete the chart.

| Day | Food added to regular menu | Measure | Calories | Minimum Amount needed for 4000 calories |
|---|---|---|---|---|
| 1 | Chocolate Milk Shakes | 11 oz serving | 355 | 12 servings |
| 2 | Steak | 1 lb with bone | 1596 | 3 steaks |
| 3 | Cheese Pizza | slice | 184 | 22 slices |
| 4 | Popcorn (plain) | cup | 25 | 160 cups |
| 5 | Apples | medium | 80 | 50 medium |
| 6 | Carrots | 1 carrot | 21 | 191 carrots |
| 7 | Lettuce | 1 head | 59 | 68 heads |
| 8 | Answers will vary | | | |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |
| 12 | | | | |
| 13 | | | | |
| 14 | | | | |
| 15 | | | | |

## YOUR GAIN - YOUR LOSS

Student Activity Sheet 2
Teacher Answer Sheet

Mad-Man won his match and decided to lose weight. He weighed in at 300 pounds. His trainer needs to help him decide how much additional exercise he needs to do. Help Mad-Man's trainer plan his program by completing the chart below.
One pound of fat is equal to 3500 calories. Add this number of calories to those you need to balance your energy requirements and you will gain one pound; subtract it and you will lose a pound.

| Calories for living | | | |
|---|---|---|---|
| A. Type of Activity | B. Approximate calories used per each pound of weight | C. Calories used per hour by a 300 pound man (column B x 300) | D. Hours of activity need to lose one pound (3500 + column C round to the nearest tenth) |
| Sleeping | 0.4 | 300 x 0.4 = 120 | 3500 + 120 = 29.2 |
| Sitting, studying, TV viewing | 0.6 | 180 | 19.4 |
| Writing | 0.7 | 210 | 16.7 |
| Eating | 0.7 | 210 | 16.7 |
| Sewing | 0.7 | 210 | 16.7 |
| Standing quietly | 0.7 | 210 | 16.7 |
| Dressing and undressing | 0.9 | 270 | 13.0 |
| Ironing clothes | 1.0 | 300 | 11.7 |
| Washing dishes | 1.0 | 300 | 11.7 |
| Typing | 1.0 | 300 | 11.7 |
| Washing clothes | 1.1 | 330 | 10.6 |
| Playing piano | 1.2 | 360 | 9.7 |
| Painting walls/furniture | 1.3 | 390 | 9.0 |
| Walking | 1.5 | 450 | 7.8 |
| Bicycle driving | 1.7 | 510 | 6.9 |
| Sweeping, vacuuming | 1.9 | 570 | 6.1 |
| Mowing lawn (power mower) | 2.0 | 600 | 5.8 |
| Skating | 2.2 | 660 | 5.3 |
| Horseback riding | 2.6 | 780 | 4.5 |
| Playing table tennis | 2.7 | 810 | 4.3 |
| Walking up hill | 3.0 | 900 | 3.9 |
| Running | 4.0 | 1200 | 2.9 |
| Active sports and games | 4.5 or more | 1350 | 2.6 |
| Swimming | 4.5 | 1350 | 2.6 |

- If Mad-Man wanted to lose 10 lbs, how would you suggest he do it? You may combine activities.

  Answer will vary

- Which activity from the chart produces the quickest weight loss? swimming
- Which produces the slowest weight loss? sleeping

Name____________________

## YOUR GAIN - YOUR LOSS

Student Activity Sheet 1

"Mad-Man" David LeRoque the wrestler needs to gain weight.
His trainer has computed that Mad-Man must eat 4000 calories per day more than he usually eats. To get his additional 4000 calories, he decides to pick a different food for each of the 15 days in his weight gaining program. Help him plan his menu. Record your answers in the chart below.

Day 1: Mad-Man decides to eat his regular menu plus 4000 calories' worth of chocolate milkshakes. 1 milkshake has 355 calories.
To find how many milkshakes are needed, divide 4000 by 355 on your calculator. Record your answer. The calculator answer, 11.267605 is between 11 and 12. Eleven milkshakes will not provide the minimum 4000 calories, so he will drink twelve.

Use the calories chart provided to select additional foods to complete the chart.

| Day | Food added to regular menu | Measure | Calories | Minimum Amount needed for 4000 calories |
|---|---|---|---|---|
| 1 | Chocolate Milk Shakes | 11 oz serving | 355 | 12 servings |
| 2 | Steak | 1 lb with bone | 1596 | |
| 3 | Cheese Pizza | slice | 184 | |
| 4 | Popcorn (plain) | cup | 25 | |
| 5 | Apples | medium | | |
| 6 | Carrots | | | |
| 7 | Lettuce | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |
| 12 | | | | |
| 13 | | | | |
| 14 | | | | |
| 15 | | | | |

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Name ____________________

## YOUR GAIN - YOUR LOSS

Student Activity Sheet 2

Mad-Man won his match and decided to lose weight. He weighed in at 300 pounds. His trainer needs to help him decide how much additional exercise he needs to do. Help Mad-Man's trainer plan his program by completing the chart below.
One pound of fat is equal to 3500 calories. Add this number of calories to those you need to balance your energy requirements and you will gain one pound; subtract it and you will lose a pound.

| Calories for living | | | |
|---|---|---|---|
| A. Type of Activity | B. Approximate calories used per each pound of weight | C. Calories used per hour by a 300 pound man (column B x 300) | D. Hours of activity needed to lose one pound (3500 ÷ column C rounded to the nearest tenth) |
| Sleeping | 0.4 | 300 x 0.4 = 120 | 3500 ÷ 120 = 29.6 |
| Sitting, studying, TV viewing | 0.6 | | |
| Writing | 0.7 | | |
| Eating | 0.7 | | |
| Sewing | 0.7 | | |
| Standing quietly | 0.7 | | |
| Dressing and undressing | 0.9 | | |
| Ironing clothes | 1.0 | | |
| Washing dishes | 1.0 | | |
| Typing | 1.0 | | |
| Washing clothes | 1.1 | | |
| Playing piano | 1.2 | | |
| Painting walls/furniture | 1.3 | | |
| Walking | 1.5 | | |
| Bicycle driving | 1.7 | | |
| Sweeping, vacuuming | 1.9 | | |
| Mowing lawn (power mower) | 2.0 | | |
| Skating | 2.2 | | |
| Horseback riding | 2.6 | | |
| Playing table tennis | 2.7 | | |
| Walking up hill | 3.0 | | |
| Running | 4.0 | | |
| Active sports and games | 4.5 or more | | |
| Swimming | 4.5 | | |

- If Mad-Man wanted to lose 10 lbs, how would you suggest he do it? You may combine activities.

  ____________________________________________________________

- Which activity from the chart produces the quickest weight loss? ______________
- Which produces the slowest weight loss? ______________

Name ____________________

## YOUR GAIN - YOUR LOSS

Student Activity Sheet 3

Plan additional activities for Mad-Man David LeRoque for 1 week. Be sure to vary his activities daily.

Complete the chart. (use your results from Student Activity Sheet 2)

**BE REALISTIC**

| Day | Activities | Hours | Pounds Lost |
|---|---|---|---|
| Sunday | | | |
| Monday | | | |
| Tuesday | | | |
| Wednesday | | | |
| Thursday | | | |
| Friday | | | |
| Saturday | | | |

At this rate, how many weeks will it take Mad-Man to lose 25 pounds? ______________

## A PIECE OF CAKE

**GRADE:** 7 - 8

**STRAND:** Number

**SKILL:** Convert fractions to decimals; add decimals, explore infinite sequences and series.

**MANAGEMENT**

**CLASS ORGANIZATION:** Small group

**TIME FRAME:** One math period

**MATERIALS:** Calculator

**VOCABULARY:** Truncate

**PREREQUISITE SKILL:** Understand fractions and decimals

**LESSON**

- **DIRECTED INSTRUCTION/GUIDED PRACTICE:**

  Give the following problem to students: *Liz is given two delicious cakes. She wants to gradually cut down on the amount of cake she eats, so she decides to eat 1 cake the first day,* $\frac{1}{2}$ *of a cake the second day,* $\frac{1}{4}$ *of a cake the third day. Each day she will continue to eat* $\frac{1}{2}$ *as much as the day before. Estimate how many days it will take her to eat the two cakes.*

  Teacher Notes:

  1) Although some is eaten each day, the second cake never gets completely eaten.
  2) Help students find the first four days' data.

- **INDEPENDENT PRACTICE:**

  Students complete the chart for 14 days and discuss their observations. Discuss with the class the fact that because the calculator display can only hold a fixed number of digits, extra decimal places get truncated. (chopped off.) Given this limitation of the calculator, if they continued on with the chart they would eventually get a zero in column B and a 2 in column A. If the computations were done with pencil and paper, without rounding, this would not occur.

- **EVALUATION:**

  Teacher observation and Student Activity Sheet. For Student Activity Sheet page 2, students observe that the answers in column C become larger but never reach 2. Liz will theoretically never finish eating the second cake. She eats smaller portions but there will always be some left.

## A PIECE OF CAKE

Student Activity Sheet page 1
Teacher Answer Sheet

| | |
|---|---|
| Column A = | Fraction of a cake eaten this day.<br>Compute by using the fraction from the previous day times 1/2. |
| Column B = | Decimal name for the fraction in Column A.<br>Compute by using your calculator to divide the numerator by the denominator. Record the number displayed. |
| Column C = | Total amount of cake eaten to date.<br>Compute by adding the amount from the previous day's Column C to the current day's Column B entry. |

| Day | A | B | C |
|---|---|---|---|
| | **Fraction of a cake eaten each day** | **Decimal name for fraction in Column A** | **Total amount of cake eaten to date** |
| 1 | 1 | 1 | 1 |
| 2 | $\frac{1}{2}$ | 0.5 | 1.5 |
| 3 | $\frac{1}{4}$ | 0.25 | 1.75 |
| 4 | $\frac{1}{8}$ | 0.125 | 1.875 |
| 5 | $\frac{1}{16}$ | 0.0625 | 1.9375 |
| 6 | $\frac{1}{32}$ | 0.03125 | 1.96875 |
| 7 | $\frac{1}{64}$ | 0.015625 | 1.984375 |
| 8 | $\frac{1}{128}$ | 0.0078125 | 1.9921875 |
| 9 | $\frac{1}{256}$ | 0.0039062 | 1.9960937 |
| 10 | $\frac{1}{512}$ | 0.0019531 | 1.9980468 |
| 11 | $\frac{1}{1024}$ | 0.0009765 | 1.9990233 |
| 12 | $\frac{1}{2048}$ | 0.0004882 | 1.9995115 |
| 13 | $\frac{1}{4096}$ | 0.0002441 | 1.9997556 |
| 14 | $\frac{1}{8192}$ | 0.000122 | 1.9998776 |

Teacher notes: Some students will find it easier to complete all of column A first before filling in the remainder of the chart. Other students will want to complete the chart filling in one day's data at a time. Either method is acceptable.

Name________________

## A PIECE OF CAKE

Student Activity Sheet page 1

Directions: Complete the chart for 14 days.

Column A = Fraction of a cake eaten this day.
Compute by using the fraction from the previous day times 1/2.

Column B = Decimal name for the fraction in Column A.
Compute by using your calculator to divide the numerator by the denominator. Record the number displayed.

Column C = Total amount of cake eaten to date.
Compute by adding the amount from the previous day's Column C to the current day's Column B entry.

| Day | A | B | C |
|---|---|---|---|
| | **Fraction of a cake eaten each day** | **Decimal name for fraction in Column A** | **Total amount of cake eaten to date** |
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |
| 12 | | | |
| 13 | | | |
| 14 | | | |

Name________________

# A PIECE OF CAKE

Student Activity Sheet page 2

A. What do you notice about your answers in Column C?

______________________________________________

______________________________________________

______________________________________________

______________________________________________

______________________________________________

B. Will she ever finish eating the second cake?________ Explain.

______________________________________________

______________________________________________

______________________________________________

______________________________________________

______________________________________________

## PARDON MY DEAR AUNT SALLY

**GRADE:** 7 - 8

**STRAND:** Algebra

**SKILL:** Use order of operations to compute.

**MANAGEMENT**

**CLASS ORGANIZATION:** Partner/Individual

**TIME FRAME:** One or two math periods

**MATERIALS:** Calculator

**VOCABULARY:** Order of operations, experiments

**PREREQUISITE SKILL:** Basic operations, exponents

### LESSON

- **DIRECTED INSTRUCTION:**
  - Teacher asks: "John put on his shoes, his socks and his pants. If this were the order in which he dressed, would it make sense? If not, why? What would be better?"

    "Now, use your calculator to solve $6 + 15 \times 5 = \boxed{\phantom{0}}$"
  - Teacher asks: "What answer did you get? Did anyone get another answer? How did you get your answers?"
  - Place the two possible responses on the board:

| | |
|---|---|
| $6 + 15 \times 5 = \boxed{\phantom{0}}$ | $6 + 15 \times 5 = \boxed{\phantom{0}}$ |
| $21 \times 5 = \boxed{\phantom{0}}$ | $6 + 75 = \boxed{\phantom{0}}$ |
| $105 = \boxed{\phantom{0}}$ | $81 = \boxed{\phantom{0}}$ |

  - Explain that mathematicians have developed rules to avoid getting two answers for this kind of problem. (Note: the "Rules for Order of Operations" page may be used as a transparency for the overhead projector.)

- **GUIDED PRACTICE:**
  - Hand out Student Activity Sheet 1 and a ruler for the order of operations.

    *Part 1 Directions: Underline the pair of numbers and the operation you will do first and then complete the problem.*

1. 8 + 9 x 7 = **71**
2. 135 - 7 x 9 = **72**
3. 29 + 58 x 32 = **1885**
4. 3 x 15 - 11 = **34**
5. 58 ÷ 2 + 63 = **92**
6. 516 ÷ 6 + 742 = **828**

Discuss and verify correct responses.

- **INDEPENDENT PRACTICE:**
  Students complete Activity Sheet 1, Part 2.

Part 2 Directions: Use the Order of Operations to solve:

1. (5 x 3) + (9 x 6) + 10 = **79**
2. 5 x (3 x 9) x (6 + 10) = **2160**
3. (5 x 3) + 9 x (6 + 10) = **159**
4. 12 x 15 + 17 x 19 = **503**
5. 12 x 15 - 17 x 6 = **78**
6. 12 ÷ 3 + 72 ÷ 9 = **12**
7. 115 ÷ 5 - (18 - 12) = **17**
8. 42 + 37 + 15 - 2 x 3 = **88**
9. 42 + 37 + (15 - 2) x 3 = **118**
10. 42 + 30 ÷ 15 x 3 = **48**

- **EVALUATION:**
  Teacher observation and Student Activity Sheets.

- **EXTENSION ACTIVITY**
  Students complete Student Activity Sheet 2.

# PARDON MY DEAR AUNT SALLY

## Transparency

## RULES FOR THE ORDER OF OPERATIONS

| | | |
|---|---|---|
| 1 ) | **PARENTHESES** | [Do all operations x, +, + -, **INSIDE** parentheses ( ) first.] |
| 2 ) | **MULTIPLY & DIVIDE** | [Multiply and divide in order. If division appears to the left of multiplication, then divide before multiplying.] |
| 3 ) | **ADD & SUBTRACT** | [ Add and subtract in order, left to right] |

The acceptable answer to
**6 + 15 x 5** would be the same as
**6 + (15 x 5)=**

or **6 + 75 = 81**

**Memory device for this rule:**

| Pardon | My | Dear | Aunt | Sally |
|---|---|---|---|---|
| a | u | i | d | u |
| r | l | v | d | b |
| e | t | i | | t |
| n | i | d | | r |
| t | p | e | | a |
| h | l | | | c |
| e | ty | | | t |
| s | | | | |
| e | | | | |
| s | | | | |

Name _______________

## PARDON MY DEAR AUNT SALLY

## RULES FOR THE ORDER OF OPERATIONS

| | | |
|---|---|---|
| 1 ) | **PARENTHESES** | [Do all operations x, ÷, +, -, and powers INSIDE of parentheses ( ) first.] |
| 2 ) | **MULTIPLY & DIVIDE** | [Multipy and divide in order. If division appears to the left of multiplication, then divide before multiplying.] |
| 3 ) | **ADD & SUBTRACT** | [Add and subtract in order, left to right.] |

The accepted answer to

6 + 15 x 5 would be the same as
6 + (15 x 5 ) =
6 + 75 = 81

**Memory device for this rule:**

| Pardon | My | Dear | Aunt | Sally |
|---|---|---|---|---|
| a | u | i | d | u |
| r | l | v | d | b |
| e | t | i | | t |
| n | i | d | | r |
| t | p | e | | a |
| h | l | | | c |
| e | ty | | | t |
| s | | | | |
| e | | | | |
| s | | | | |

Name____________________

## PARDON MY DEAR AUNT SALLY

Student Activity Sheet 1

**Part 1 Directions:** Underline the pair of numbers and operation you will do first and then complete the problem.

1. 8 + 9 x 7 = ____________________
2. 15 - 7 x 9 = ____________________
3. 29 + 58 x 32 = ____________________
4. 3 x 15 - 11 = ____________________
5. 58 + 2 + 63 = ____________________
6. 516 + 6 + 742 =____________________

**Part 2 Directions:** Use the Order of Operations to compute.

1. (5 x 3) + (9 x 6) + 10 = ______________________________
2. 5 x (3 x 9) x (6 + 10) = ______________________________
3. (5 x 3) + 9 x (6 + 10)= ______________________________
4. 12 x 15 + 17 x 19 = ______________________________
5. 12 x 15 - 17 x 6 = ______________________________
6. 12 + 3 + 72 + 9 = ______________________________
7. 115 + 5 - (18 - 12) = ______________________________
8. 42 + 37 + 15 - 2 x 3 = ______________________________
9. 42 + 37 + (15 - 2) x 3 = ______________________________
10. 42 + 30 + 15 x 3 = ______________________________

Name____________________

## PARDON MY DEAR AUNT SALLY
Student Activity Sheet 2

### RULES FOR ORDER OF OPERATIONS WITH EXPONENTS

| | |
|---|---|
| 1 ) **PARENTHESES** | [ Do all operations x, +, +, -, and powers **INSIDE** of parentheses ( ) first.] |
| 2 ) **EXPONENTS** | [Find value of any powers (exponents)] |
| 3 ) **MULTIPLY & DIVIDE** | [Multipy and divide in order. If division appears to the left of multiplication, then divide before multiplying.] |
| 4 ) **ADD & SUBTRACT** | [Add and subtract in order, left to right.] |

Memory device for this rule:

| ( ) | $n^2$ | X | + | + | − |
|---|---|---|---|---|---|
| Please | Excuse | My | Dear | Aunt | Sally |
| a | x | u | i | d | u |
| r | p | l | v | d | b |
| e | o | t | i | | t |
| n | n | i | d | | r |
| t | e | p | e | | a |
| h | n | l | | | c |
| e | t | y | | | t |
| s | s | | | | |
| e | | | | | |
| s | | | | | |

*Rewrite the exponents:* $2^3 = 2 \times 2 \times 2$

**Example:** $4 + 2^3$ is the same as $4 + 8 = 12$

$(2 + 4)^3$ is the same as $6^3 = 216$

1. $7 + 3^2 = 16$
2. $(7 + 3)^2 = 100$
3. $7^2 + 3 = 52$
4. $49^2 + 35 = 2436$
5. $(5 + 3) + 9 \times 6 + 10^2 = 162$
6. $(49 + 35)^2 = 7056$

**Write and solve 5 new problems on the back of this page.**

## CAUTIOUS CALORIES

**GRADE:** 7 - 8

**STRAND:** Algebra/Logic

**SKILL:** Write and solve an equation.

**MANAGEMENT**
**CLASS ORGANIZATION:** Groups of four

**TIME FRAME:** One math period

**MATERIALS:** Calculators

**VOCABULARY:** Equation, calories

**PREREQUISITE SKILL:** Equation solving

**LESSON:**
**DIRECTED LESSON/GUIDED PRACTICE:**

Teacher Notes:

Students work in groups of four to solve the following problems. They define the problem, estimate an answer, write an equation, solve the equation, and then write the answer to the problem.

Hand out Student Activity Sheet 1. Work the problems with your class.

1. Laurie is dieting, and she can have about 625 calories for lunch. A hamburger without a bun has approximately 340 calories and an average single french fry has about 15 calories. If Laurie decides to eat the hamburger without a bun, how many french fries can she eat to have a total of about 625 calories?

**$Z$ = number of french fries Laurie may eat.**

$$\begin{aligned}
\text{Hamburger} + \text{french fries} &= 625 \text{ calories}\\
340 \text{ calories} + 15 \text{ calories each} \times Z &= 625 \text{ calories}\\
340 + 15 \times Z &= 625 \text{ calories}\\
15 \times Z &= 625 - 340 \text{ calories}\\
15 \times Z &= 285 \text{ calories}\\
Z &= 285 \div 15\\
Z &= 19 \text{ french fries}
\end{aligned}$$

Laurie may eat 19 french fries.

(Students may also solve this problem using a calculator and trial and error by trying different numbers of french fries until a correct solution is found.)

2. Bob wants to eat about 795 calories for lunch. If he eats a 6-ounce hamburger without a bun, how many french fries may he eat?

$Y$ = number of french fries that Bob may eat.
Hamburger = 75 calories x 6 ounces = 450 calories

$$\begin{aligned}
&\text{Hamburger + french fries} = 795 \text{ calories}\\
&450 \text{ calories} + 15 \times Y \text{ calories} = 795 \text{ calories}\\
&15 \times Y = 795 - 450 \text{ calories}\\
&15 \times Y = 345 \text{ calories}\\
&Y = 345 \div 15\\
&Y = 23 \text{ french fries}
\end{aligned}$$

Bob may eat 23 french fries.

3. David wants to eat a 6-ounce hamburger on a bun. How many french fries may he eat to have a total of 795 calories?

$W$ = number of french fries that David may eat.
Hamburger = 75 calories x 6 ounces = 450 calories.

$$\begin{aligned}
&\text{Hamburger + bun + french fries} = 795 \text{ calories}\\
&450 \text{ calories} + 165 + 15 \times W = 795 \text{ calories}\\
&15 \times W = 795 - (450 + 165) \text{ calories}\\
&15 \times W = 795 - 615 \text{ calories}\\
&15 \times W = 180 \text{ calories}\\
&W = 180 \div 15\\
&W = 12 \text{ french fries}
\end{aligned}$$

David may eat 12 french fries.

4. Nikki is planning to have a 1065 calorie lunch. She ordered a 6-ounce hamburger on a bun. How many french fries may she eat?

$N$ = Number of french fries Nikki may eat.
6-ounce hamburger = 75 calories x 6-ounces = 450 calories
bun = 165 calories and french fries = 15 calories each

$$\begin{aligned}
&\text{Hamburger + bun + french fries} = 1065 \text{ calories}\\
&450 \text{ calories} + 165 \text{ calories} + 15 \times N = 1065 \text{ calories}\\
&15 \times N = 1065 - (450 + 165)\\
&15 \times N = 1065 - 615\\
&15 \times N = 450\\
&N = 450 \div 15\\
&N = 30
\end{aligned}$$

Nikki may eat 30 french fries.

- **INDEPENDENT PRACTICE:**
  Hand out and have students complete Student Activity Sheet 2.

Tell students to discuss within their group different lunches they may create for dieters. Use the above information to create lunches for about: 1000 calories, 1500 calories, 2500 calories, and 3000 calories. Note: 3500 calories equal 1 pound of body fat. Discuss group results.

- **EVALUATION:**
  Teacher observation

Name_______________________

## CAUTIOUS CALORIES

Student Activity Sheet 1

**Eating 3,500 calories will create 1 pound of body fat**

Directions: Solve the following problems by first defining the problem, estimating an answer, writing an equation, solving the equation, and then writing the answer to the problem.

1. Laurie is dieting, and she can have about 625 calories for lunch. Her hamburger without a bun has approximately 340 calories, and an average single french fry has about 15 calories. If Laurie decides to eat the hamburger without a bun then how many french fries can she eat to have a total of about 625 calories?

   Use the following information to solve problems 2 - 4.

   **Hamburger = 75 calories an ounce**
   **Single french fry = 15 calories**
   **Hamburger bun = 165 calories**

2. Bob wants to eat about 795 calories for lunch. If he eats a 6-ounce hamburger without a bun then how many french fries may he eat?

3. David wants to eat a 6-ounce hamburger with a bun. How many french fries may he eat to have a total of 795 calories?

4. Nikki is planning to have a 1065 calorie lunch. She ordered a 6-ounce hamburger with a bun. How may french fries may she eat?

Name____________________

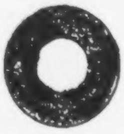
## CAUTIOUS CALORIES

Student Activity Sheet 2

Discuss different lunches you may create with your group . Use the information below to create these lunches for: 1000 calories, 1500 calories, 2500 calories, and 3000 calories. Note: eating 3500 calories will create 1 pound of body fat.

---

**Bacon-cheeseburger = 400 calories**
**Hamburger with tomato, lettuce, onion = 265**
**15 potato chips = 160 calories**
**1/2 cup potato salad = 180 calories**
**Beef burrito = 466 calories**
**Beef tacos = 372 calories**
**7-ounce Sloppy Joe sandwich = 473 calories**
**1 glazed donut = 151 calories**
**6-ounce cola = 77 calories**



| **1000 calories menu** | **1500 calories menu** |
| --- | --- |
| | |
| **2500 calories menu** | **3000 calories menu** |
| | |

# WHAT ARE MY NUMBERS?

**GRADE:** 7 - 8

**STRAND:** Algebra/Logic

**SKILL:** Solve problems using equations, logic, guess and check, and/or finding patterns.

**MANAGEMENT**
**CLASS ORGANIZATION:** Small groups

**TIME FRAME:** Two math periods

**MATERIALS:** Calculators

**VOCABULARY:** Equations, consecutive

**PREREQUISITE SKILL:** Logical reasoning skills

**LESSON:**
- **DIRECTED LESSON AND GUIDED PRACTICE:**

Teacher Notes:

Hand out Student Activity Sheet. Students work in small groups to solve the following problems from the activity sheet.

Find the numbers:

1. The sum of two consecutive whole numbers is 547. Use guess and check strategy, or notice that 547 ÷ 2 = 273.5 (round down and round up) answer: **273, 274**

2. The sum of three consecutive whole numbers is 870.
870 divided by 3 = 290, answer: **289, 290, 291**

- **INDEPENDENT PRACTICE:**

3. The sum of two consecutive odd numbers is 924.
924 divided by 2 = 462, answer: must be odd, **461, 463**

4. The sum of three consecutive odd numbers is 591.
591 divided by 3 = 197, answer: **195, 197, 199** See note on item 6.

5. The sum of four consecutive whole numbers is 90. 90 divided by 4 = 22.5 (count up and count down from this central number to obtain the sequence of 4 numbers) answer: **21, 22, 23, 24**

6. The sum of three consecutive even numbers is 42. When the last number is subtracted from two times the middle number, the answer is the first number.

42 divided by 3 = 14: answer must be **12, 14, 16** (2 times the middle number is 14 x 2 = 28, and last number subtracted from 28 - 16 = 12).

Note: This applies to any three consecutive even or odd numbers.

Note: **Students may solve each of these problems in different ways.** For the teacher's benefit an algebraic solution to each problem is included.

7. One number is 18 more than another. The sum is 24.

$n = $ 1st number

$n + 18 = $ 2nd number

$(n) + (n + 18) = 24$

$2n + 18 = 24$

$2n = 24 - 18$

$2n = 6$

$\mathbf{n = 3}$

21 + 3 = 24, and 21 is 18 more than 3. So the numbers are 3 and 21.

8. If twice a number is decreased by 20, the result is 16.

$2n - 20 = 16$

$2n = 20 + 16$

$2n = 36$

$\mathbf{n = 18}$

9. The sum of 10 and 3 times a number is 25.

$10 + 3n = 25$

$3n = 25 - 10$

$3n = 15$

$\mathbf{n = 5}$

10. Twice a number is increased by 6, the answer is 18.

$2n + 6 = 18$

$2n = 18 - 6$

$2n = 12$

$\mathbf{n = 6}$

11. Three less than three times a number is 42.

$3n - 3 = 42$

$3n = 42 + 3$

$3n = 45$

$\mathbf{n = 15}$

12. If 8 times a number is decreased by 20, the result is 4 times 11.

$4 \times 11 = 8n - 20$

$44 = 8n - 20$

$44 + 20 = 8n$

$64 = 8n$

$\mathbf{n = 8}$

13. Triple a mystery number, and then increase it by 40. The result is 11 times the mystery number you started with.

$3n + 40 = 11n$

$11n - 3n = 40$

$8n = 40$

$\mathbf{n = 5}$

Name__________

## WHAT ARE MY NUMBERS?

Student Activity Sheet

Find the numbers:

1. The sum of two consecutive numbers is 547.

_______________

2. The sum of three consecutive numbers is 870.

_______________

3. The sum of two consecutive odd numbers is 924.

_______________

4. The sum of three consecutive odd numbers is 591.

_______________

5. The sum of four consecutive numbers is 90.

_______________

6. The sum of three consecutive even numbers is 42. When the last number is subtracted from two times the middle number, the answer is the first number.

_______________

7. One number is 18 more than another. The sum is 24.

____________

8. If twice a number is decreased by 20, the answer is 16.

____________

9. The sum of 10 and 3 times a number is 25.

____________

10. Twice a number is increased by 6, the answer is 18.

____________

11. Three less than three times a number is 42.

____________

12. If 8 times a number is decreased by 20, the result is 4 times 11.

____________

13. Triple a number, and then increase by 40, the result is 11 times the mystery number.

____________

# MULTIPLE MADNESS

GRADE: 7 - 8

STRAND: Number

SKILL: Find least common multiples.

## MANAGEMENT

CLASS ORGANIZATION: Pairs

TIME FRAME: One or two math periods

MATERIALS: Calculator

VOCABULARY: Multiples, least common multiple, common multiples

PREREQUISITE SKILL: Multiples and least common multiple

## LESSON

- **DIRECTED INSTRUCTION:**

  Review with students the use of the constant function to find multiples of a number. e.g. $\boxed{\text{on/c}}$ $\boxed{3}$ $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ or 0 + 3 $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ... displays the multiples of 3.

- **GUIDED PRACTICE:**

  Divide the class into pairs, A's and B's. The A's number is 250, and the B's number is 175.

1. Student A enters $\boxed{0}$ $\boxed{+}$ $\boxed{250}$ $\boxed{=}$ on their calculators.

2. Student B enters $\boxed{0}$ $\boxed{+}$ $\boxed{175}$ $\boxed{=}$ on their calculators.
   Student A now has 250 on the calculator display. Student B has 175.

3. **The student with the smaller number in the calculator display presses the equal sign.**

4. Repeat step 3 until the numbers in the displays of both groups' calculators match. (When Student A passes Student B, student B presses $\boxed{=}$ until matching or passing Student A, etc.)

Calculator display

| Group A Calculators | Group B Calculators |
|---|---|
| *175 | 250 |
| 350 | *250 |
| *350 | 500 |
| 525 | *500 |
| *525 | 750 |
| *700 | 750 |
| 875 | *750 |
| *875 | 1000 |
| 1050 | *1000 |
| *1050 | 1250 |
| *1225 | 1250 |
| 1400 | *1250 |
| *1400 | 1500 |
| 1575 | *1500 |
| *1575 | 1750 |
| 1750 | 1750 |

* Indicates which Student presses the $\boxed{=}$ sign next

Explain that since Student A was finding multiples of 175 and Student B was finding multiples of 250, the Least Common Multiple of 175 and 250 must be 1750.

- **INDEPENDENT PRACTICE:**

  Hand out Student Activity Sheet. Students work in pairs to complete the worksheet.

  Discuss the fact that since you are finding the multiples of the two numbers, when they match you have found the least common multiple. The number of times each student hits the = sign (including original entry) indicates what the original number must be multiplied by to find the Least Common Multiple.

- **EVALUATION:**

  Responses to questions about Least Common Multiple

- **HOME ACTIVITY:**

  Write a word problem that can be solved by finding the least common multiple of two numbers.

  (Example: Machine A prints a poster every 18 minutes, Machine B every 24 minutes. How long will it be when they print their first poster simultaneously? Least common multiple of 18 and 24 = 72 minutes.)

## MULTIPLE MADNESS

Student Activity Sheet
Teacher Answer Sheet

Directions:
A. First student enters 0 + First Number = on a calculator.
B. Second student enters 0 + Second Number = on second calculator.
C. Student with the smaller number in the calculator display, presses =
D. Repeat step C until the numbers in the display of both calculators match.
E. As soon as the numbers in the display of both calculators match, you have identified the Least Common Multiple of the two original numbers..

1.

| First Number | Second Number | Least Common Multiple |
|---|---|---|
| 35 | 77 | 385 |
| 210 | 54 | 1890 |
| 25 | 200 | 200 |
| 143 | 55 | 715 |
| 217 | 155 | 1085 |

2. Can you tell by looking at the numbers which person will need to press = more times? Explain.

The person with the lower number will always press = more times.

Using the first example: note that it takes 11 (35's) but only 5 (77's) to equal 385. The 11 and the 5 are the # of times = had to be pressed.

3. How could you find the Least Common Multiple of 3 numbers using calculators? You can use 3 calculators and follow a similar process as above. Again, the student with the smallest number in the display presses =. Eventually all three calculators will be displaying the same number. That number is the Least Common Multiple for the original 3 numbers.

4. Experiment to find 2 numbers between 50 and 60 whose Least Common Multiple is found when you and your partner alternately press = (No person presses = twice in a row while still following rule C)

Answer will vary. Two numbers 51 & 52. Their L.C.M. is 2652

5. A) Use 11 and 13 for your starting numbers. Use tally marks. Count how many times each of you pressed the $\boxed{=}$ (Including the original 0 + 11 $\boxed{=}$ or 0 $\boxed{+}$ 13 $\boxed{=}$)

What does this number represent?

It indicates what your original number must be multiplied by to get the L.C.M.

B) Divide the Least Common Multiple by the number of times you pressed the equal sign. What does this number represent?

It's your original number.

Name ______________________

## MULTIPLE MADNESS
Student Activity Sheet

Directions:
A. First student enters $\boxed{0}$ $\boxed{+}$ $\boxed{\text{First Number}}$ $\boxed{=}$ on a calculator.
B. Second student enters $\boxed{0}$ $\boxed{+}$ $\boxed{\text{Second Number}}$ $\boxed{=}$ on second calculator.
C. Student with the smaller number in the calculator display, presses $\boxed{=}$
D. Repeat step C until the numbers in the display of both calculators match.
E. When the numbers in the display of both calculators first match, you have identified the Least Common Multiple.

1.

| First Number | Second Number | Least Common Multiple |
|---|---|---|
| 35 | 77 | |
| 210 | 54 | |
| 25 | 200 | |
| 143 | 55 | |
| 217 | 155 | |

2. Can you tell by looking at the numbers which person will need to press $\boxed{=}$ more times? Explain.

______________________________________________

______________________________________________

3. How could you find the Least Common Multiple of 3 numbers using calculators? ____________________________________

______________________________________________

______________________________________________

4. Experiment to find 2 numbers between 50 and 60 whose Least Common Multiple is found when you and your partner alternately press $\boxed{=}$ (No person presses $\boxed{=}$ twice in a row while still following rule C)

______________________________________________

5. A. Use 11 and 13 for your starting numbers. Use tally marks. Count how many times each of you pressed $\boxed{=}$ (Including the original 0 + 11 = or 0 + 13 =) What does this number represent? __________

B. Divide the Least Common Multiple by the number of times you pressed the $\boxed{=}$ sign. What does this number represent? ______________

## FAMOUS DATES

**GRADE:** 7 - 8

**STRAND:** Number

**SKILL:** Identify prime and composite numbers. Use square root and memory keys.

**MANAGEMENT**

**CLASS ORGANIZATION:** Pairs

**TIME FRAME:** One math period

**MATERIALS:** Calculator

**VOCABULARY:** Factor, prime number, composite number

**PREREQUISITE SKILLS:** Prime numbers to 50

**LESSON**

- **DIRECTED INSTRUCTION:**
  1. Hand out Student Activity Sheet
  2. Work through the first Famous Date example with the class:

> Find out if 1987 is prime or composite. (The 200th anniversary of the U.S. Constitution was celebrated in 1987.)

1. Press the Memory Clear key.

2. Enter the Famous Date 1987 on the calculator.
   Display: 1987.

3. Press the Memory plus (M+) key.
   Display: M 1987.

4. Press the square root key.
   Display: M 44.575778 Record the whole number only (44) in column B of the Student Activity Sheet.

5. Record in Column C, all the prime numbers from 1 through the number that was recorded in Column B. Prime numbers are listed on the top of the Activity Sheet. Record: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, and 43.

6. Press the Memory recall (MR or R-CM) key.
   Display: M 1987.

7. Press the Divide key
   Display: M 1987.

8. Enter one of the prime numbers recorded in Column C. (Try 2)
   Display: M 2.

9. Press the = key.
   Display: M 993.5

10. Check the display to see if it is a whole number. If the display is a whole number then the famous date is a composite number. Write "composite" in Column D. If the display is not a whole number, then select another prime number from Column C, and repeat the process from step 6.

    The number is prime if after dividing the famous date by all the primes in Column C, you did not get a whole number quotient.

11. Continue dividing 1987 by each of the primes in Column C.

    Note: 1987 is a prime number. Write prime in column D.

12. Press the memory clear key, (on many calculators you may need to press R-CM twice to clear the memory. The M on the display should disappear.) Use the Student Activity Sheet and determine if 1969, 1937, 1215, and the year you were born, are prime or composite numbers.

- **EXTENSION:**
  List 5 or more famous dates from history, science, sports, or entertainment, and find out if these dates are prime or composite numbers.

- **EVALUATION:**
  Teacher observation and Student Activity Sheets.

- **TEACHER NOTE:**
  1. This lesson is based on the Fundamental Theorem of Arithmetic: Every whole number greater than 1 is either a prime or a product of primes.

  2. At least one of the prime factors of a composite number must be less than or equal to the square root of the number. For example, 39 is a composite number. The prime factors of 39 are 3 and 13. The square root of 39 is about 6.2449979. In this instance, the prime factor 3 is smaller than the square root of 39.

## FAMOUS DATES

Teacher Answer Sheet

**Prime numbers between 1 and 50:**
**2 3 5 7 11 13 17 19 23 29 31 37 41 43 47**

| A | B | C | D | E |
|---|---|---|---|---|
| **Famous Dates** | **Square Root (Whole No.)** | **Primes less than number in column B** | **'Prime' or 'Composite'** | **Famous Event** |
| 1987 | 44 | 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 | Prime | 200th Anniversary of U.S. Constitution |
| 1969 | 44 | 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 | Composite | Armstrong Walked on Moon |
| 1937 | 44 | 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 | Composite | Disney created Mickey Mouse |
| 1215 | 34 | 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 | Composite | Magna Carta |
| Your birth year | | | | Year you were born |
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Name ____________________

## FAMOUS DATES
Student Activity Sheet

**Prime numbers between 1 and 50:**
**2 3 5 7 11 13 17 19 23 29 31 37 41 43 47**

| A | B | C | D | E |
|---|---|---|---|---|
| Famous Dates | Square Root (Whole No.) | Primes less than number in column B | 'Prime' or 'Composite' | Famous Event |
| 1987 | | | | 200th Anniversary of U.S. Constitution |
| 1969 | | | | Armstrong Walked on Moon |
| 1937 | | | | Disney created Mickey Mouse |
| 1215 | | | | Magna Carta |
| Your birth year | | | | Year you were born |
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## INTERESTing

| | |
|---|---|
| **GRADE:** | 7 - 8 |
| **STRAND:** | Number |
| **SKILL:** | Apply knowledge of percent. |

**MANAGEMENT**

| | |
|---|---|
| **CLASS ORGANIZATION:** | Pairs |
| **TIME FRAME:** | One or two math periods |
| **MATERIALS:** | Calculator, Student Record Sheets |
| **VOCABULARY:** | Percent, interest, annual |
| **PREREQUISITE SKILL:** | Percent, decimals, rounding, and calculator constant function |

**LESSON**

- **DIRECTED INSTRUCTION:**

  Solve the following problem with the class:
  What will be the value of your account after 1 year, if you put $600.00 in a bank that pays 5.4% annual interest?

**TO FIND THE YEARLY VALUE OF AN AMOUNT OF MONEY DEPOSITED IN THE BANK:**

- **Press in the percent of interest paid. Do not use the percent key!**
  Press 5.4 on the calculator.

- **Press $\boxed{+}\ \boxed{1}\ \boxed{0}\ \boxed{0}\ \boxed{=}$ (This gives you the percent that you will have after one year.)**
  The calculator display will read $\boxed{1}\ \boxed{0}\ \boxed{5}\ \boxed{.}\ \boxed{4}$

- **Press $\boxed{+}\ \boxed{1}\ \boxed{0}\ \boxed{0}\ \boxed{=}$ (Dividing by 100 changes the percent to its equivalent decimal.)**
  The calculator display will read $\boxed{1}\ \boxed{.}\ \boxed{0}\ \boxed{5}\ \boxed{4}$

- **Press $\boxed{x}\ \boxed{\text{the amount of money}}\ \boxed{=}$. (This will find the value after one year.)**
  Press $\boxed{x}\ \boxed{6}\ \boxed{0}\ \boxed{0}\ \boxed{=}$ . The calculator display will read 632.4 which means $632.40 will be the value of the $600.00 deposit after 1 year at 5.4 % interest.

$$\boxed{\boxed{5}\boxed{.}\boxed{4}\boxed{+}\boxed{1}\boxed{0}\boxed{0}\boxed{+}\boxed{1}\boxed{0}\boxed{0}\boxed{x}\boxed{6}\boxed{0}\boxed{0}\boxed{=}\boxed{6}\boxed{3}\boxed{2}\boxed{.}\boxed{4}}$$

## TO FIND THE VALUE YEAR AFTER YEAR OF THE MONEY ORIGINALLY DEPOSITED:

- **Press $\boxed{=}$ . Each time you press $\boxed{=}$, it will show you the next year's total.**

  The calculator display reads 632.4 (value after 1 year.) Press $\boxed{=}$ and 666.5496 will be displayed, representing the value of the money after two years. Press $\boxed{=}$ again and 702.54327 will be displayed representing the value of the money after three years.

  Rounded to the nearest dollar, 702.54327 = $703.00 So, the value of $600 deposited in the bank at 5.4% interest = $703 after 3 years.

  **Continue pressing $\boxed{=}$ to examine future totals.**

  Hand out Student Activity Sheet 1:
  Now you are given a present of $10,000 to invest. You are curious about how fast the money will grow. You go to your local bank and they tell you they can now give you 7% annual interest.

  Have students estimate how many years it would take for $10,000 deposited in the bank to be worth over $100,000. ____________________

  Hand out Student Record Sheet 1. Work Column A with students.

  As you work the problem you may wish to make new estimates. Use your calculator and the rule below to find the yearly value of your money deposited in the bank.

  RULE: Enter in order on your calculator:
  (Percent + 100) + 100 x Amount of money =
  (Then keep pressing the = sign for additional years)

  Record the results on Student Record Sheet 1 in column A, to see how many years it would take before the original $10,000 becomes worth over $100,000.

- **GUIDED PRACTICE:**

  Column B: Another bank offers 9% interest if you promise not to take out your money for at least 10 years. Estimate how long it would take for your $10,000 to exceed $250,000 at the 9% interest rate.____________________

  Use the interest rule and your calculator. Fill in the information on Student Record Sheet 1, Column B. See how close your estimate is.

- **INDEPENDENT PRACTICE:**

  Column C: A friend says, "Did you know that banks once paid 15% interest?" You think to yourself, "With that rate I would have been able to earn more money from my investment." Estimate how long it would have taken for your $10,000 to become at least $1,000,000 at the 15% rate. ____________________

  Use the interest rule and your calculator. Fill in the information on Student Record Sheet 1, Column C.

- **EXTENSION:**

  Pass out Student Activity Sheet 2.
  Choose an amount of money smaller than $60,000 and choose a percent for the interest rate.

  Amount of money ____________________

  Percent ____________________

  Estimate how long it would take you to become a millionaire. ________________

  Use your calculator and the interest rule to complete the chart. How close was your estimate?

- **EVALUATION:**

  Teacher observation and Student Activity Sheets.

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## INTERESTIng

### Student Activity Sheet 1

Answer Key: Rounded to the nearest dollar.

| Year | Value of $10,000 invested at an interest rate of | | |
|---|---|---|---|
| | Column A<br>7 % | Column B<br>9 % | Column C<br>15 % |
| 1 | 10,700 | 10,900 | 11,500 |
| 2 | 11,449 | 11,881 | 13,225 |
| 3 | 12,250 | 12,950 | 15,209 |
| 4 | 13,108 | 14,116 | 17,490 |
| 5 | 14,026 | 15,386 | 20,114 |
| 6 | 15,007 | 16,771 | 23,131 |
| 7 | 16,058 | 18,280 | 26,600 |
| 8 | 17,182 | 19,926 | 30,590 |
| 9 | 18,385 | 21,719 | 35,179 |
| 10 | 19,672 | 23,674 | 40,456 |
| 11 | 21,049 | 25,804 | 46,524 |
| 12 | 22,522 | 28,127 | 53,502 |
| 13 | 24,098 | 30,658 | 61,528 |
| 14 | 25,785 | 33,417 | 70,757 |
| 15 | 27,590 | 36,425 | 81,371 |
| 16 | 29,522 | 39,703 | 93,576 |
| 17 | 31,588 | 43,276 | 107,613 |
| 18 | 33,799 | 47,171 | 123,755 |
| 19 | 36,165 | 51,417 | 142,318 |
| 20 | 38,697 | 56,044 | 163,665 |
| 21 | 41,406 | 61,088 | 188,215 |
| 22 | 44,304 | 66,586 | 216,447 |
| 23 | 47,405 | 72,579 | 248,914 |
| 24 | 50,724 | 79,111 | 286,252 |
| 25 | 54,274 | 86,231 | 329,189 |
| 26 | 58,074 | 93,992 | 378,568 |
| 27 | 62,139 | 102,451 | 435,353 |
| 28 | 66,488 | 111,671 | 500,656 |

| Year | Value of $10,000 invested at an interest rate of | | |
|---|---|---|---|
| | Column A<br>7 % | Column B<br>9 % | Column C<br>15 % |
| 29 | 71,143 | 121,722 | 575,754 |
| 30 | 76,123 | 132,677 | 662,117 |
| 31 | 81,451 | 144,618 | 761,435 |
| 32 | 87,153 | 157,633 | 875,650 |
| 33 | 93,253 | 171,820 | 1,006,998 |
| 34 | 99,781 | 187,284 | |
| 35 | 106,766 | 204,140 | |
| 36 | | 222,512 | |
| 37 | | 242,538 | |
| 38 | | 264,367 | |

NAME ____________________

## INTERESTing

Student Activity Sheet 1

A. You are given a present of $10,000, which you want to invest. You are curious about how fast the money will grow if invested. You go to your local bank and they tell you they can now give you 7% annual interest.

Estimate how many years it would take for $10,000 deposited in the bank to be worth over $100,000. ____________________

As you work the problem you may wish to make new estimates. Use your calculator and the rule below to find the annual value of your money deposited in the bank.

> RULE: Enter in order on your calculator:
> (Percent + 100) ÷ 100 x Amount of money =
> (Then keep pressing the = sign for additional years)

Record the results on Student Record Sheet 1, Column A, to see how many years it would take before the original $10,000 becomes worth over $100,000.

B - Another bank offers 9% interest if you promise not to take out your money for at least 10 years. Estimate how long it would take for your $10,000 to become $250,000 at the 9% interest rate. ______________________________

Use the interest rule and your calculator. Fill in the information on Student Record Sheet 1, Column B. See how close your estimate is.

C - You meet a friend who says, "Did you know that bank once paid 15% interest?" You think to yourself, with that rate, I would have been able to earn more money from my investment. Estimate how long it would have taken for your $10,000 to become $1,000,000 at the 15% rate.

______________________________

Use the interest rule and your calculator. Fill in the information on Student Record Sheet 1, Column C.

Name ______________________

## INTERESTling
Student Activity Sheet 1

**RULE:**
**ENTER ( % OF INTEREST + 1 0 0 ) + 1 0 0 X AMOUNT OF MONEY = ...**

| Year | Value at $10,000 invested at an interest rate of | | |
|---|---|---|---|
| | Column A<br>7 % | Column B<br>9 % | Column C<br>15 % |
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
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| 21 | | | |
| 22 | | | |
| 23 | | | |
| 24 | | | |
| 25 | | | |
| 26 | | | |
| 27 | | | |
| 28 | | | |

Name ____________________

## INTERESTing

Student Activity Sheet 1, page 2

RULE:
ENTER ( % OF INTEREST + 1 0 0 ) + 1 0 0 X AMOUNT OF MONEY = ...

| Year | Value at $10,000 invested at an interest rate of | | |
|---|---|---|---|
| | Column A<br>7 % | Column B<br>9 % | Column C<br>15 % |
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NAME ____________________

## INTERESTing
Student Activity Sheet 2

Choose an amount of money smaller than $60,000 and choose a percent for the interest rate. Amount of money ____________________ Percent ____________________

Estimate how long it would take to become a million dollars. ____________

Use your calculator and the interest rule to complete the chart.

| End of Year | Value | End of Year | Value |
|---|---|---|---|
| 1 | | 26 | |
| 2 | | 27 | |
| 3 | | 28 | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |
| 12 | | | |
| 13 | | | |
| 14 | | | |
| 15 | | | |
| 16 | | | |
| 17 | | | |
| 18 | | | |
| 19 | | | |
| 20 | | | |
| 21 | | | |
| 22 | | | |
| 23 | | | |
| 24 | | | |
| 25 | | | |

How close was your estimate? ____________________

# ROCK ' N MATH

**GRADE:** 7 - 8

**STRAND:** Number

**SKILL:** +, -, x, ÷ of decimals and percent

## MANAGEMENT

**CLASS ORGANIZATION:** Small groups

**TIME FRAME:** One to three math periods

**MATERIALS:** Calculator

**VOCABULARY:** Profit, loss, expense, balance, public liability

**PREREQUISITE SKILL:** Basic operations of decimals, percent, and interpreting remainders

## LESSON

- **DIRECTED INSTRUCTION:**

**SUGGESTED SCHEDULE:**

Day 1
- Use Student Activity Sheet 1 as a motivation for this lesson. Organize the class into groups of four. Students will brainstorm and estimate costs to form a group and arrange for a concert. Later, students will be given fixed costs, profit and losses, and have an opportunity to make decisions. Answers will vary.
- Assign Home Activity Sheet. Be sure students also take home both pages of Student Activity Sheet 1.

Day 2
- In their small groups, students discuss the "reasonableness" of the estimates and compare the findings of each group.
- Hand out Student Activity Sheet 2, "Cost Sheet for Arena Rental." Tell students they are computing the costs for a concert based on fixed and estimated costs. Discuss each service with them. Students complete the Cost Sheet for Arena Rental, Project 1. Discuss results.

- **GUIDED PRACTICE:**

Day 3.
- Hand out Student Activity Sheets 3 and 4. Groups complete the activity and discuss the results.
- Hand out Student Activity Sheet 5 and assist as needed, especially in explaining how to subtract expenses from a Balance Column.
- Discuss results.

- **INDEPENDENT PRACTICE:**
  - Hand out Student Activity Sheet 6. Students complete the charts.
  - Discuss the results.

- **EVALUATION:**

  Teacher observation and Student Activity Sheets.

- **HOME ACTIVITY:**
  - Home Activity Sheet done as part of Student Activity Sheet 1.

## ROCK ' N MATH
## COST SHEET FOR ARENA RENTAL
Student Activity Sheet 2, 3, and 4
Teacher Answer Sheet

(Required expenses are listed. See Student Sheet for Questions.)

| Service | Project 1 cost | Project 2 cost | Project 3 cost |
|---|---|---|---|
| Police | 3000.00 | 3000.00 | 3000.00 |
| Phone switchboard operator | 75.00 | 75.00 | 75.00 |
| Event coordinator | 175.00 | 175.00 | 175.00 |
| Ticket takers,crowd management and ushers | 3150.00 | 3150.00 | 3150.00 |
| Setup and take down | 2500.00 | 2500.00 | 2500.00 |
| Unarmed guards (contemporary services) | 5500.00 | 5500.00 | 5500.00 |
| Fire safety inspector | 400.00 | 400.00 | 400.00 |
| Use of electric sign near the freeway | 150.00 | 150.00 | 150.00 |
| Stage hands before, during, and after the performance | 18,000.00 | 18,000.00 | 18,000.00 |
| Subtotal fixed costs | 32,950.00 | 32,950.00 | 32,950.00 |
| | | | |
| Spotlights ($110 for each spotlight) | 1100.00 | 1100.00 | answer will vary |
| | | | |
| Operator for spotlights 1 operator per spotlight (operators cost $125.00 each) | 1250.00 | 1250.00 | answer will vary |
| | | | |
| Insurance (Public Liability)($.25 for each person attending paid or unpaid | 3750.00 | 1750.00 | 2500.00 |
| | | | |
| Arena rental $7000 or 10% of the money taken in from ticket sales, whichever amount is greater | 22,500.00 | 10,500.00 | answer will vary |
| Total cost | 61,550.00 | 47,550.00 | answer will vary |
| | | | |
| Money taken in on ticket sales: ☐ Number of tickets x cost per ticket | 225,000.00 | 105,000.00 | answer will vary |
| | | | |
| Profit or loss = Money taken in minus all costs | 163,450.00 | 57,450.00 | answer will vary |

### ROCK ' N MATH
Student Activity Sheet 5
Teacher Answer Sheet

| PROJECT 4: Enter totals in appropriate columns | EXPENSE | BALANCE |
|---|---|---|
| Assume that your group brought in $105,000 from selling tickets. | | 105,000 |
| It cost your group 47,550 to use the arena. What is your profit? Enter in the balance column. | 47,550 | 57,450 |
| You have an agent who arranged for your group playing the arena. He charges 10% of the amount of profit that you make. (0.10 x profit). How much money would your agent be given? (Enter in the expense column). How much money would your group have left after paying the agent fee? Subtract expense from balance and enter in the balance column. | 5,745 | 51,705 |
| It cost your group $3000 to travel to Los Angeles and back with all the equipment. How much is left after paying off travel expenses? | 3,000 | 48,705 |
| A full page ad in the Los Angeles Times cost your group 32,497.68 each day. How much money is left after paying of the ad which you luckily ran for only one day? | 32,497.68 | 16,207.32 |
| You split the money that is left among the 5 members of your band. How much money do you get? | | 3,241.46 |
| Your tax man tells you will have to pay about 45% of the money you received in taxes. How much will you have to pay in taxes? (0.45 x amount of money you received). Enter in expense column. How much money will you have left after you have paid your taxes? (Enter in Balance column.) | 1,458.66 | 1782.80 |

## ROCK'N MATH

Student Activity Sheet 6
Teacher Answer Sheet

The people who heard the concert loved it so much that your group received a recording contract with a famous record company. The record company agreed to give your group $0.60 for every record they sell.

| | Workspace | Answers |
|---|---|---|
| How many records would need to be sold for your group to make $100,000? | 100000 ÷ .60 =<br>166,666.66 | 166,667<br>records |
| How many records would need to be sold for each person in the group to make $50,000. (Remember your group has 5 people in it.) | 5 x 50,000 =<br>250,000<br>250,000÷.60= | 416,667<br>records |
| If your group's record sold 2,000,000 copies: How much money would your group receive? | 2,000,000 x .60= | $1,200,000.00 |
| How much money would you receive? | 1,200,000÷5 | $240,000 |
| Do you still want to form a rock band? | | |

State the reasons for your decision. ______________________________

Name____________________

## ROCK'N MATH

Student Activity Sheet 1 page 1

**PROJECT:**

You and your friends are interested in forming a rock group. Discuss and estimate what your costs would be to form a group and to put on a rock concert. Decide what musical instruments and other electronic equipment your group would need.

| Instruments or Equipment | Estimated Costs |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | Total cost for instruments |

Are there special outfits you will need to wear? If yes, write down your estimates of the cost of your outfit and compute the total cost of outfits for your group.

| Outfits or accessories | Estimated Costs |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | Total cost for outfits |

Will there be a cost for a place to practice or can you use your house or garage? If you need to pay for a practice location, list the estimated cost to obtain the place you would want to use.

| Practice location | Estimated Costs |
|---|---|
| | |

NAME____________________

## ROCK'N MATH

### Student Activity Sheet 1 page 2

| Total from Student Activity Sheet 1 page 1 | EXPENSE | BALANCE |
|---|---|---|
| A. Total cost for instruments | | |
| B. Total cost for outfits | | |
| C. Cost for practice location | | |
| D. A + B + C = Total Estimated Costs = | | |
| E. Your school principal agrees to let you play in the school auditorium for your debut concert. How many people can fit in your school auditorium or assembly area? ________<br>F. How many free tickets do you plan to give to the friends and family of your group? ________<br>G. To figure the number of tickets available for sale, subtract F from E and enter here. ________ | | |
| H. Estimate what it would cost to print the tickets. | | |
| I. How much would you charge for a ticket to your school performance?________ | | |
| J. If you sell all the tickets that you expect to sell, how much money will you take in? (Number of tickets x price per ticket) | | |
| K. What is the expense total of all the items you have listed so far? (Amount on lines D and H) | | |
| L. Will you be taking in enough to pay for all of your expenses? If yes, how much profit will you make? Profit = balance less expense. (Amount on line J - amount on line K = profit) | | |
| M. If not, how many concerts would you have to put on before you start making a profit? (Number of concerts x amount on line J) | | |
| N. How much is that for each member of your group? Profit + number of members in your group. | | |

Name______________

## ROCK ' N MATH

### Home Activity Sheet

Bring home Student Activity Sheet. Share your estimated expenses with your parents, and call music stores to find if your original estimates should be changed. Recompute the answers to all these questions using your new information. Write the new answers in brackets next to your original estimates.

Analyze your estimated costs:

- Which estimated instrument costs were reasonable?

______________________________________________

______________________________________________

______________________________________________

______________________________________________

______________________________________________

______________________________________________

- Analyze the costs of outfits and accessories. How close were your estimated costs? If you need to economize, what will you eliminate?

______________________________________________

______________________________________________

______________________________________________

______________________________________________

______________________________________________

______________________________________________

- Was your estimate of cost for a practice location reasonable? Why or why not?

______________________________________________

______________________________________________

______________________________________________

______________________________________________

______________________________________________

______________________________________________

Name____________________

# ROCK ' N MATH
## COST SHEET FOR ARENA RENTAL
Student Activity Sheet 2

**PROJECT 1:**

Use all of the information in the paragraph below to complete the Project 1 Cost column. You are computing the costs for a concert based on fixed and estimated costs. Fifteen thousand people bought tickets and will attend the concert. The tickets cost $15 each and we assume all will be sold. How much money will we receive from the ticket sales? ____________________

| Service | Project 1 cost |
|---|---|
| Police | 3000.00 |
| Phone switchboard operator | 75.00 |
| Event coordinator | 175.00 |
| Ticket takers,crowd management and ushers | 3150.00 |
| Setup and take down | 2500.00 |
| Unarmed guards, (contemporary services) | 5500.00 |
| Fire safety inspector | 400.00 |
| Use of electric sign near the freeway | 150.00 |
| Stage hands before, during, and after the performance | 18000.00 |
| **Subtotal-fixed costs** | |
| Spotlights ($110 for each spotlight) (10 used) | 1100.00 |
| Operator for spotlights: one operator per spotlight (operators cost $125.00 each) | |
| Insurance (Public liability) $.25 for each person attending paid or unpaid | |
| Arena rental $7000 or 10% of the money taken in from ticket sales, whichever amount is greater. | |
| **Total cost** | |
| Money taken in on ticket sales: Number of tickets x cost per ticket | |
| **Profit or Loss** = Money taken in minus all costs | |

How do you know your Profit or Loss sum is reasonable? ____________

______________________________________________

______________________________________________

How can you increase your profit? ____________________

______________________________________________

______________________________________________

Name____________________

### ROCK ' N MATH
### COST SHEET FOR ARENA RENTAL
Student Activity Sheet 3

**PROJECT 2:**

We charged $15 for a ticket. It rained the day of the concert and we sold only 7000 tickets. How much money did we take in on ticket sales? ________________

Using the same fixed costs in Project 1, using the 7000 tickets sold, the $15 price per ticket, and the ten spotlights needed, compute the Project 2 column of the Arena Cost Sheet.

| **Service** | **Project 2 cost** |
|---|---|
| Police | 3000.00 |
| Phone switchboard operator | 75.00 |
| Event coordinator | 175.00 |
| Ticket takers,crowd management and ushers | 3150.00 |
| Setup and take down | 2500.00 |
| Unarmed guards, (contemporary services) | 5500.00 |
| Fire safety inspector | 400.00 |
| Use of electric sign near the freeway | 150.00 |
| Stage hands before, during, and after the performance | 18,000.00 |
| **Subtotal fixed costs** | |
| Spotlights ($110 for each spotlight) | 1100.00 |
| Operator for spotlights: one operator per spotlight (operators cost $125.00 each) | |
| Insurance (Public liability) $.25 for each person attending paid or unpaid | |
| Arena rental — $7000 or 10% of the money taken in from ticket sales, whichever amount is greater | |
| **Total cost** | |
| Money taken in on ticket sales: Number of tickets x cost per ticket | |
| **Profit or Loss** = Money taken in minus all costs | |

Analyze the differences between Project 1 and 2. ________________

________________________________________

________________________________________

Name________________

# ROCK ' N MATH
## COST SHEET FOR ARENA RENTAL
Student Activity Sheet 4

**PROJECT 3:**

Estimate the amount you would have chosen for the price of tickets. _______

If 10,000 people bought tickets and attended, what amount of money could you take in from ticket sales?

Using the same fixed costs as in Project 1 and 2, compute the Project 3 column of the Arena Cost Sheet to find the profit or loss for the group.

| Service | | Project 3 cost |
|---|---|---|
| Police | | 3000.00 |
| Phone switchboard operator | | 75.00 |
| Event coordinator | | 175.00 |
| Ticket takers,crowd management and ushers | | 3150.00 |
| Setup and take down | | 2500.00 |
| Unarmed guards, (contemporary services) | | 5500.00 |
| Fire safety inspector | | 400.00 |
| Use of electric sign near the freeway | | 150.00 |
| Stage hands before, during, and after the performance | | 18,000.00 |
| | **Subtotal fixed costs** | |
| Spotlights ($110 for each spotlight) (10 used) | | 1100.00 |
| Cperator for spotlights: one operator per spotlight (operators cost $125.00 each) | | |
| Insurance (Public liability) $.25 for each person attending paid or unpaid | | |
| Arena rental | $7000 or 10% of the money taken in from ticket sales, whichever amount is greater | |
| | **Total cost** | |
| Money taken in on ticket sales:<br>Number of tickets x cost per ticket | | |
| **Profit or Loss** = Money taken in minus all cost | | |

What would be the lowest price you could charge for tickets and still make a profit?______________________________________________

______________________________________________

______________________________________________

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Name ______________

## ROCK 'N MATH

Student Activity Sheet 5

| **PROJECT 4:** Enter totals in appropriate columns | **EXPENSE** | **BALANCE** |
|---|---|---|
| Assume your group brought in $105,000 from selling tickets. | | $105,000 |
| It cost your group $47,550 to use the arena. Subtract your expense from $105,000 to get your profit. Enter the expense and the new balance. | | |
| You have an agent who arranged for your group to play in the arena. He charges 10% of the amount of profit that you make. (0.10 x profit). How much money did you give your agent? (Enter in the expense column)<br>How much money did your group have left after paying the agent fee? Subtract expense from balance to find your new balance. | | |
| It cost your group $3000 to travel to Los Angeles and back with all the equipment. How much was left after paying the travel expenses? | | |
| A full page ad in the Los Angeles Times cost your group $32,497.68 each day. How much money was left after paying off the ad which you ran for only one day? | | |
| You split the money that was left among the 5 members of your band. How much money do you get? | | |
| Your tax man said you will have to pay about 45% of the money you received in taxes. How much will you have to pay in taxes? (0.45 x amount of money you received) Enter in expense column<br>How much money will you have left after you have paid your taxes? Enter in balance column. | | |

Name____________________

## ROCK'N MATH
Student Activity Sheet 6

The people who heard the concert loved it so much that your group received a recording contract with a famous record company. The record company agreed to give your group $0.60 for every record they sell.

| | Workspace | Answers |
|---|---|---|
| How many records would need to be sold for your group to make $100,000? | | |
| Each of your five - member group wishes to make $50,000. How many records need to be sold? | | |
| If your group's record sold 2,000,000 copies:<br>How much money would your group receive? | | |
| How much money would you receive? | | |
| Do you still want to form a rock band? | | |

State the reasons for your decision. ________________________________________

________________________________________________________________

________________________________________________________________

________________________________________________________________

________________________________________________________________

________________________________________________________________

# DASHING DOWN THE ROAD

| | |
|---|---|
| GRADE: | 7 - 8 |
| STRAND: | Number |
| SKILL: | Multiplication and division of whole numbers and decimals. |

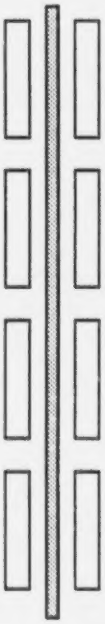
## MANAGEMENT

| | |
|---|---|
| CLASS ORGANIZATION: | Small groups |
| TIME FRAME: | Two math periods |
| MATERIALS: | Calculator |
| VOCABULARY: | None |
| PREREQUISITE SKILL: | Multiply and divide whole numbers, interpret remainders, round to the nearest tenth, hundredth, whole number |

## LESSON

- **DIRECTED INSTRUCTION:**

  Pretend that you are the chief engineer for the highway department. You need to arrange to paint the dividing stripe on Route 5. There will be three stripes as shown. What information do you need in order to accomplish this task?



  **Teacher-led discussion**

  Students discuss the problem presented and should come up with a list of things they need to know such as:

  1. How long is Route 5?
  2. How many stripes do you need to paint?
  3. How much paint will you need?
  4. How many people will you need to hire?
  5. How long will it take to complete the job?
  6. How much money will it cost to complete the job?

  Hand out Student Activity Sheet 1 and work together.

- **GUIDED PRACTICE:**

  Hand out Student Activity Sheet 2. Observe the groups and assist as needed.

- **INDEPENDENT PRACTICE:**

  Hand out Student Activity Sheet 3 (parts 1 and 2) and have the groups work them independently.

- **EVALUATION:**

  Teacher observation and Student Activity Sheets.

- **HOME ACTIVITY:**

  Hand out Student Activity Sheet 4.

## DASHING DOWN THE ROAD

### Student Activity Sheet 1
### Teacher Answer Sheet

Determine the amount of paint needed for the solid center stripe.
Facts:

- Paint is sold by the gallon.
- One gallon of paint will cover a surface area of 107 square feet.
- The stripe is 4" wide.
- One mile = 5280 feet.

Picture the shape of the stripe. Notice that it is a long rectangle. Use the formula, area = length x width, to find the total area to be covered. Use the facts to determine how many gallons of paint are needed to paint the center stripe down
Route 5.

Route 5 stretches from Mexico to Canada through California, Oregon, and Washington states. The route is approximately 1400 miles long.

**HOW MANY GALLONS ARE NEEDED TO PAINT A SOLID LINE STRIPE?**

| Solid Center Stripe | Width in Feet Round to nearest hundredth | Area of Stripe in Square Feet Per Mile | Area of Route in Square Feet | Number of Gallons Needed |
|---|---|---|---|---|
| FACT<br>4 inches wide 1400 miles long | 4 inches must be converted to feet | Area= Length x Width | 1400 mile route | 1 gallon covers 107 square feet |
| WORKSPACE | 4/12 of a foot<br>4/12 = .33 feet | L x W<br>5280 feet per mile x .33 foot width = 1742.4 square feet | 1742.4 square feet painted per mile x 1400 mile route = 2,439,360 square feet | 2,439,360 square foot area to be painted ÷ 107 square feet covered per gallon = 22,797.757 gallons |
| ANSWERS | .33 foot | 1742.4 square feet = Area of painted stripe per mile | 2,439,360 square feet of area of painted stripe for 1400 mile route | = 22,798 gallons of paint needed |

## DASHING DOWN THE ROAD
### Student Activity Sheet 2
### Teacher Answer Sheet

Determine the amount of paint needed for broken-line stripes.
Facts:

- Broken-line stripes: 12 feet long and 4 inches wide, and separated by 36 feet of unmarked road.
- Since 12 out of every 48 feet are painted, only 1/4 the amount of paint that was used on the solid stripe is needed.
- Remember to figure enough paint for **two** broken-line stripes.

**HOW MANY GALLONS ARE NEEDED FOR A BROKEN-LINE STRIPE?**

| Broken-Line Stripe | Width in Feet Round to nearest Hundredth | Length in Feet of Broken-Line Stripe Per Mile* | Area of Broken-Line Stripe Per Mile L X W | Area of Route in Square Feet | Number of Gallons Needed for One Stripe | Number of Gallons for Two Stripes |
|---|---|---|---|---|---|---|
| WORKSPACE | 4/12 ft.<br>4/12 = .33 | Every 12' out of 48' are painted 12/48 = .25 .25 area to be painted x 5280 feet per mile = 1320 feet painted per mile | 1320 foot of stripe per mile x .33 width of stripe = 435.6 square feet (Area of broken-line stripe per mile) | 435.6 square foot area of broken-line stripe x 1400 mile route = 609,840 square ft. | 609,840 square foot area to be painted + 107 square feet covered per gallon 5699.44 gallons needed | 5699.44 gallons needed x 2 broken-line stripes = 5699.44 x 2 = 11,398.88 gallons needed |
| ANSWERS | .33 foot | 1320 ft. painted per mile | | 609,840 square ft. | 5699.44 gallons needed | 11,399 gallons needed to paint 2 stripes (interpret remainder) |

*Reminder: Since only 12 out of 48 feet are painted 12/48 or 1/4 feet times the number of feet per mile (5,280) equal the painted length of the broken-line strip. 1/4 x 5,280 or 5,280 + 4.

Explore another way to compute this answer using the reminder above. How would you calculate it?
1. Take the number of gallons needed for a solid stripe, and multiply by 1/4 or divide by 4. This gives the amount for 1 broken line stripe. Multiply by 2 for 2 broken line stripes. 2. Since 1 broken-line stripe uses 1/4 the paint needed, then 2 broken-line stripes = 2 x 1/4 or 1/4 + 1/4. 2 broken-line stripes use 1/2 the paint of a solid line strip. 1/2 x 22,798 = .50 x 22,798 = 11,399. Or 1/4 x 22,798 = .25 x 22,798 = 5699.5 + another .25 x 22,798 = 5,6995 + 5,6995 = 11,399. Or 22,798 + 4 = 5,6995 then multiply by 2.

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## DASHING DOWN THE ROAD

Student Activity Sheet 3
Teacher Answer Sheet

Facts:

- Stripes are painted by a truck.
- The truck travels at a speed of 7 miles per hour while painting.

Step 1
How many **hours** will it take to paint the three stripes? (1 solid stripe and 2 broken-line stripes.)
Note: It takes the same time for the truck to travel while painting a solid of a broken-line stripe. Figure the time for one stripe and multiply by three.

**HOW LONG WILL IT TAKE?**

| Truck Speed | Length of Route | Number of Hours Per Stripe | Number of Hours for 3 Stripes |
|---|---|---|---|
| 7miles per hour | 1400 miles | 1400 miles ÷ 7mph = 200 hours | 200 hours x 3 stripes = 600 hours |

Step 2
How many days will it take to paint the stripes?
Enter the number of hours to paint the 3 stripes in the box below.

**HOW MANY DAYS WILL IT TAKE?**

| Number of Hours for 3 Stripes | Number of Days for a Truck Driving 8 Hrs. Per Day |
|---|---|
| 600 hours for 3 stripes | 600 hours ÷ 8 hours driving a day = 75 days |

Step 3
How many painting trucks will you need to use?
Fact: The job needs to be completed in one month (20 working days).
Fact: As one person drives the painting truck, another person puts out traffic cones so cars do not drive on wet paint. A second truck will follow later and pick up the cones. Twice as many trucks are needed to complete the job.

**HOW MANY TRUCKS DO YOU NEED?**

| Number of Days for 1 Painting Truck | Number of Painting Trucks Needed to Finish in 20 days | Number of Trucks Needed to Pick up Cones |
|---|---|---|
| 75 days | 75 ÷ 20 = 3.75 = 4 trucks needed to finish in 20 days (interpret remainders) | 4 x 2 = 8 8 trucks needed |

## DASHING DOWN THE ROAD

### Student Activity Sheet 3 PART 2

### Teacher Answer Sheet

Compute the total cost of the truck fuel and the paint for the job. Do not compute the cost of the people for the job since they are being supplied by another department.

Step 4
How many people are needed to complete the job?
Fact: Two people for each truck.

**PEOPLE NEEDED TO COMPLETE THE JOB?**

| Number of Trucks Needed to paint and pick up cones | Number of people needed |
|---|---|
| 8 | 16 |

Step 5
Facts: Diesel fuel costs $.99 per gallon.
Trucks get about 9 miles per gallon.

**COST OF GASOLINE**

| Total Number of Miles for each Stripe | Total Number of Miles for for All Stripes* | Number of Gallons Needed Round to nearest gallon | Cost of Gasoline |
|---|---|---|---|
| 1400 | 1400 miles x 6 stripes = 8400 miles | 8400 + 9 = 933.33= 934 gallons | 934 gallons x .99 per gallon = $924.66 |

*Multiply the number of miles by 6 because there are 3 stripes and 3 sets of cones to be picked up, which require 1 truck trip each.

Step 6
Fact: Paint costs $13 per gallon.
Refer to the number of gallons needed to paint the three stripes from Student Activity Sheet 1-Step 1 and find the total cost of paint.

**COST OF PAINT**

| Number of Gallons Needed for all 3 Stripes | Total Cost for 3 Stripes | Total Cost of Gasoline | Total Cost of Paint and Gasoline |
|---|---|---|---|
| 22,798 + 11,399 = 34,197 gallons | 34,197 x $13 = $444,561 | $924.66 | $924.66 + 444,561 = $445,485.66 |

Name ____________________

## DASHING DOWN THE ROAD

Student Activity Sheet 1

Determine the amount of paint needed for the solid center stripe.
Facts:

- Paint is sold by the gallon.
- One gallon of paint will cover a surface area of 107 square feet.
- The stripe is 4" wide.
- One mile = 5280 feet.

Picture the shape of the stripe. Notice that it is a long rectangle. Use the formula, area = length x width, to find the total area to be covered. Use the facts to determine how many gallons of paint are needed to paint the center stripe down Route 5.

Route 5 stretches from Mexico to Canada through California, Oregon, and Washington states. The route is approximately 1400 miles long.

### HOW MANY GALLONS ARE NEEDED TO PAINT A SOLID LINE STRIPE?

| **Solid Center Stripe** | **Width in Feet Round to nearest hundredth** | **Area of Stripe in Square Feet Per Mile** | **Area of Route in Square Feet** | **Number of Gallons Needed** |
|---|---|---|---|---|
| **FACT**<br>4 inches wide 1400 miles long | 4 inches must be converted to feet | Area= Length x Width | 1400 mile route | 1 gallon covers 107 square feet |
| **WORKSPACE** | | | | |
| **ANSWERS** | | | | |

Name____________________

## DASHING DOWN THE ROAD

Student Activity Sheet 2

Determine the amount of paint needed for broken-line stripes.
Facts:

- Broken-line stripes: 12 feet long and 4 inches wide, and separated by 36 feet of unmarked road.
- Since 12 out of every 48 feet are painted, only 1/4 the amount of paint that was used on the solid stripe is needed.
- Remember to figure enough paint for **two** broken-line stripes.

**HOW MANY GALLONS ARE NEEDED FOR A BROKEN-LINE STRIPE?**

| Broken-Line Stripe | Width in Feet Round to nearest Hundredth | Length in Feet of Broken-Line Stripe Per Mile* | Area of Broken-Line Stripe Per Mile L X W | Area of Route in Square Feet | Number of Gallons Needed for One Stripe | Number of Gallons for Two Stripes |
|---|---|---|---|---|---|---|
| WORKSPACE | | | | | | |
| ANSWERS | | | | | | |

*Reminder: Since only 12 out of 48 feet are painted, 12/48 or 1/4 feet times the number of feet per mile (5,280) equal the painted length of the broken-line strip. 1/4 x 5,280 or 5,280 ÷ 4

Explore another way to compute this answer using the reminder above. How would you calculate it?

______________________________________________

______________________________________________

______________________________________________

______________________________________________

______________________________________________

Name____________________

## DASHING DOWN THE ROAD

### Student Activity Sheet 3 page 1

Facts:

- Stripes are painted by a truck.
- The truck travels at a speed of 7 miles per hour while painting.

Step 1

How many **hours** will it take to paint the three stripes? (1 solid stripe and 2 broken-line stripes.)

Note: It takes the same time for the truck to travel while painting a solid of a broken-line stripe. Figure the time for one stripe and multiply by three.

**HOW LONG WILL IT TAKE?**

| Truck Speed | Length of Route | Number of Hours Per Stripe | Number of Hours for 3 Stripes |
|---|---|---|---|
| | | | |

Step 2

How many days will it take to paint the stripes?

Enter the number of hours to paint the 3 stripes in the box below.

**HOW MANY DAYS WILL IT TAKE?**

| Number of Hours for 3 Stripes | Number of Days for a Truck Driving 8 Hrs. Per Day |
|---|---|
| | |

Step 3

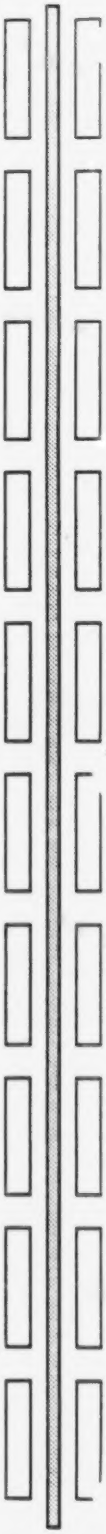
How many painting trucks will you need to use?

Fact: The job needs to be completed in one month (20 working days).

Fact: As one person drives the painting truck, another person puts out traffic cones so cars do not drive on wet paint. A second truck will follow later and pick up the cones. Twice as many trucks are needed to complete the job.

**HOW MANY TRUCKS DO YOU NEED?**

| Number of Days for 1 Painting Truck | Number of Painting Trucks Needed to Finish in 20 days | Number of Trucks Needed to Pick up Cones |
|---|---|---|
| | | |



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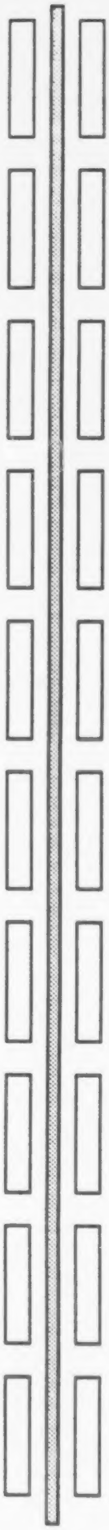
Name ____________________

## DASHING DOWN THE ROAD

Student Activity Sheet 3 page 2



Compute the total cost of the truck fuel and the paint for the job. Do not compute the cost of labor for the job since they are being supplied by another department.

Step 4
How many people are needed to complete the job?
Fact: Two people per truck.

### PEOPLE NEEDED TO COMPLETE THE JOB?

| Number of Trucks Needed to Paint and Pick up Cones | Number of People Needed |
|---|---|
| | |

Step 5
Facts: Diesel fuel costs $.99 per gallon.
Trucks get about 9 miles per gallon.

### COST OF FUEL

| Total Numbe of Miles for each Stripe | Total Number of Miles for for All Stripes* | Number of gallons Needed (Round to nearest Gallon) | Cost of fuel |
|---|---|---|---|
| | | | |

*Multiply the number of miles by 6 because there are 3 stripes and 3 sets of cones to be picked up.

Step 6
Fact: Paint costs $13 per gallon.
Refer to the number of gallons needed to paint the three stripes from Student Activity Sheet 1-Step 1 and find the total cost of paint.

### COST OF PAINT

| Number of Gallons Needed for all 3 Stripes | Total Cost for 3 Stripes | Total Cost of Gasoline | Total Cost of Paint and Gasoline |
|---|---|---|---|
| | | | |

Name ______________________

## DASHING DOWN THE ROAD

Student Activity Sheet 4

You have become so famous that people want to see the house where you grew up and where you went to school. In order to make it easy for the Famous Mathematicians' Tour Bus to point out your school and your house, they need to have a solid green line painted to mark the route between the two locations. Estimate the distance from your famous home site to your celebrated school site and use the mileage figure to calculate the number of gallons needed and the cost of the paint. The Famous Mathematicians' Tour Company will pay you for the job.

**HOW MUCH DOES IT COST TO PAINT A SOLID STRIPE FROM HOME TO SCHOOL?**

| Estimated Distance from Home to School | Width In Feet | Stripe Area In Square Feet per Mile | Area of Route In Square Feet | Number of Gallons Needed (107 square ft) covered per Gallon | Cost ($13 per Gallon) |
|---|---|---|---|---|---|
| Length:<br>____ miles<br>Width:<br>4 inches | | | | | |

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# CHAPTER 4 ASSESSMENT:

## NUMBER AND ALGEBRA

1. Estimate, then solve.
   a. How many years are there in 1,000,000 days?
   b. How many years are there in 1,000,000 hours?
   c. How many years are there in 1,000,000 minutes?

   Student responses:
   a. 1,000,000 days = 2739.726 = 2739 complete years + 265 days
   b. 1,000,000 hours ÷ 24 ÷ 365 = 114.15524 = 114 years
   c. 1,000,000 minutes ÷ 60 ÷ 24 ÷ 365 = 1.9 years (about 2 years)

2. How long will it take you to read a million words? Make an estimate. Determine a strategy and carry out your plan to solve the problem. Interpret your results and write alternate ways in which this answer can be found.

   Student response will vary. All responses should mention obtaining data for a smaller number of words.

3. The total price of a package of hamburger is $10.27. The number of pounds and the price per pound on the label is smudged. Complete three possible labels that include weight and price per pound.

   Student response will vary. In all cases the number of pounds multiplied by the price per pound must round to $10.27. The reasonableness of answers needs to also be discussed.

4. Sparkling apple juice comes in three different sizes: 12, 32, and 48 fluid ounces. Today the market showed them priced as follows: 12 oz for $.55, 32 oz for $1.29, and 48 oz for $1.69. Which is the best size to buy? Explain.

   $.55 ÷ 12 oz = .0458333
   $1.29 ÷ 32 oz = .0403125
   $1.69 ÷ 48 oz = .0352083

   The 48 oz package is the least expensive per ounce. The best to buy may also take into account the size of the package and how often you drink apple juice.

5. Which would you rather have, $5,000 or a magic dime that doubles every day for 27 days? Why is it the better choice? How much more money would you have?

Student response: Magic Dime

| Day | Amount of Money |
|---|---|
| 1 | .10 |
| 2 | .20 |
| 3 | .40 |
| 4 | .80 |
| 5 | 1.60 |
| 6 | 3.20 |
| 7 | 6.40 |
| 8 | 12.80 |
| 9 | 25.60 |
| 10 | 51.20 |
| 11 | 102.40 |
| 12 | 204.80 |
| 13 | 409.60 |
| 14 | 819.20 |

| Day | Amount of Money |
|---|---|
| 15 | 1638.40 |
| 16 | 3276.80 |
| 17 | 6553.60 |
| 18 | 13107.20 |
| 19 | 26214.40 |
| 20 | 52428.80 |
| 21 | 104857.60 |
| 22 | 209715.20 |
| 23 | 419430.40 |
| 24 | 838860.80 |
| 25 | 1677721.60 |
| 26 | 3355443.20 |
| 27 | 6710886.40 |
|  |  |

From the 17th day on, the amount of money from the magic dime is more than $5000. On the 27th day you have $6710886.40 that is $6705886.40 more than $5,000.

6. Complete the chart and use it to find the answer to 343 x 2401 =.

$7^1 =$

$7^2 =$

$7^3 =$

$7^4 =$

$7^5 =$

$7^6 =$

$7^7 =$

$7^8 =$

$7^9 =$

Student responses:

$7^1 = 7$

$7^2 = 49$

$7^3 = 343$

$7^4 = 2{,}401$

$7^5 = 16{,}807$

$7^6 = 117{,}649$

$7^7 = 823{,}543$

$7^8 = 5{,}764{,}801$

$7^9 = 40{,}353{,}607$

$343 \times 2401 = 7^3 \times 7^4 = 7^{3+4} = 7^7 = 823{,}543$

7. A person playing Table Tennis will use about 2.7 calories per hour for each pound of weight. Running uses 4.0 calories per hour for each pound of weight. If Steve weighs 90 pounds, describe, in words, how he can use exactly 846 calories playing table tennis and/or running.

Student responses may vary. The easiest solution is to run for one hour and play table tennis for two hours. Table tennis uses 2.7 x 90 = 243 calories per hour for a 90 lb. person. Running uses 4 x 90 = 360 calories per hour for a 90 lb. person. Therefore, two hours of table tennis and one hour of running uses 2 x 243 + 360 = 846 calories.

8. Elisabeth is 4 feet away from a wall. Each time she takes a step she moves $\frac{1}{2}$ the remaining distance to the wall. Complete the chart below.

| STEP NUMBER | DISTANCE TO WALL |
|---|---|
| | |
| | |
| | |
| | |
| | |

Explain what happens as the process continues.

Student responses:

| STEP NUMBER | DISTANCE TO WALL |
|---|---|
| 0 | 4 |
| 1 | 2 |
| 2 | 1 |
| 3 | .5 |
| 4 | .25 |
| 5 | .125 |

As the process continues, Elisabeth gets closer to but never touches the wall.

9. Explain why an order of doing operations is necessary. Demonstrate, using at least two examples.

Student responses will vary. All responses should refer to the fact that many expressions would have several different answers if there was not an order for doing operations.

10. Place parentheses to make this sentence true: 23 x 39 + 50 x 73 = 149, 431

    Student response.
    23 x (39 + 50) x 73 = 149,431

11. Write the order of operations.

    Student response.
    Parentheses (left to right)
    Exponents (left to right)
    Multiplication and division (left to right)
    Addition and subtraction (left to right)

12. By placing parentheses, see how many different solutions you can obtain for 15 + 12 x 16 + 24 - 8. Show your work.

    Student responses could include:
    (15 + 12) x 16 + 24 - 8 = 448
    15 + (12 x 16) + 24 - 8 = 223
    15 + 12 x (16 + 24) - 8 = 487
    15 + 12 x (16 + 24 - 8) = 399
    (15 + 12) x (16 + 24) - 8 = 1072
    (15 + 12) x (16 + 24 - 8) = 864

13. Hamburger patties have 75 calories an ounce. A single french fry has 15 calories. Hamburger buns have 165 calories.

    Bob eats a hamburger in a bun with 2 (6 oz) patties inside. How many fries can he eat to keep his calorie intake to about 1365 calories?

    Student response.
    2 (6 oz) patties have 2 x 6 x 75 = 900 calories. 1 bun has 165 calories. The hamburger and bun together have 1065 calories. 1365 - 1065 = 300 calories remain for french fries. 300 + 15 = 20 fries may by eaten.

14. Find the missing number: 3 times a number + 7 = 841.

    Student response.
    3N + 7 = 841
    3N = 841 - 7
    3N = 834
    N = 834 + 3
    N = 278

15. Find the 2 consecutive numbers whose sum is 891.

    Student response.
    One possible solution is to say 891 + 2 = 445.5 so the numbers are 445 and 446.

16. Write how to find the least common multiple of 11 and 13 using two calculators.

Student responses.
Use one calculator to show multiples of 11 and the other to show multiples of 13. The calculator with the smallest number in its display increases to the next multiple of its number until the numbers in both calculators match. The L.C.M. is 143.

17. Use your calculator to find out if the following numbers are primes.

1991, 1993

Student responses.

For 1991 Check to see if any of the primes up to $\sqrt{1991}$ are divisors of 1991. Check 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 to see if they are divisors. 1991 ÷ 11 = 181, so 1991 is not a prime.

For 1993 Check to see if any of the primes up to $\sqrt{1993}$ are divisors of 1993. Check 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43. 1993 is a prime.

18. The local bank pays 8% interest each year on money in a time account. If you deposited $200 and leave it in your account for 12 years, how much money will your account be worth. (Interest is added to your account at the end of each year.)

Student response.
[(8 + 100) ÷ 100] x 200 = = = = = = = = = = = = $503.63

19. A television set is on sale for 23% off at Store A. The regular price is $410. The same television set is on sale for 35% off at store B. There its regular price is $430. Explain which is a better buy.

Student response.
Store A $94.30 off. 410 - 94.30 = $315.70 sale price.
Store B $150.50 off. 430 - 150.50 = $279.50 sale price.
Store B is a better buy.